BETTING ON LEVERAGE[1]

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October 22, 2019

ABSTRACT

Combining capital structure theory with asset pricing theory of leverage-constrained investors, we explain why CAPM beta is negatively related to abnormal stock returns. Adjusting for mismeasurement in portfolio rankings by accounting for cross-sectional differences in firm leverage we show: a) leverage-constrained investors tilt towards high-beta stocks with high leverage, b) significant risk-adjusted returns to a portfolio of high-leverage firms which is long low-beta assets and short high-beta assets, and c) no evidence of a beta anomaly in tests of a model of leverage-constrained investors. Our results demonstrate that leverage constraints have practical implications for investors and testing asset pricing models.

JEL Classification: G01, G11, G12, G14, G15

Keywords: beta, betting against beta, CAPM, asset beta, unlevered beta, stock returns.

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[1] We thank Turan Bali, Ron Balvers, Jarred Williams, faculty from the University of South Florida, McMasters University, and audience members at Penn State University for comments and suggestions. We also thank Zihan Ye for research assistance.

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I. Introduction

In 1972, Black, Jensen, and Scholes uncovered one of the first asset pricing anomalies by sorting stocks into portfolios based on beta from the capital asset pricing model (CAPM). Referred to as the "beta anomaly" or the "low-risk effect", high-beta assets exhibit low abnormal returns while low-beta assets generate high abnormal returns. The list of subsequent papers attempting an explanation of the anomaly is long.[5] A recent and contentious explanation for the beta anomaly is Frazzini and Pedersen's (2014, FP hereafter) general equilibrium model of asset prices set in an economy with leveraged-constrained investors. In the FP model, constrained investors seeking higher expected returns bid up prices of high-beta stocks, thus placing downward pressure on their expected returns and alphas. Of the considerable number of papers attempting to either refute or affirm FP's empirical results and concomitant betting-against-beta factor (BAB), none provide a direct test of their model.

We supply this test. Implementing a procedure suggested by Gibbons (1982) within a generalized method of moments (GMM) framework, we reject the FP model. The estimation technique allows for the non-linear relation between abnormal returns and market risk in the FP model while reducing estimation error in the parameters. For 20 portfolios sorted on CAPM beta, a likelihood ratio test and the Gibbons, Ross, Shanken (1989) test both reject the null hypothesis that alphas are jointly zero for both the CAPM and the FP model. Alphas from both the FP model and CAPM are negatively correlated with market risk. The Gauss-Newton method we employ provides consistent estimates of the sorted-portfolio betas but cannot account for measurement error in the estimated betas used to form the portfolios. If the measurement error in the asset

[5] See, e.g., Blume and Friend (1973), Fama and MacBeth (1973), Lakonishok and Shapiro (1986) and Fama and French (1992, 1993, 2016), Roll and Ross (1994), Jagannathan and Wang (1996), Ang, Hodrick, Xing, and Zhang (2006, 2009), Baker, Bradley, and Wurgler (2011), and Jylha, Suominen and Tomunen (2018).

rankings is related to the behavior of leverage-constrained investors, then our initial test of FP's model is tenuous.

For a partial solution to measurement error in the beta rankings, we appeal to Modigliani and Miller's (1958, M&M hereafter) Proposition I and hypothesize that investors prohibited from investing with 'homemade leverage' bid up firms for which high CAPM betas reflect high levels of financial leverage rather than firms investing in systematically risky assets.[6] That is, if investor constraints are driving the inverse relation between alphas and betas as suggested by FP, the intuition of M&M suggests this behavior should manifest itself as significant mispricing of equities for which high CAPM beta reflects financial leverage. However, double sorting assets into portfolios based on beta and leverage reveals that the only high-beta firms to produce statistically significant negative abnormal returns are those with the *least* amount of leverage. This counterintuitive result leads us to believe mismeasurement of ranking betas may induce the rejection of the FP model.

Because CAPM betas are estimated to rank assets and again estimated to form abnormal returns, we isolate the impact of mismeasurement in the ranking betas using a boot-strap experiment which avoids the second estimation of beta needed to create abnormal returns. Here we evaluate the impact on the Sharpe ratio of a leverage-constrained investor who tilts away from being fully invested in the market portfolio by rebalancing to add a small investment in either a high- or low-beta asset. Contrary to the prediction of FP that leverage constrained investors should chase high-beta assets, our experiment reveals that an investor can achieve a higher Sharpe ratio by tilting her portfolio toward low-beta assets relative to a portfolio tilted toward high-beta assets.

[6] M&M Proposition I states that the value of the firm is independent of the percentage of debt in its capital structure. Its proof requires investors without leverage constraints. Intuitively, if financial leverage increases value, then investors could simply invest on margin in the less valuable unlevered firm. Prohibiting investors from investing borrowed funds should thus result in a preference for levered firms among those who prefer levered returns.

Controlling for firm leverage does not change these results. We take these results as further evidence of mismeasurement in beta rankings.

Mismeasurement of ranking betas can occur for purely statistical reasons, e.g. Welch (2019) or from inherent shortcomings of the CAPM. Early work from Hamada (1972), Merton (1974) and Galai and Masulis (1976) demonstrate that equity betas reflect financial and operational leverage, as well as the systematic risk of corporate investment. Beta estimates contaminated by firm leverage bias estimates of true stock risk [Drobetz, Meier and Seidel, (2014)] and cause beta instability [DeJong and Collins, (1985)]. Moreover, Ferguson and Shockley (2003) show that betas estimated using an equity-only market proxy are understated and the bias is more pronounced among high-leverage firms.

In our setting, we are concerned with accounting for firm-level financial leverage and its varying impact on beta rankings. When ranking firms, we replace CAPM betas with estimates for asset betas corrected for cross-sectional variation in financial leverage. We are guided by the theory of Gomes and Schmid (2010, hereafter GS) who show that traditional CAPM beta is a function of age, growth options, and default probability as well as financial leverage.[7] Specifically, we estimate the systematic risk of firm assets with the fitted value of a cross-sectional regression of traditional CAPM beta on leverage, growth options relative to firm age, and default probability relative to the size of the firm. We then rank firms based on these fitted betas and form portfolios. Accounting for leverage in ranking betas has a significant impact on a firm's ranking and on the results of our experiments, *e.g.* on average 21% of firms ranked in the lowest beta quintile move

[7] To validate the GS model variables for our setting, we run cross-sectional and multivariate regressions and find that the time-series of CAPM alphas of beta-sorted portfolios are significantly related to these firm characteristics after controlling for the Fama-French size, book-to-market and momentum factors, and a measure of the aggregate leverage constraint in Boguth and Simutin (2018). We also find that among these firm characteristics, leverage is the dominant explanatory factor.

to the highest beta quintile after making the GS leverage adjustment while 73% of firms ranked by beta into the highest quintile move to a lower quintile after accounting for GS leverage adjustment.

Sorting stocks into portfolios using leverage-adjusted asset-betas reconciles our intuition with the data. For 20 portfolios sorted on our fitted asset betas, our tests continue to reject the null hypothesis that alphas are jointly zero for the CAPM but cannot reject the null that alphas are jointly zero for the FP model. Sorting on our fitted betas, we find that alphas from both the FP model and CAPM are now *positively* correlated with market risk, although the FP alphas are much closer to zero. Consistent with predictions from M&M and the FP model, double sorting assets into portfolios based on adjusted beta and leverage reveals that abnormal returns increase with both beta ranking and firm leverage. Indeed, high adjusted-beta firms in the high leverage quintile earn the largest abnormal return (0.8% per month). Our boot-strap experiment reveals that constrained investors on average achieve a higher Sharpe ratio by tilting towards high adjusted-beta stocks with high leverage rather than high adjusted-beta stocks with low-leverage. Finally, we highlight the importance of considering firm leverage by showing that a simple portfolio that is long high-adjusted-beta stocks with high-leverage and short high-adjusted-beta stocks with low-leverage generates an alpha that is 30% larger than the BAB factor alpha.

Overall, we document three novel contributions to the literature. First, we provide an empirical test of the FP theory of leverage-constrained investors. Although a host of recent papers (reviewed in Section I below) critically review the work of Frazzini and Pedersen (2014), we believe that we are first to formally test their model. Ignoring cross-sectional differences in leverage, we reject the FP model. Second, we find no remaining evidence of the negative relation between abnormal returns and market risk after adjusting ranking betas for firm-level financial leverage prior to sorting. Adjusting betas for cross-sectional differences in firm leverage, our results support the prediction that investor leverage constraints are important for understanding the cross-section of stock returns. Third, we show that once ranking betas are corrected for leverage, a simple portfolio that is long high-beta stocks with high-leverage and short high-beta stocks with low-leverage generates a larger alpha than the BAB factor. Our results suggest betting on leverage when betting against beta.

Our results also provide evidence supporting the theory of Gomes and Schmid. Leverage constraints of investors matter for asset prices through corporate capital structure (corporate finance policies), not through systematic asset risk (corporate investment policies). Utilizing financial leverage (rather than investing in riskier assets) can thus make firms more attractive during periods when the average level of investors' leverage constraint is binding. Our results support recent asset pricing and corporate finance theories and suggest that leverage-constrained investors should look specifically to high-beta firms with high-leverage to increase their utility. For instance, our results provide empirical support for the model of Baker, Hoeyer, and Wurgler (2017). Baker *et al.* predict that leverage is negatively related to asset risk, find supporting empirical evidence, and prescribe optimal capital structure based on asset risk.[8]

Section II motivates our study with prior literature. Section III describes the data. Section IV describes the empirical methods for our experiments. Section V presents our results and we conclude in section VI.

II. Literature Review and Hypothesis Development

In addition to the leverage-constrained investor solution to the beta anomaly offered by FP, recent work by Cederburg and O'Doherty (2016), Bali, Brown, Murray and Tang (2017), and Liu, Stambuagh, and Yuan (2018) offer alternative explanations for this negative risk-return

[8] Baker *et al.* neither test the FP model nor attempt to resolve the beta anomaly, but study how capital structure should be set in the presence of the anomaly. In a related paper, Baker and Wurgler (2015) discuss the unintended consequences of stricter bank capital requirements associated with the inverse relationship between bank risk and bank cost of capital.

phenomenon. Building on Lewellen and Nagel (2006), Cederburg and O'Doherty (2016) hypothesize that the anomaly is due to a bias in unconditional alpha. They demonstrate that in a version of the CAPM with time-varying parameters, conditioning beta on the cross-sectional dispersion of leverage, along with cross-sectional dispersions of other firm characteristics such as investment and idiosyncratic volatility, helps explain the alphas of beta-sorted portfolios.

Bali *et al*. (2017) and Liu *et al.* (2018) likewise argue that the beta anomaly is driven by idiosyncratic risk rather than systematic risk. Bali *et al*. (2017) suggest a behavioral explanation showing that the negative relation between alpha and market risk disappears after controlling for characteristics of lottery stocks [Brunnermeier, Gollier, and Parker (2007), Barberis and Huang (2008)]. They argue that the demand by lottery investors for stocks with high probabilities of large short-term price increases are partially generated by a stock's sensitivity to the overall market beta.[9] Liu *et al.* (2018) document a positive correlation between CAPM beta and idiosyncratic volatility (IVOL)[10] and examine IVOL's role in FP's BAB strategy. These authors find that the performance of the BAB strategy is largely unrelated to the beta anomaly.[11] Asness, Frazzini, Gormsen, and Pedersen (2018) respond with additional tests designed to differentiate between these idiosyncratic risk explanations and the systemic leverage constraints of the FP model and find stronger support for the FP theory.

Other papers critical of the FP model or the BAB factor include Gilbert, Hrdlicka, Kalodmios, and Siegel (2014) who argue that FP's results follow from their use of daily data,

[9] Bali *et al*. (2017) identify lottery demand as the average of the five highest daily returns of the given stock in a given month. Excess returns to a portfolio long high-beta stocks and short low-beta stocks (High- Low beta portfolio) disappear when the portfolio is constrained to be neutral to lottery demand. Antoniou, Doukas, and Subrahmanyam (2016) offer a similar behavioral explanation based on time-variation in unsophisticated "noise" trading among high beta stocks.

[10] Ang *et al.* (2006, 2009) previously document stocks with low idiosyncratic volatility have high risk-adjusted returns.

[11] The BAB strategy takes a levered net-long position to achieve a zero beta. As a result, Liu *et al.* (2018) find the BAB strategy produces positive alpha where there is no beta anomaly but zero alpha where there is. In a related paper, Irvine, Kim and Ren (2018) find that the BAB factor is insufficient to control for the beta anomaly in mutual fund performance.

Welch (2019) who finds FP's beta estimation is highly sensitive to outliers, and Novy-Marx and Velikov (2018) who argue that the performance of the BAB factor is driven by the equal-weighting of stock returns and that BAB earns positive returns by tilting towards profitability and investment, exposures for which it is fairly compensated. Geppert and Zhao (2018) make a similar argument about the types of stocks that drive the beta anomaly while Ayash, Bednarek and Patel (2017) and Campbell and Kassa (2018) attribute the performance of the BAB factor to market segmentation. Collectively, these papers can be taken as evidence against the FP solution to the beta anomaly, but none represent formal tests of the FP model.[12]

Related papers that complement FP (2014) include Adrian, Etula, and Muir (2014) who provide a leverage factor that correlates with the BAB portfolio. Their evidence suggests that leverage represents funding constraints and that the BAB factor comoves with leverage as predicted. Boguth and Simutin (2018) provide a measure for the tightness of the leverage constraints in the FP model, Christoffersen and Simuntin (2017) provide evidence that pension fund managers tilt their portfolios toward high-beta stocks, and De (2017) modifies the BAB trading strategy to capitalize on behavioral bias among retail investors. Although these papers can collectively be taken as evidence consistent with the FP model, none provide a formal test.

Our contribution to this expanding strand of literature is a formal test of the FP model, both with and without adjustments suggested by corporate finance theory to account for measurement error in ranking betas. To the extent that leverage-constrained investors explain the high price (low return) of high equity beta stocks, as suggested by FP, we show that once we account for the measurement error in portfolio rankings it is high-beta firms with high financial leverage that are overpriced. Our results do not refute Welch's (2019) argument that outliers in the daily data bias

[12] Also related is work from Malamud and Vilkov (2018) who distinguish between current stock exposure to the market (myopic beta) and the future return on the efficient portfolio (non-myopic beta) and find that betting against non-myopic beta generates superior performance relative to FP's original betting against CAPM beta strategy.

FP's estimates of beta. However, because the beta anomaly appears in papers using monthly data (such that accounting for outliers will not likely resolve the anomaly), we consider measurement error in ranking betas for additional insight. Statistically, we argue that it is not the unconditional alphas that are biased, as in Cederburg and O'Doherty (2016), but measurement error in ranking betas due to ignoring theoretically and empirically known relations between CAPM beta and cross-sectional differences in leverage.

Our empirical test of the FP model utilizes predictions from the continuous-time model provided by Gomes and Schmid (2010) of the relation between expected returns on equity and corporate capital structure where both corporate investment and financing decisions are endogenous. Their model provides a link between equity beta and corporate investment and financial policies. The results of their paper indicate that it is crucial to consider growth options when examining the cross-sectional relation between leverage and equity returns. We extend that warning to the relation between firm-level financial leverage and abnormal equity returns. We therefore employ the variables from the GS model to adjust equity betas in a formal test of the FP model of constrained investors.

III. Data

For direct comparability our data construction follows Frazzini and Pedersen (2014).[13] We utilize a sample of U.S. equities and employ monthly data over the same sample period: January 1963 to December 2015. We further replicate the non-standard method of estimating a firm's beta employed by FP. We use daily rolling regressions of excess returns on market excess returns. Ranking beta estimates are calculated with the following formula: $\hat{\hat{\beta}}_i^{ts} = \hat{\rho}\left(\hat{\sigma}_i / \hat{\sigma}_m\right)$ where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are the volatility estimates for the stock and the market and the $\hat{\rho}$ is the correlation between

[13] Sources and formation methodologies for all our data are fully described in the appendix.

the firm and market returns. Volatilities are estimated by using one year rolling standard deviations of daily log stock returns and correlations are estimated by using five year rolling daily overlapping three-day log returns, $r_{i,t}^{3d} = \sum_{k=0}^{2} \ln(1 + r_{t+k}^{i})$. At least 120 trading days of non-missing data and at least three years (750 trading days) of non-missing data is needed to estimate volatilities and correlations, respectively. Finally, betas are adjusted in order to reduce the impact of outliers in the following way: $\beta_{i,t}^{Levered} = w_i \hat{\hat{\beta}}_i^{ts} + (1 - w_i \hat{\beta}^{xs})$ where $\hat{\beta}^{xs}$ is the cross-sectional mean. We set w=0.6 and $\hat{\beta}^{xs}$=1, again following FP.

To verify that our sample of firms and beta estimation methodology matches that of FP, we compare the summary statistics and alphas from the CAPM, 3- and 4-factor models for our measure of the BAB factor with those obtained from the BAB data employed by FP and available from AQR Capital Management[14]. In Panel A of Table 1 the distributions of the AQR BAB factor and our BAB factor are almost identical. Our BAB factor has a modestly higher mean, standard deviation, and Sharpe ratio, with slightly fatter tails. Even so, the two BAB factors have a time-series correlation of 96%. In Panel B, we show our BAB factor produces slightly higher measures of alpha across the three models, *i.e.* our BAB alphas are a basis point higher than those produced by FP's BAB factor available from AQR.

[Table 1 Here]

Table 2 contains a replication of FP's Table 3 for our sample. Qualitatively, we find similar results to those in FP. Our portfolios are rebalanced every month and sorted by the previous month's beta. As in FP, we see in row 1 that beta sorted portfolio returns increase then decrease with market risk. Also consistent with previous literature we see in rows 3, 6, and 9, CAPM, Fama-

[14] https://www.aqr.com/Insights/Datasets/Betting-Against-Beta-Equity-Factors-Monthly

French 3-factor and Carhart 4-factor alphas shrink moving from portfolios of low beta stocks to high beta stocks.

[Table 2 Here]

For our measure of a firm's leverage we calculate the market value debt-to-equity ratio adjusted for taxes using marginal tax rates from Graham (1996a, 1996b) and fixed-cost (non-cancelable, long-term) operating leases as suggested by Cornaggia, Franzen, and Simin (2013).[15] To adjust ranking betas for leverage we use a measure of a firm's growth options relative to the age of the firm, as well as the default probability of the firm. Growth options are measured using the market value of assets to book value of assets. For the age variable we merge data from Jovanovic and Rousseau (2001) and Jay Ritter's webpage. We use the founding date of the firm, then year of incorporation, then listing date if no founding date is available. We estimate the expected default probability (EDF) derived from the Merton (1974)/KMV model as in Duan and Simonato (2017) and detail the methodology in the appendix. Throughout the paper we measure the size of a firm by its market capitalization. Summary statistics for these data are in the appendix.

IV. Empirical Methods

In this section, we outline our estimation framework and testing procedures. We begin with a discussion of how we adjust ranking betas then we describe the Gauss-Newton estimation method of the FP general equilibrium model and define the joint tests of alpha used to evaluate model performance.

A. Adjustments to Ranking Betas

We sort stocks into portfolios using three measures of lagged beta. We use the betas calculated as in FP as the benchmark sorting betas. These provide nearly identical rankings as

[15] As of January 2019, U.S. firms must recognize fixed-costs leases on the balance sheet; see FASB Accounting Standards Codification topic 842 "Leases". Previously, this leverage was discoverable only from disclosure notes.

using OLS estimates of the CAPM and we refer to them as traditional betas, $\left(\beta^{Levered}\right)$. We also create ranking betas using the standard textbook method of un-levering beta first described in Hamada (1972). We create this adjusted beta $\left(\beta^{H}\right)$ every month as

$$\beta_{i,t}^{H} = \frac{\beta_{i,t}^{Levered}}{\left[1+\left(1-Tax\right)*\dfrac{Debt}{Equity}_{t-1}\right]}. \tag{1}$$

For our primary version of adjusted beta $\left(\beta^{GS}\right)$, we follow the decomposition of equity beta into four components presented in Gomes and Schmidt (2010). The first component captures the risk of assets in place. The second represents the risk of levering up equity cash flows. The third part of beta is related to default risk relative to the value of the firm. The final component reflects the risk stemming from growth options relative to the age of the firm. The intuition for the last term is that debt financing is often used for investment in growth options with different amounts of risk. Since the number and risk of growth options depends on the age of the firm, the betas of mature firms with fewer and lower-risk growth opportunities are impacted differently by leverage than the betas of younger firms with higher-risk growth options.

To calculate β^{GS} we first regress the cross-section of traditional $\beta^{Levered}$ at time *t* on the time-series average over the previous 60-months of the asset-specific leverage; the ratio of market value of assets to book value of assets, relative to the age of the asset; and the default probability relative to the log of the assets' market capitalization.[16] Using the coefficients from that cross-sectional regression we form adjusted betas as

$$\beta_{i,t}^{GS} = \hat{\gamma}_{1,tln}\left(1+\frac{D}{E}_{i,t-1}\right)+\hat{\gamma}_{2}\left(\frac{GO_{i,t-1}}{Age_{i,t-1}}\right)+\hat{\gamma}_{3}\left(\frac{100*DefProb_{i,t-1}}{ln\left(Size_{i,t-1}\right)}\right). \tag{2}$$

[16] In several of the cross-sectional regressions, the lack of variability in the measure of default produces a singular covariance matrix. To avoid this problem, we do not include an intercept in the cross-sectional regressions.

Here

$\frac{D}{E}_i$ = leverage defined by the average lagged market value debt-to-equity ratio adjusted for taxes and operating leases of the firms in portfolio *i*,

GO_i = growth options defined by the average market value of assets to book value of assets of the firms in portfolio *i*,

Age_i = the average age of the firms in portfolio *i*,

$DefProb_i$ = the average default probability from the Merton (1974)/KMV model of the firms in portfolio *i*, and

$Size_i$ = the average market capitalization of the firms in portfolio *i*.

B. Estimating the Non-linear FP Model

Equilibrium in the Frazzini and Pedersen (2014) model produces the following linear relation between expected returns and risk.

$$E_t\left(r_{i,t+1}\right)-r_f-\varphi_t=\beta_t\left[E_t\left(r_{M,t+1}\right)-r_f-\varphi_t\right] \tag{3}$$

where $r_{i,t+1}$ is the return on asset *i*, $r_{M,t+1}$ is the return on the market factor, r_f is the risk-free return with the parameters β_t, which represents market risk, and φ_t which is the average Lagrange multiplier from the portfolio constraint having the economic interpretation of a measure of the tightness of leverage constraints in the economy.

The theory of FP implies a CAPM represented by the following statistical model.

$$R_{i,t}=\varphi\left(1-\beta_i\right)+\beta_i R_{M,t}+\eta_{i,t}. \tag{4}$$

where:

$R_{i,t}$ = the return on firm *i* in period *t* in excess of the return on a risk-free asset,

$R_{m,t}$ = the excess return on the market portfolio at time *t*,

$\beta_i = \text{cov}(R_{i,t}, R_{m,t})/\text{var}(R_{m,t})$, and

$\eta_{i,t}$ = a random disturbance, $\eta \sim N(0,\sigma^2)$.

Compared with the classic Sharpe-Lintner version of the CAPM,

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \eta_{i,t} \quad i = 1, \ldots, N, \text{ and } t = 1, \ldots, T, \tag{5}$$

the FP model places a non-linear restriction on the CAPM alpha. In particular, $\alpha_i = \varphi(1-\beta_i)$. Non-linearity in the parameters, namely the multiplication of φ_t and β_t, make the FP model difficult to estimate. To circumvent the non-linearity problem, we follow an estimation method similar to the one-step Gauss-Newton procedure of Gibbons (1982).

Using a Taylor series expansion about consistent estimates of φ_t and β_t, we are able to linearize the non-linear term as,

$$\varphi\beta_i \cong \hat{\varphi}\hat{\beta}_i + \hat{\beta}_i(\varphi - \hat{\varphi}) + \hat{\varphi}\left(\beta_i - \hat{\beta}_i\right) = \varphi\hat{\beta}_i + \hat{\varphi}\beta_i - \hat{\varphi}\hat{\beta}_i\,. \tag{6}$$

Here we use a consistent estimate of $\hat{\beta}$ from the unrestricted model in equation (5) estimated by Ordinary Least Squares (OLS). Note that the functional form of equation (6) is identical to the statistical model representing the CAPM without a risk-free rate developed by Black (1972). In Black's model φ represents the return on a zero-beta asset. Given the similarity of the FP and Black models, we estimate $\hat{\varphi}$ using the functional form for an estimate of the zero-beta rate in the Black version of the CAPM found in Black, Jensen and Scholes (1972). We find our consistent estimate of $\hat{\varphi}$ using a generalized least squares (GLS) version of the Black, Jensen, and Scholes (1972) estimator, $\hat{\varphi} = \dfrac{\hat{\alpha}'\hat{\Sigma}^{-1}\left(\iota_N - \hat{\beta}\right)}{\left(\iota_N - \hat{\beta}\right)'\hat{\Sigma}^{-1}\left(\iota_N - \hat{\beta}\right)}$ where $\hat{\Sigma}^{-1}$ is the covariance matrix of the errors from the OLS regressions of (5).

Substituting equation (6) into equation (4) allows for linearization of the FP model,

$$\left(R_i - \hat{\varphi}\hat{\beta}_i \iota_T\right) \cong \varphi\left(1-\hat{\beta}_i\right)\iota_T + \beta_i\left(R_M - \hat{\varphi}\iota_T\right) + \hat{\eta}_i \ , i = 1, \ldots, N. \tag{7}$$

Using equation (7) we estimate φ and β with a multivariate seemingly unrelated regression model (SURM) using a general linear hypothesis across equations. Specifically, we estimate

$$\begin{bmatrix} R_i - \hat{\varphi}\hat{\beta}_1\iota_T \\ R_i - \hat{\varphi}\hat{\beta}_2\iota_T \\ \vdots \\ R_i - \hat{\varphi}\hat{\beta}_N\iota_T \end{bmatrix} = \begin{bmatrix} R_M - \hat{\varphi}\iota_T & 0 & \cdots & 0 & \left(1-\hat{\beta}_1\right)\iota_T \\ 0 & R_M - \hat{\varphi}\iota_T & \cdots & 0 & \left(1-\hat{\beta}_2\right)\iota_T \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & & R_M - \hat{\varphi}\iota_T & \left(1-\hat{\beta}_N\right)\iota_T \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_N \\ \varphi \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}$$

or more compactly,

$$y^* = Z\delta + \hat{\eta}^* \tag{8}$$

where:

$$y^{*\,'} = \left(R_1 - \hat{\varphi}\hat{\beta}_1\iota_T, R_2 - \hat{\varphi}\hat{\beta}_2\iota_T, \ldots, R_N - \hat{\varphi}\hat{\beta}_N\iota_T\right),$$

$$Z = \left[I_N \otimes \left(R_M - \hat{\varphi}\iota_T\right) \vdots \left(\iota_N - \hat{\beta}\right) \otimes \iota_T\right],$$

$$\delta' = \left(\beta_1, \beta_2, \ldots, \beta_N, \varphi\right).$$

The SURM estimator for δ is then defined by

$$\hat{\delta} = Z'\left[\left(\hat{\Sigma}^{-1} \otimes I_T\right)Z\right]^{-1} Z'\left(\hat{\Sigma}^{-1} \otimes I_T\right)y^* *. \tag{9}$$

We use the generalized method of moments (GMM) with a Newey-West spectral density estimate with $T^{1/3}$ lags to estimate equations (5), (8), and (9).

C. Direct Tests of the FP Model

We compare the fit of our estimations of the CAPM in equation (5) and the FP model in equation (4) to restricted versions of the models without an intercept using a large sample likelihood ratio test (LRT). The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model. If the model fits are close, the null hypothesis is not rejected. Since we are comparing the models with and without an intercept, a *p*-value less than 0.05 indicates that we can reject that the alphas of the 20 sorted portfolios are jointly different from zero. The LRT is defined as

$$-2ln\lambda = Tln = \left|\hat{\Sigma}_r\right|\left[ln - \left|\hat{\Sigma}_u\right|\right], \; -2ln\lambda \sim \chi^2_{N-1}, \tag{10}$$

where:

$\left|\hat{\Sigma}_r\right|$ = the determinant of the contemporaneous covariance matrix estimated from the residuals of the restricted model and

$\left|\hat{\Sigma}_u\right|$ = the determinant of the contemporaneous covariance matrix estimated from the residuals of the unrestricted model.

We also use the finite sample test of Gibbons, Ross, Shanken (1989), (GRS).

$$GRS = \left(\frac{T-N-1}{N}\right)\left[1+\frac{E(mkt)^2}{\sigma^2_{Mkt}}\right]^{-1}\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha}, \qquad GRS \sim F_{N,T-N-1} \tag{11}$$

Where $\hat{\Sigma} =$ a consistent estimate of the covariance matrix of the residuals.

V. Empirical Evidence

In this section we present our empirical evidence. We begin by demonstrating the relation between the cross-section of abnormal returns and the variables we use to adjust beta rankings. We then demonstrate the impact of accounting for leverage on portfolio rankings, portfolio sorts of abnormal returns, and show how Sharpe ratios across portfolios sorted on beta and leverage conform to the combined intuition of FP and M&M only when portfolio rankings are adjusted for leverage. We also document that a simple spread portfolio of high leverage firms produces a larger abnormal return than the BAB factor. We conclude this section with our formal tests of the CAPM and the FP model in equation (4).

A. Are Abnormal Returns Related to Leverage?

In this section we relate CAPM alpha with the variables we use to adjust ranking betas both through time and cross-sectionally. We generate a time-series of CAPM alpha for each of 20 portfolios sorted on the lagged traditional beta. The alpha time-series are intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy. Next, we estimate a series of cross-sectional and multivariate seemingly unrelated regressions to test whether these CAPM alphas are related to our measures of leverage, growth options, and default probability used in our adjusted beta rankings. In cross-sectional regressions of alpha reported in Panel A of Table 3, we observe that leverage is most often significantly related to alpha with p-values $\leq$ 0.05 in 56% of the cross-sections based on White's standard errors. Growth options and default probability are significant in 39% and 35% of samples respectively. The average of the absolute value of the t-statistics is 3.20 for leverage, 1.80 for the growth option proxy, and 1.89 for the default risk measure.

We also regress the portfolio alphas on our measures of leverage, growth options, and default probability within a seemingly-unrelated-regression equation (SURE) using GMM. Using the GMM SURE to estimate the multivariate system produces standard errors that are adjusted for cross-equation correlation and correct for heteroscedasticity using White's standard errors. As controls we include the Fama-French risk-factors, momentum, and a measure of the aggregate leverage constraint (LCT) from Boguth and Simutin (2018)[17]. Panel B of Table 3 contains the average of the absolute value of the t-statistics across the 20 portfolios. In this experiment, each of

[17] LCT is a measure of leverage constraint tightness that strongly and significantly predicts returns of FP's betting-against-beta factor. Boguth and Simutin (2018) find LCT alone explains 19% of the variation in future annual BAB returns.

the firm characteristics is statistically related to alpha; controlling for ubiquitous risk-factors ignored by the CAPM and a measure of the aggregate level of leverage constraints does not mitigate their relevance. As before, the magnitude of leverage is greater than the magnitudes of growth options or default risk.

[Table 3 Here]

The results in Table 3 reveal that the cross-section of abnormal returns of portfolios, formed by rankings that do not account for leverage, is significantly related to our measures of firm leverage, growth and default risk. Under the assumption that the asset rankings are related to the behavior of leverage constrained investors, we take this as evidence of possible mismeasurement in the rankings due to ignoring the importance of accounting for leverage.

B. Does Accounting for Leverage in Rankings Matter?

In this section we evaluate the impact of adjusting CAPM betas for leverage. We first demonstrate the impact on portfolio rankings. For each cross-section, we calculate the percentage of firms that are ranked in a particular $\beta^{Levered}$ quintile that move to the highest beta quintile after adjusting for leverage via the textbook $\left(\beta^{H}\right)$ and the GS $\left(\beta^{GS}\right)$ methods. In Panel A of Table 4 we present the mean, median, and standard deviation of the time-series of re-rankings when firms are re-ranked as high β^{GS} and in Panel B of Table 4 are the summary statistics when firms are re-ranked as high β^{H}. In Panel A, between 16.3% and 21.17% of firms on average move from a lower beta ranking to the highest beta ranking when adjusting for leverage using the GS method while 73.34% (= 100-26.66) of firms ranked into the highest beta portfolio move to lower beta ranked portfolios using the GS method. The results are not nearly as dramatic when adjusting for leverage by the classic textbook method where the vast majority of firms moving to the highest beta quintile come from quintile 4 on average and only 37.78% (= 100-62.22) of firms in the highest traditional beta portfolio are re-ranked to lower beta portfolios.

[Table 4 Here]

Next, we show that sorting on leverage-adjusted rankings decreases the likelihood of finding a negative relationship between CAPM alpha and beta. We generate a time-series of CAPM alpha for three sets of 20 portfolios sorted on (1) the lagged traditional beta $\left(\beta^{Levered}\right)$, (2) the standard textbook method of un-levering beta first described in Hamada (1972) $\left(\beta^{H}\right)$, and (3) betas adjusted for cross-sectional difference in leverage based on the theory of Gomes and Schmidt (2010) $\left(\beta^{GS}\right)$. As in the previous section, the alpha time-series are intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy.

[Table 5 Here]

In Table 5, we report the percentage of the 433 cross-sections of alpha exhibiting a negative slope across the 20 beta-sorted portfolios measured by regressing each cross-section of alphas on a linear trend. We find that 65% of the cross-sections of alphas for the portfolios sorted by $\beta^{Levered}$ are negative with 52% of the cross-sections having a significantly negative slope based on White's standard errors to correct for heteroscedasticity. Sorting firms into portfolios using β^{H} rankings we find that 44% of the cross-sections exhibit a negative slope where 28% are statistically significant. Sorting using β^{GS} rankings these percentages drop to 24% of the cross-sections having a negative slope where 18% are statistically significant. This significant decrease in the percentage of cross-sections with a negative relation between abnormal returns and beta ranking indicates the importance of accounting for firm level leverage when sorting portfolios.

In Table 6 we present alphas for quintiles of portfolios sorted on the three different ranking betas and leverage defined as the lagged market value debt-to-equity ratio adjusted for taxes and (non-cancelable, long-term) operating leases. The alphas are intercepts from a regression of excess portfolio returns on the equally weighted excess portfolio returns of the market proxy, where the market proxy and the risk-free rate are both from Ken French's database. All alphas are scaled by 100 and bold alphas indicate significance at the 5% level using Newey-West standard errors with six lags.

[Table 6 Here]

The intuition from FP is that leverage-constrained investors seek high-beta assets to increase their returns. The increased price pressure on high-beta assets results in lower expected returns and suggests significant mispricing by the CAPM for portfolios of high-beta stocks. The intuition gleaned from M&M suggests that leverage-constrained investors seek highly levered firms to increase their expected returns. Together these views of the behavior of leverage-constrained investors indicate that the CAPM should misprice high-beta firms with high leverage. In Panel A of Table 6, the alphas for portfolios sorted on traditional CAPM beta $\left(\beta^{Levered}\right)$ decrease moving from low-beta to high-beta stocks; generally producing a monotonically decreasing trend in alpha (except for firms in the fourth leverage portfolio, where the correlation is non-linear). These results are consistent with the results in Table 2. Moving from low-leverage to high-leverage portfolios, alphas increase. However, there is little evidence that high-beta stocks are mispriced by the CAPM. In the highest beta ranking, the only significant mispricing occurs for the portfolio of firms with the least amount of leverage (alpha = -0.42, *t*-statistic = -1.85). Ranking stocks based on CAPM beta produces abnormal returns that are inconsistent with the combined intuition of FP and M&M. Sorting on $\beta^{Levered}$, the 'beta anomaly' is an artifact of mispricing by the CAPM of low-beta firms.

Ranking firms on the textbook adjustment of betas $\left(\beta^{H}\right)$ mitigates the beta anomaly for firms with lower leverage to some degree, but overall this adjustment to the sorting variable has little impact on the anomalous relation between abnormal returns and beta. Alphas for low-beta firms are significant at the 5% level and high-beta firms are not statistically different from zero except in the lowest leverage quintile. For firms with the highest leverage rankings, the relation between abnormal returns and asset beta is non-monotonic. Ranking stocks based on "unlevered" CAPM beta produces abnormal returns that are inconsistent with the intuition of both FP and M&M.

When ranking firms on betas adjusted cross-sectionally for leverage $\left(\beta^{GS}\right)$ the beta anomaly is not preset for firms in the low-leverage quintile where all the alphas across beta quintiles are statistically indistinguishable from 0. For firms in the second leverage quintile, the beta anomaly is present, but in the remaining leverage quintiles alphas increase as beta increases and all the abnormal returns are statistically significant. Ranking firms on β^{GS} is the only ranking that generates portfolios where the CAPM significantly under prices high-leverage/high-beta firms.

We also compare the performance of the leveraged BAB factor with a simple spread portfolio using stocks sorted by β^{GS}. In Table 1 we demonstrate that our estimate of the BAB factor produced an alpha nearly identical to the alpha of the BAB factor from AQR over the period originally used in the FP paper. We re-estimate the BAB and AQR-BAB alphas for the sample with reliable leverage data, January of 1970 to December of 2015.

[Table 7 here]

For these data the AQR-BAB and our estimate of BAB continue to produce significant alphas. The ARQ-BAB (BAB) alpha is .009 (.010) with a robust *t*-statistic of 4.20 (4.68) and an R^2 of .008

(.005). However, using portfolios of stocks sorted on β^{GS}, a simple spread portfolio going long low-beta stock in the high leverage quintile and shorting high-beta stocks also from the highest leverage quintile increase the abnormal returns to 0.012 (a 29% change from .009) with a t-statistic of 4.09 and much larger R^2 of .039. This result highlights in the importance of accounting for leverage in portfolio rankings and suggests betting on leverage when betting against beta.

The results above demonstrate the impact of sorting firms into portfolios using leverage adjusted beta. Accounting for firm leverage in the portfolio rankings generates portfolios with abnormal returns that match the intuition of how leverage-constrained investors' trading behavior should impact returns provided by FP and M&M. However, these comparisons require estimation of beta at two stages; sorting betas and portfolio betas. We isolate the impact of accounting for leverage in ranking betas in a bootstrap experiment utilizing Sharpe ratios to avoid the second stage estimation. The combined intuition from FP and M&M suggests that constrained investors seek to add high-beta/high-leverage stocks, rather than high-beta/low-leverage stocks to their portfolios. If true, an investor who tilts from holding 100% of her portfolio in the market to a portfolio with 99% invested in the market and 1% invested in high-beta stock should see a larger percentage change in her Sharpe ratio when the high-beta stock has more leverage compared to tilting towards a high-beta stock with low leverage. We check this via a bootstrap exercise.

In our exercise we use all firms in our sample from the CRSP-Compustat database between 1970 and 2015 with at least 60 months of data. For each of the 493 60-month windows, we sort firms independently based on their average traditional beta and average financial leverage defined as the natural logarithm of one plus the tax-and-operating-lease-adjusted market value of debt to market value of equity ratio (as defined in the data appendix). Firms are ranked into leverage and beta quintiles based on their average leverage and beta over the 60 months. We rank firms first on beta, and then rank firms on leverage within each beta quintile. We perform the exercise twice, once where we rank firms on $\beta^{Levered}$ and again were firms are ranked using β^{GS}.

In each five-year period we calculate the Sharpe ratio, S_{MKT}, of the market proxy obtained from Ken French's website. We also calculate the Sharpe ratios for portfolios where the investor invests 99% of her wealth in market proxy and 1% in each firm ranked as (1) high-beta (*HB*) or low-beta (*LB*), (2) high- and low-beta firms with high leverage (*HBHL* and *LBHL*), and (3) high- and low-beta firms with low leverage (*HBLL* and *LBLL*) respectively. We then estimate the bootstrap expected value of the percentage change in the squared Sharpe ratio of going from being fully invested in the market to each of these six other portfolios by drawing with replacement 5000 samples of percentage changes. The means of the bootstrapped percentage changes distributions are in Table 8.[18]

[Table 8 here]

Counter to the intuition in FP, we see in Panel A of Table 8 that when sorting firms on traditional beta ($\beta^{Levered}$), tilting away from full investment in the market towards stocks in the *LB* quintile produces an expected improvement in the Sharpe ratios of 1.18%. This is larger than the expected increase in Sharpe ratios achieved by tilting towards *HB* stocks (0.28%). Accounting for leverage in the ranking beta by using β^{GS} we find results consistent with the implication of FP that leveraged constrained investors should tilt towards stocks in the *HB* quintile where the expected change in the Sharpe ratio is 1.05% rather than tilting toward the *LB* quintile which produces an expected increase in the Sharpe ratio of 0.73%. [19]

[18] We winsorize the percentage change distributions at the 0.01% to reduce the impact of a small number of large percentage changes.

[19] Using a test of stochastic dominance proposed by Linton, Maasoumi and Whang (2005), we reject the null that distribution of HB Sharpe ratios dominates the LB distribution (p-value = 0.00) when stocks are sorted by $\beta^{Levered}$, but we are unable to reject the null that distribution of HB Sharpe ratios dominates the LB distribution (p-value = 0.21) when stocks are sorted on β^{GS}. Linton, Maasoumi and Whang (2005) propose a procedure for estimating the critical values of the extended Kolmogorov-Smirnov tests of Stochastic Dominance of arbitrary order in the general *K*-prospect case. Their test allows for serially dependence and accommodates general dependence among the prospects

Accounting for dispersion of leverage within each beta quintile does little to change the above results. When stocks are sorted on $\beta^{Levered}$, low-beta stocks continue to provide more of an increase to Sharpe ratios regardless of firm leverage as can be seen in Panels B and C of Table 8. When the sorting variable incorporates the cross-sectional dispersion in leverage (β^{GS}) we see the largest expected increase in Sharpe ratios occurring when tilting towards firms in the *HBHL* portfolio (1.20) and the smallest expected increase in Sharpe ratios occur when tilting towards firms in the *LBLL* portfolio. The Sharpe ratio results indicate the benefit from adjusting for firm leverage in the sorting betas.

C. Formal Model Tests

The full sample market model alphas for the three sets of beta adjusted portfolios are displayed in Figure 1.

[Figure 1 Here]

As expected, the CAPM alpha decreases moving from the low- to the high-traditional-beta portfolio. However, adjusting beta for leverage per Hamada $\left(\beta^{H}\right)$ produces sorted portfolios with alphas that first increase with market risk, then decrease across the 20 portfolios. Finally, the portfolios sorted on β^{GS} exhibit clearly non-decreasing alphas. We therefore elect to focus on the traditional beta, $\left(\beta^{Levered}\right)$, and leverage adjusted betas, $\left(\beta^{GS}\right)$, rankings of monthly beta-sorted portfolio returns.

We begin by estimating a time-varying version of φ_t using the Black, Jensen, and Scholes (1972) estimator noted above. For both the set of 20 portfolios sorted by CAPM beta and β^{GS} we estimate the CAPM alpha, beta, and the inverse of error covariance matrix $\hat{\Sigma}_r$ using 60-month

being ranked. *P*-values are based on a centered boot-strap.

rolling windows. For each window we then estimate the GLS cross-sectional regression to obtain $\hat{\varphi} = \frac{\hat{\alpha}'\hat{\Sigma}^{-1}\left(\iota_N - \hat{\beta}\right)}{\left(\iota_N - \hat{\beta}\right)'\hat{\Sigma}^{-1}\left(\iota_N - \hat{\beta}\right)}$. For each traditional beta sorted portfolio, we create a time series of FP alphas as $\hat{\alpha}_{FP,i} = \hat{\varphi}_t\left(1 - \hat{\beta}_{t,i}\right)$ where $\hat{\beta}_{t,i}$ are the rolling CAPM betas. For each β^{GS} sorted portfolio we create FP adjusted alphas as $\hat{\alpha}_{FPGS,i} = \hat{\varphi}_t\left(1 - \hat{\beta}_{t,i}\right)$ where the $\hat{\beta}_{t,i}$ are the rolling CAPM betas for the β^{GS} sorted portfolios. Figure 2 plots the average FP alphas for these two sets of portfolios along with the average rolling window CAPM alphas.

[Figure 2 Here]

The average of the time-varying alphas produces graphs like those of the full sample alphas in Figure 1. Alphas of $\beta^{Levered}$ sorted portfolios (CAPM alphas) fall as market risk increases while that trend is reversed if stocks are sorted into portfolios using betas that account for firm-leverage (CAPM AdjAlphas). Using a *t*-test that accounts for the large amount of autocorrelation built into the rolling window time-series, only two of the average CAPM alphas are not significantly different from zero and all the CAPM AdjAlphas are significantly different from zero.[20] Accounting for investor's leverage constraints shrinks the $\hat{\alpha}_{FP,i}$ (FP alphas) for all of the portfolios but does little to remove the negative relation between abnormal returns and market risk. Only 12 of the 20 portfolios exhibit $\hat{\alpha}_{FP,i}$ significantly different from zero. We do find that combining the FP model alphas for portfolios sorted using betas adjusted for firm level leverage eliminates the

[20] To test if the rolling window alphas are different from zero, we regress each time-series of 433 alpha on a vector of ones and use the autocorrelation-heteroskedasticity-consistent (HAC) standard errors from Newey and West (1987), with an automatic lag selection procedure. The number of lags is chosen by computing the autocorrelations of the estimated residuals and truncating the lag length when the sample auto-correlations become "insignificant". Specifically, we compute 60 sample autocorrelations and compare the values with a cutoff at two approximate standard errors: $2/\sqrt{T}$, where T is the sample size. The number of lags chosen is the minimum lag length at which no higher order autocorrelation is larger than two standard errors.

negative relation between abnormal returns and market risk. The $\hat{\alpha}_{FPGS,i}$ (FP AdjAlphas) are all close to zero with only three being significantly different from zero.

The evidence in Figure 2 is compelling but the estimation of the time-varying FP alphas may suffer from several potential statistical issues. To avoid these problems, we estimate the multivariate seemingly unrelated regression described above. The SURM procedure is attractive since it produces consistent and asymptotically efficient estimates. The procedure also avoids the errors-in-variables problem by simultaneously estimating φ and β. Finally, the procedure produces more precise estimators since we use the full contemporaneous covariance matrix of the errors. We form FP alphas based using $\hat{\varphi}$ and $\hat{\beta}$ from our estimation of equation (9). These alphas along with the CAPM alphas in Figure 1 are presented in Figure 3.

[Figure 3 here]

For both sets of portfolios, the FP model produces smaller alphas on average than the CAPM alphas. When applied to the beta-sorted portfolio the alphas, labeled FP Alphas, are shifted down with half falling below zero. As can be seen in the figure, when applied to beta-sorted portfolios where beta does not account for the GS model leverage variables the FP alphas continue to decrease with market risk and are significantly different from zero based on the LRT and GRS test results in Table 9.

[Table 9 here]

When the FP model is applied to beta sorted portfolios where beta accounts for the GS model leverage variables $\left(\beta^{GS}\right)$, the FP alphas marginally increase with market risk and we fail to reject that they are jointly different from zero using either the LRT or GRS test results in Table 10.

[Table 10 here]

We then split our sample into two equally sized sub-samples and repeat our experiment. Alphas for the beta sorted portfolios are presented in Figure 4.

[Figure 4 here]

We find similar patterns for alpha across the $\beta^{Levered}$ and β^{GS} sorted portfolios in both sub-samples. CAPM alphas decrease across portfolios sorted on the traditional measure of market risk. CAPM alphas increase with the market risk adjusted for the impact of leverage and are not significantly different from zero based on the LRT tests results in Table 11. Alphas implied by the FP model are smaller in both sub-sample for portfolio sorted on either traditional or adjusted measures of risk, but the FP alphas on the adjusted-beta-sorted portfolios display a slight increase in both sub-samples and are not statistically different from zero based on the LRT tests in Table 11.

[Table 11 here]

VI. Conclusions

Our results highlight the importance of accounting for firm level leverage when sorting stocks into portfolios using a measure of market risk. To separate the market risk that stems from firm leverage (corporate finance policy) from systematic asset risk (corporate investment policy), we un-lever traditional betas based on variables suggested by Gomes and Schmid's (2010) continuous-time capital structure model. Ranking stocks on leverage adjusted betas generates portfolios with risk and return characteristics reflecting the intuition that leverage-constrained investors demand high-beta assets with high leverage. We find that sorting on adjusted beta, high-beta assets with high leverage exhibit significant mispricing, earn higher returns than low-beta stocks, and produce higher portfolio Sharpe ratios compared to high-beta/low leverage firms. We reject the classic CAPM, which ignores the leverage constraints of investors, when we sort stocks into portfolios using leverage-adjusted betas. However, we cannot reject the linear FP model which accounts for the leverage constraints of investors when we price leverage-adjusted beta sorted portfolios.

Our contributions to the literature are novel empirical support for the FP model and an explanation for the persistent anomalous relationship between CAPM alpha and beta based on our empirical implementation of the intuition found in the Gomes and Schmid (2010) theory. Beyond the implications for asset pricing models, our results are further relevant to leverage-constrained investors. Blume and Keim (2012) show that the proportion of U.S. public equities managed by institutions has risen from about 7% of market capitalization in 1950, to about 67% in 2010. Considering this increased trading by institutions, many of which face leverage constraints, our work commends accounting for firm-level leverage in the development of new asset pricing models.

We do not rule out the importance of increases in high frequency trading which may be related to the lottery-based explanation of Bali, *et al*. (2017). And our leverage adjustments do imbue some time-variation in beta consistent with the findings of Cederburg and O'Doherty (2016). However, because our results provide support for a rational explanation, rather than behavioral or statistically motived explanation, of the long-standing anomaly that sorting firms on beta produces portfolios of assets that exhibit higher (lower) expected abnormal returns with lower (higher) market risk, we believe our paper provides an important contribution to the literature.

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Table 1: Comparison Statistics

Panel A contains univariate summary statics for the betting-against-beta (BAB) factor available from AQR Capital Management, LLC and our replication of the factor, as well as their Sharpe ratios and the correlation between the two measures. Panel B contains the intercept coefficient, t-statistics and R-squared from regressing the BAB factors on the excess market return, the Fama-French three factors, and the Fama-French three factor model plus momentum respectively. All data are monthly over the period January, 1963 to December, 2015 and all the explanatory variables and the risk free rate are from Ken French's web page.

Panel A: Summary Statistics		
	AQR BAB	BAB
Mean	0.007	0.008
Median	0.007	0.008
Standard Deviation	0.031	0.032
Skewness	-0.747	-0.726
Kurtosis	8.952	10.957
Autocorrelation	0.136	0.129
Sharpe Ratio	0.262	0.284
Correlation		0.96
Panel B: Regress BAB on CAPM and FF models		
	AQR BAB	BAB
CAPM alpha	0.007	0.008
t-stat	7.537	7.909
R^2	0.013	0.006
3-factor alpha	0.007	0.008
t-stat	7.484	7.886
R^2	0.014	0.006
4-factor alpha	0.005	0.006
t-stat	5.591	6.016
R^2	0.087	0.078

Table 2: U.S. Equities Returns, 1963 - 2015

This Table shows average monthly portfolio returns. The first row across columns 1 through 10 contain average returns of beta sorted portfolios. Portfolios are formed each month by ranking all stock using the past month's beta. Betas used for sorting are calculated as in Frazzini and Pederson (2014). The ranked stocks are sorted into equally sized portfolios each month and the returns are equally (Panel A) or value (panel B) weighted. Each panel contains the intercept coefficient, t-statistics (in parentheses) and R-squared from regressing the portfolio monthly excess return on the excess market return, the Fama-French three factors, and the Fama-French three factor model plus momentum respectively. All data are monthly over the period June, 1963 to December, 2015 and all the explanatory variables and the risk free rate are from Ken French's web page. The parentheses contain t-statistics. Alphas are scaled by 100.

Panel A: Equally-Weighted Returns

	Low Beta	2	3	4	5	6	7	8	9	High beta
Mean	0.009	0.009	0.010	0.010	0.010	0.009	0.009	0.009	0.008	0.008
	(6.15)	(5.37)	(5.52)	(4.99)	(4.57)	(4.04)	(3.82)	(3.15)	(2.68)	(1.92)
CAPM alpha	0.639	0.516	0.546	0.512	0.404	0.341	0.320	0.220	0.105	-0.153
	(5.96)	(5.02)	(5.35)	(5.20)	(3.90)	(3.09)	(2.70)	(1.66)	(0.61)	(-0.63)
R^2	0.491	0.622	0.682	0.747	0.762	0.765	0.772	0.765	0.708	0.625
3-factor alpha	0.439	0.290	0.323	0.295	0.181	0.119	0.092	-0.009	-0.106	-0.356
	(5.03)	(3.66)	(4.45)	(4.48)	(2.66)	(1.77)	(1.24)	(-0.11)	(-0.93)	(-2.00)
R^2	0.674	0.783	0.845	0.891	0.901	0.917	0.914	0.911	0.876	0.807
4-factor alpha	0.465	0.344	0.364	0.327	0.266	0.216	0.240	0.196	0.202	0.105
	(5.21)	(4.28)	(4.93)	(4.88)	(3.92)	(3.27)	(3.41)	(2.59)	(1.99)	(0.65)
R^2	0.675	0.786	0.847	0.892	0.906	0.922	0.926	0.930	0.907	0.850

Panel B: Value-Weighted Returns

	Low Beta	2	3	4	5	6	7	8	9	High beta
Mean	0.010	0.008	0.009	0.009	0.010	0.010	0.010	0.011	0.011	0.015
	(6.38)	(6.16)	(6.47)	(5.77)	(6.12)	(5.38)	(5.13)	(4.82)	(4.23)	(4.41)
CAPM alpha	0.791	0.559	0.586	0.546	0.573	0.506	0.503	0.538	0.431	0.646
	(5.88)	(5.75)	(6.35)	(6.31)	(6.78)	(6.17)	(5.94)	(5.91)	(4.00)	(3.75)
R^2	0.324	0.489	0.584	0.692	0.743	0.800	0.823	0.838	0.824	0.749
3-factor alpha	1.020	0.798	0.882	0.938	1.024	1.049	1.123	1.268	1.297	1.845
	(6.25)	(5.88)	(6.17)	(6.02)	(6.18)	(5.78)	(5.69)	(5.75)	(5.27)	(5.74)
R^2	0.004	0.004	0.000	0.005	0.011	0.025	0.036	0.053	0.083	0.132
4-factor alpha	1.088	0.883	0.967	0.988	1.049	1.078	1.145	1.327	1.296	1.781
	(6.62)	(6.48)	(6.77)	(6.33)	(6.29)	(5.89)	(5.73)	(5.97)	(5.12)	(5.25)
R^2	0.003	0.011	0.017	0.015	0.012	0.023	0.026	0.051	0.041	0.046

Table 3: Explaining Alphas

This table contains regressions using the CAPM alpha for each of 20 portfolios sorted on the lagged traditional beta. The alpha time-series are intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy. Panel A contains the percentage of times we find a significant coefficient in the cross-sectional regressions of portfolio alphas on our leverage adjustment variables based on the GS model. Panel C (a) contains the average of the absolute value of the *t*-statistics from regressing the 20 portfolio alphas on our measures of leverage, growth options, and default probability within a seemingly-unrelated-regression equation. Columns (b) and (c) add controls from the four-factor model and the measure of aggregate leverage constraints LCT. All standard errors are adjusted for cross-equation correlation and correct for heteroscedasticity using White's standard errors.

Panel A: Percentage of Significant Coefficients

Leverage	Growth Options	Default Probability
56%	39%	35%

Panel B: Average |*t*-statistic|

	(a)	(b)	(c)
Intercept	1.998	1.942	1.45
Lev	5.78	5.38	5.09
GO	2.32	2.26	2.44
PD	4.64	4.68	4.95
Mkt		0.30	0.36
SMB		2.13	1.99
HML		2.26	2.11
UMD		0.70	0.65
LCT			0.43

Table 4: Leverage Adjustment and Portfolio Rankings

This table contains summary statistics of the percentage of firms in each cross-section previously ranked into quintiles based on traditional beta $\left(\beta^{Levered}\right)$ that are re-ranked to the highest beta quintile when sorting on betas adjusted for leverage via textbook $\left(\beta^{H}\right)$ and GS $\left(\beta^{GS}\right)$ methods. All data are monthly over the period January 1970 to December 2015.

Panel A: Percentage of Firms Re-ranked as High GS Beta					
	1	2	3	4	5
Mean	21.17	16.30	16.81	19.40	26.66
Median	22.16	16.52	16.94	19.19	25.51
Standard deviation	4.41	2.82	2.02	2.42	6.36

Panel B: Percentage of Firms Re-ranked as High Hamada Beta					
	1	2	3	4	5
Mean	0.00	0.01	5.34	32.61	62.22
Median	0.00	0.00	2.79	34.04	61.67
Standard deviation	0.00	0.10	6.32	5.78	10.70

Table 5: Cross-Sectional Slopes of Alphas

This table contains regressions using the CAPM alpha for each of 20 portfolios sorted on the lagged traditional beta as the left-hand-side variable. The alpha time-series are intercepts from 60-month rolling window estimations of the market model using the S&P500 excess return as the market proxy. The first column contains the percentage of times we find a negative coefficient (percentage significant at 5%) in the 433 cross-sectional regressions of portfolio alphas on a linear trend where the portfolios are sorted on traditional beta $\left(\beta^{Levered}\right)$. The next two columns are the results when sorting on betas adjusted for leverage via textbook $\left(\beta^{H}\right)$ and GS $\left(\beta^{GS}\right)$ methods. All standard errors are adjusted for cross-equation correlation and correct for heteroscedasticity using White's standard errors.

Percentage of Significant Slopes		
$\beta^{Levered}$ Rankings	β^{H} Rankings	β^{GS} Rankings
65%	44%	24%
(52%)	(28%)	(18%)

Table 6: Sorted Portfolio CAPM Alphas

Alphas*100 for the double sorted portfolios. Panel A contains the alphas for the equally weighted portfolios sorted on traditional beta $\left(\beta^{Levered}\right)$. Panel B contains alphas for portfolios sorted on betas adjusted for leverage via textbook $\left(\beta^{H}\right)$ and GS $\left(\beta^{GS}\right)$ methods.. Portfolios are sorted each month on last month's beta then on last month's leverage defined as the lagged market value debt-to-equity ratio adjusted for taxes and operating leases. Alphas are the intercepts from a regression of excess portfolio returns on the excess returns of a market proxy. The market proxy and the risk-free rate are both from Ken French's data base. All alphas are scaled by 100. Bold alphas are significant at the 5% level using Newey-West standard errors with 12 lags.

Panel A: $\beta^{Levered}$ Rankings

	Low Beta	2	3	4	High Beta
Low Leverage	**0.53**	**0.40**	0.24	0.05	**-0.42**
2	**0.63**	**0.42**	**0.29**	0.12	-0.22
3	**0.69**	**0.51**	**0.37**	0.21	-0.06
4	**0.49**	**0.62**	**0.44**	**0.36**	-0.08
High Leverage	**0.83**	**0.72**	**0.63**	**0.44**	0.14

Panel B: β^{H} Rankings

	Low Beta	2	3	4	High Beta
Low Leverage	**0.62**	**0.48**	**0.49**	0.27	**-0.4**
2	**0.50**	**0.66**	**0.47**	**0.32**	-0.02
3	**0.81**	**0.54**	**0.32**	**0.28**	-0.05
4	**0.79**	**0.65**	**0.42**	**0.32**	-0.23
High Leverage	**0.85**	0.19	0.26	-0.01	-0.14

Panel C: β^{GS} Rankings

	Low Beta	2	3	4	High Beta
Low Leverage	0.28	0.13	0.07	-0.07	-0.38
2	**0.40**	**0.36**	**0.30**	0.21	0.19
3	**0.29**	**0.37**	**0.34**	**0.49**	**0.48**
4	**0.25**	**0.44**	**0.40**	**0.45**	**0.63**
High Leverage	**0.41**	**0.33**	**0.55**	**0.58**	**0.83**

Table 7: Spread Portfolios

This tables contains alphas (times 100) from regressing spread portfolios on the market proxy. ARQ-BAB is the betting-against-beta factor from the ARQ webpage. BAB is our estimation of the BAB factor following Frazzini and Pederson (2014). The BOL portfolio is long Low-beta/high-leverage stocks and short high-beta/high-leverage stocks when stocks are sorted on using β^{GS}. BOL is equally weighted and all data are from the period January 1970 to December 2015. The market proxy is from Ken French's web page.

	Alpha	*t*-stat	R^2
ARQ-BAB	0.9	4.20	0.008
BAB	1.0	4.68	0.005
BOL	1.2	4.09	0.039

Table 8: Bootstrapping Sharpe Ratios

This table contains the average of the expected percentage change in the constrained investor's Sharpe ratio when tilting away from the market portfolio across the 5,000 bootstrap samples. Panel A contains the expected percentage increase in Sharpe ratio when tilting towards firms ranked as high (*HB*) or low (*LB*) traditional beta ($\beta^{Levered}$) or high cross-sectionally adjusted beta (β^{GS}). Panels B and C show the expected change in the Sharpe ratio when tilting towards firms selected from the portfolios created from double sorting firms on betas then leverage.

Panel A: Beta Sorts		
	HB	*LB*
$\beta^{Levered}$	0.28	1.18
β^{GS}	1.05	0.73
Panel B: High and Low Beta Firms with High Leverage		
	HBHL	*LBHL*
$\beta^{Levered}$	0.13	1.35
β^{GS}	1.20	0.99
Panel C: High and Low Beta Firms with Low Leverage		
	HBLL	*LBLL*
$\beta^{Levered}$	0.25	1.15
β^{GS}	1.09	0.48

Table 9: Joint Tests of Alpha

This table contains test statistics and associated *p-values* comparing the fit of our estimations of the CAPM to restricted versions of the CAPM without an intercept using a large sample likelihood ratio test (LRT) and the finite sample Gibbons, Ross, Shanken (1989) test. The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model while the GRS is a joint test that the alphas are equal to zero. A *p*-value less than 0.05 indicates that we can reject that the unrestricted and restricted model fits are close for the LRT test and that the alphas of the 20 sorted portfolios are jointly different from zero for the GRS test. The column labelled β^{CAPM}-Alphas uses portfolios sorted on traditional betas. The column labelled β^{GS}-Alphas uses portfolios sorted on GS adjusted betas. The column labelled β^{H}-Alphas uses portfolios sorted on unlevered betas.

	β^{CAPM}-Alphas	β^{GS}-Alphas	β^{H}-Alphas
LRT	97.76	46.65	60.69
p-value	0.000	0.000	0.000
	β^{CAPM}-Alphas	β^{GS}-Alphas	β^{H}-Alphas
GRS	4.03	2.34	3.09
p-value	0.000	0.001	0.000

Table 10: Joint Tests of FP Alpha

This table contains test statistics and associated *p-values* comparing the fit of our estimations of the FP version of the CAPM

$$R_{i,t} = \varphi(1 - \beta_i) + \beta_i R_{M,t} + \eta_{i,t}$$

to restricted versions of the FP CAPM without an intercept using a large sample likelihood ratio test (LRT) and the finite sample Gibbons, Ross, Shanken (1989) test. The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model while the GRS is a joint test that the alphas are equal to zero. A *p*-value less than 0.05 indicates that we can reject that the unrestricted and restricted model fits are close for the LRT test and that the alphas of the 20 sorted portfolios are jointly different from zero for the GRS test. The column labelled $\beta^{Levered}$-FPAlphas uses portfolios sorted on traditional betas. The column labelled β^{GS}-FPAlphas uses portfolios sorted on GS adjusted betas.

	$\beta^{Levered}$-FPAlphas	β^{GS}-FPAlphas
LRT	77.28	17.30
p-value	0.000	0.570
	$\beta^{Levered}$-FPAlphas	β^{GS}-FPAlphas
GRS	1.566	0.284
p-value	0.056	0.992

Table 11: Joint Tests of FP Alpha, Sub-Samples

This table contains test statistics and associated *p-values* comparing the fit of our estimations of the FP version of the CAPM

$$R_{i,t} = \varphi(1 - \beta_i) + \beta_i R_{M,t} + \eta_{i,t}$$

to restricted versions of the FP CAPM without an intercept using a large sample likelihood ratio test (LRT) and the finite sample Gibbons, Ross, Shanken (1989) test splitting our sample into two equally sized sub-samples. The LRT compares the statistical fit of the unrestricted model with the fit of the restricted model while the GRS is a joint test that the alphas are equal to zero. A p-value less than 0.05 indicates that we can reject that the unrestricted and restricted model fits are close for the LRT test and that the alphas of the 20 sorted portfolios are jointly different from zero for the GRS test. The column labelled β^{CAPM}-Alphas contains the results from the CAPM when the portfolios are sorted on FP betas. The column labelled β^{CAPM}-FPAlphas uses portfolios sorted on traditional betas. The column labelled β^{GS}-FPAlphas uses portfolios sorted on GS adjusted betas. The column labelled β^{H}-Alphas uses portfolios sorted on unlevered betas.

	β^{CAPM}-Alphas	β^{CAPM}-FPAlphas	β^{GS}-FPAlphas
	First Sub-Sample		
LRT	2.87	36.73	22.11
p-value	0.000	0.009	0.279
	Second Sub-Sample		
LRT	2.25	45.74	28.89
p-value	0.002	0.001	0.068

Figure 1: Alphas of Beta Sorted Portfolios

This figure contains the alphas of the 20 sorted portfolios estimated from the CAPM over our sample. The solid line represents the alphas when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas when the portfolios are sorted on lagged betas adjusted by the GS variables. The dotted line represents alphas when the portfolios are sorted on lagged unlevered (Hamada) betas.

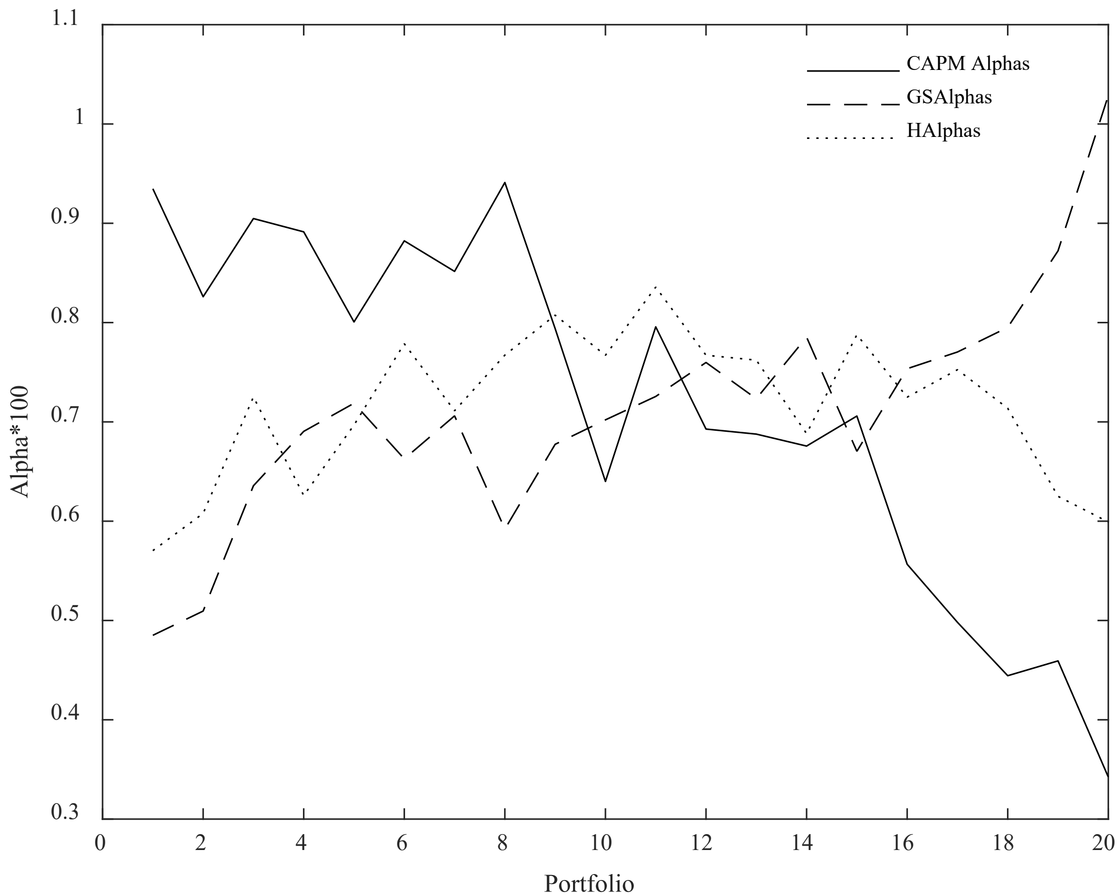

Figure 2: Time-Varying Alphas of Beta Sorted Portfolios

This figure contains the average alphas of the 20 sorted portfolios estimated from 60-month rolling window estimates of the CAPM and the FP model over our sample. We estimate a time-series of the leverage constraint parameter φ_t using, $\hat{\varphi}_t = \frac{\hat{\alpha}'\hat{\Sigma}^{-1}(\iota_N - \hat{\beta})}{(\iota_N - \hat{\beta})'\hat{\Sigma}^{-1}(\iota_N - \hat{\beta})}$. The dotted line represents the alphas from the CAPM when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas from the FP model when the portfolios are sorted on lagged traditional betas. The dashed/dotted line represents alphas from the CAPM when the portfolios are sorted on lagged betas adjusted by the GS variables. The solid line represents alphas from the FP model when the portfolios are sorted on lagged betas adjusted by the GS variables.

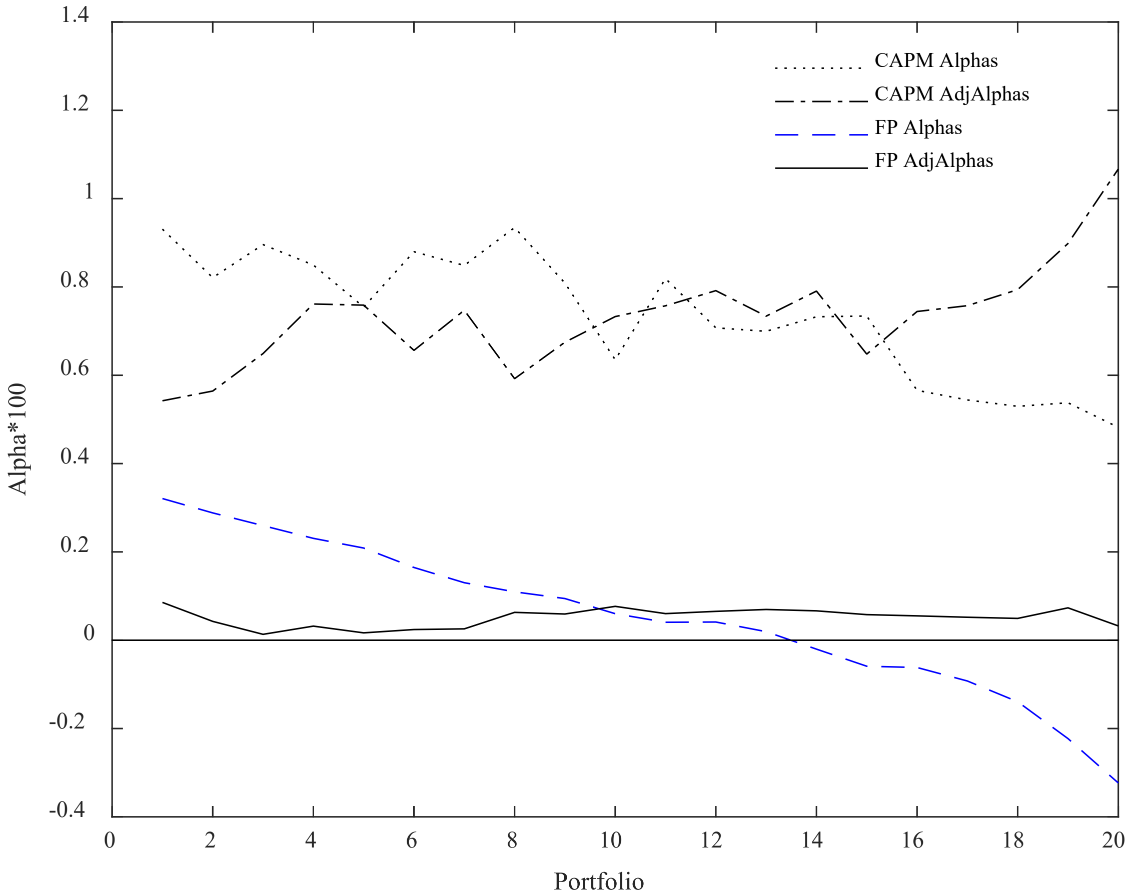

Figure 3: Alpha from the FP Model

This figure contains the alphas of the 20 sorted portfolios estimated from the CAPM and the FP model over our sample. The dotted line represents the alphas from the CAPM when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas from the FP model when the portfolios are sorted on lagged traditional betas. The dashed/dotted line represents alphas from the CAPM when the portfolios are sorted on lagged betas adjusted by the GS variables. The solid line represents alphas from the FP model when the portfolios are sorted on lagged betas adjusted by the GS variables.

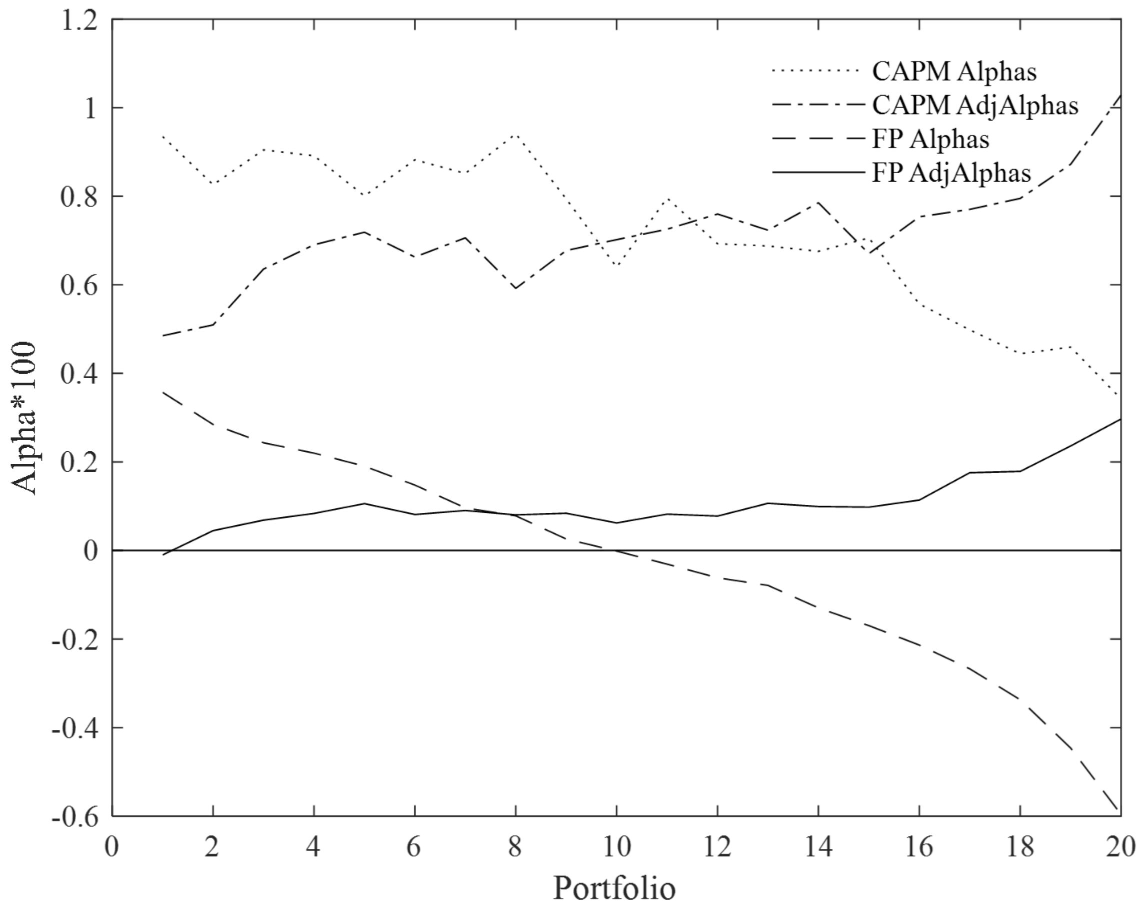

Figure 4: Alphas from the CAPM and FP Model

This figure contains the alphas of the 20 sorted portfolios estimated from the CAPM and the FP model two evenly sized subsamples. The dotted line represents the alphas from the CAPM when the portfolios are sorted on lagged traditional betas. The dashed line represents the alphas from the FP model when the portfolios are sorted on lagged traditional betas. The dashed/dotted line represents alphas from the CAPM when the portfolios are sorted on lagged betas adjusted by the GS variables. The solid line represents alphas from the FP model when the portfolios are sorted on lagged betas adjusted by the GS variables.

Panel A: First Sub-Sample

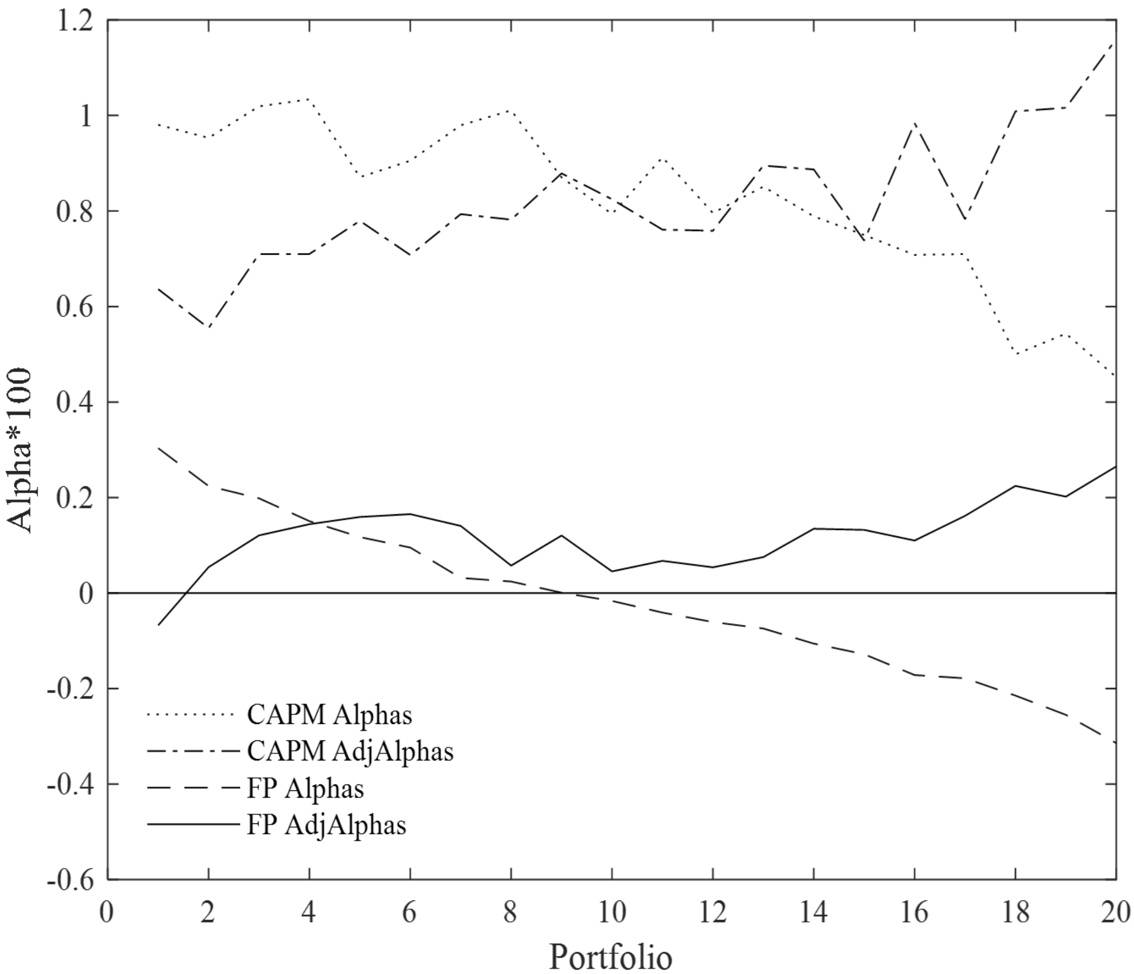

Figure 4: Alphas from the CAPM and FP Model (continued)

Panel B: Second Sub-Sample