Pareto Improving Segmentation of Multi-product Markets

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joint with Ron Siegel

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Market Segmentation

Firms can segment the market based on available data

- age, sex, location, browsing history, ...

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Effects of segmentation on consumers?

- Segmentation can harm consumers
- Can segmentation benefit consumers?

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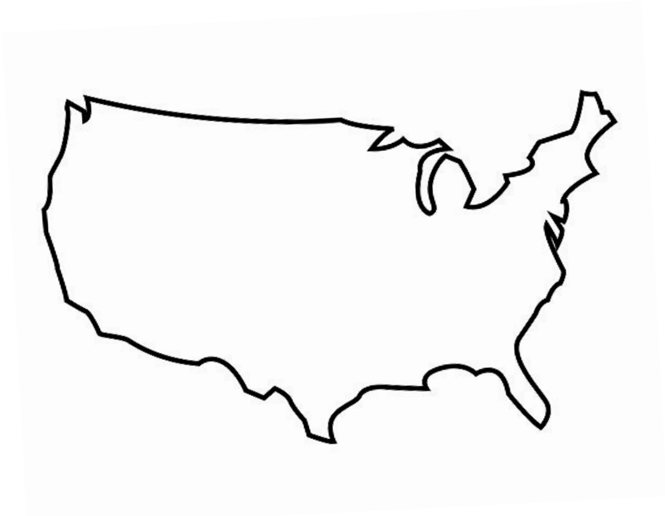

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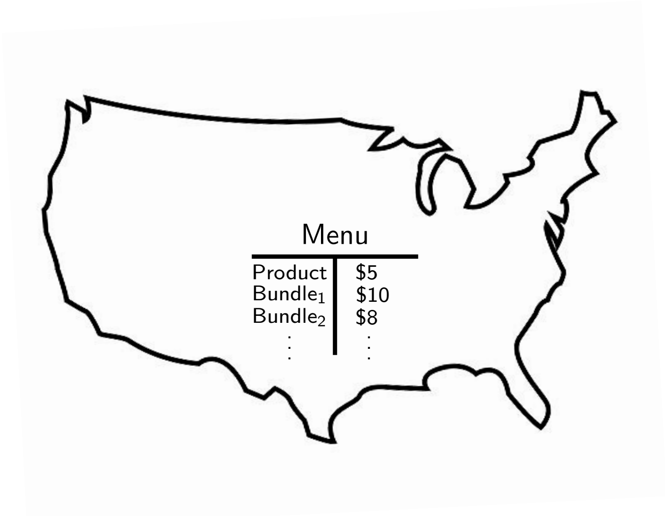

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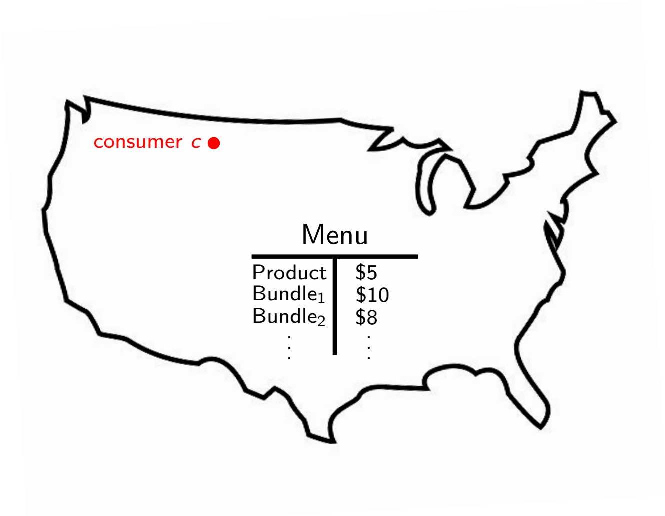
Un-segmented market: $CS(c)$



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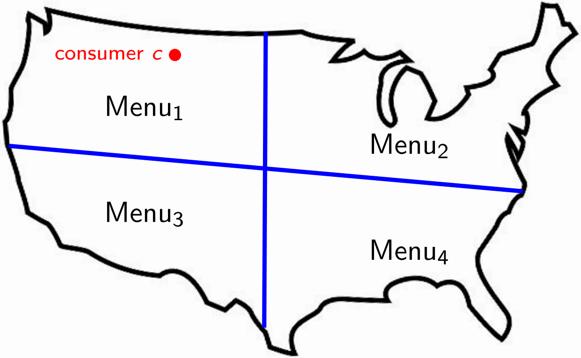

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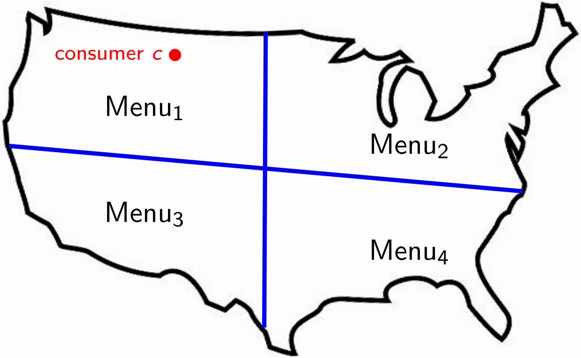

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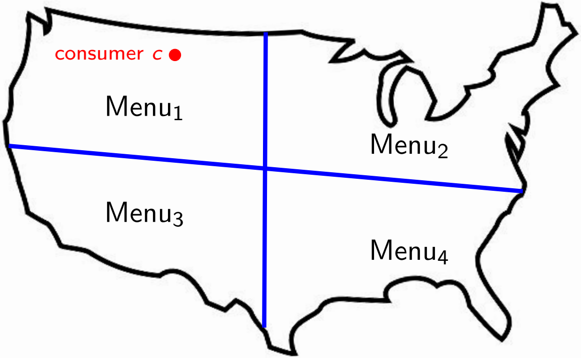
∃? segmentation  s.t. $CS(c,$  $) \geq CS(c)$ for all c,

& $>$ for some c



A Single Product Example (with Unit Demands)

"Market":	$1 - q$	q
	●	●
Valuation v:	1	2

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A Pareto Improving (PI) segmentation exists if market q is inefficient

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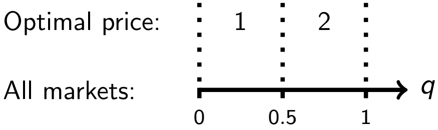

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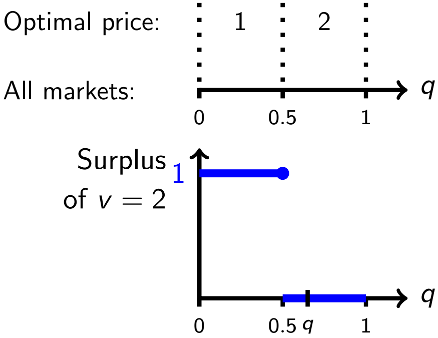

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A Pareto Improving (PI) segmentation exists if market q is inefficient

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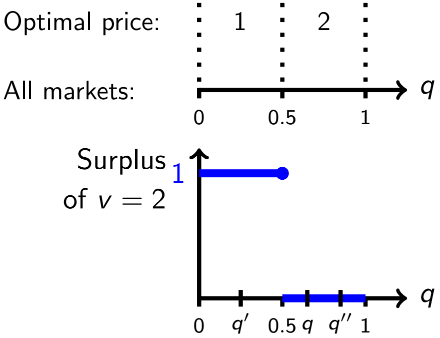

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A Pareto Improving (PI) segmentation exists if market q is inefficient

- $q \in (0.5, 1)$: Segment to $q' \leq 0.5$ and $q'' > q$
- Holds with any number of valuations

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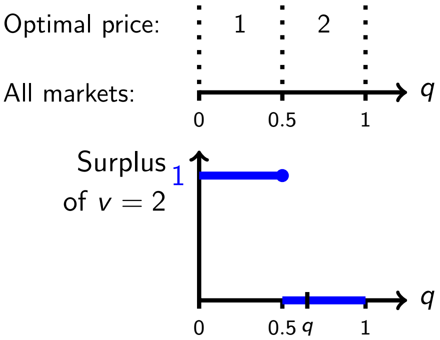

Screening Example: Qualities L and H

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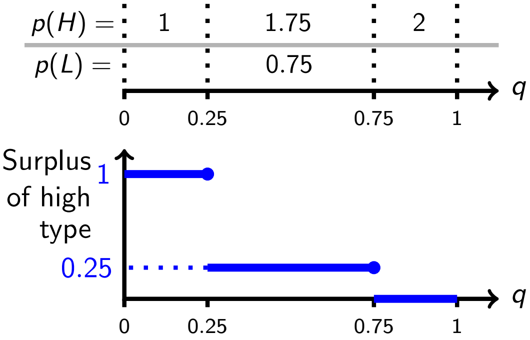

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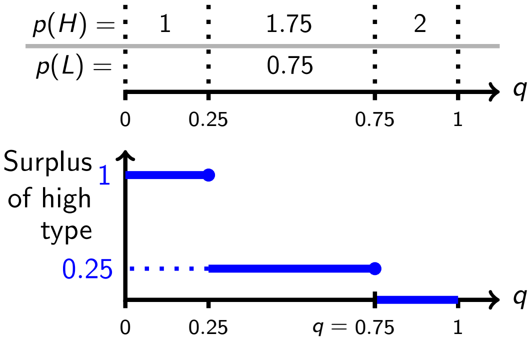

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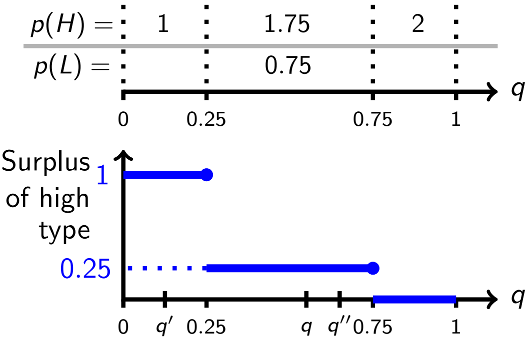

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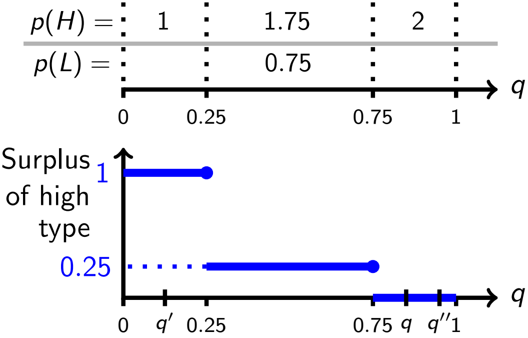

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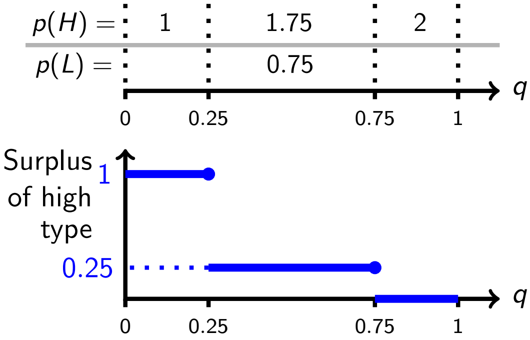

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Two types, any # of qualities (characterize optimal mechanisms)

- PI segmentation exists for all but finitely many inefficient markets

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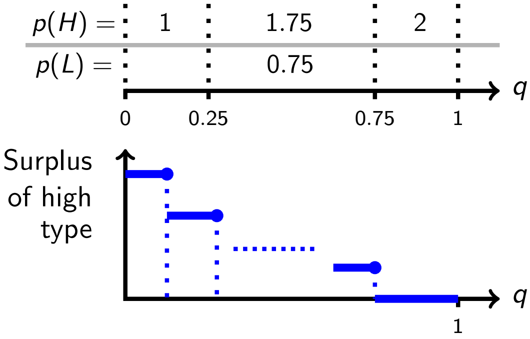

Model

- Types: $t \in T$ (finite)
- Alternatives: $a \in A$ (finite)
  - e.g., qualities, quantities, configurations
  - e.g., bundles: $\{1\}, \{2\}, \{1, 2\}$
- Valuations: $v(t, a)$
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Mechanism: $a : T \to \Delta(A)$, $p : T \to R$

- IC: $E[v(t, a(t))] - p(t) \geq E[v(t, a(t'))] - p(t')$
- IR: $E[v(t, a(t))] - p(t) \geq 0$

Markets and Segmentations

Market $f \in \Delta(T)$

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- $CS(t, f)$: surplus of type t in "the" optimal mechanism for market f
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($f' \succ_{CS} f$ if 1 and 2)

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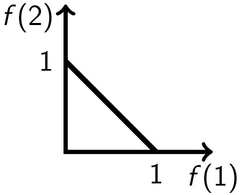

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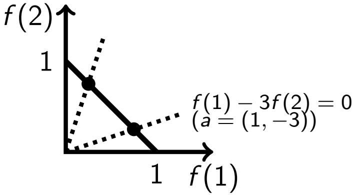

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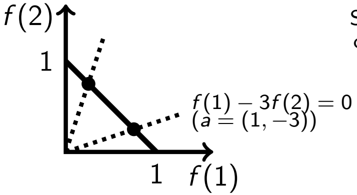

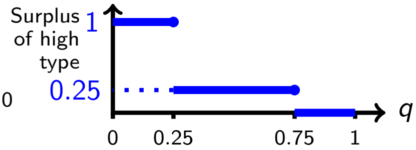

Proof Outline

Given inefficient market f

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Define f_2 s.t. $f = \epsilon f_1 + (1 - \epsilon) f_2$

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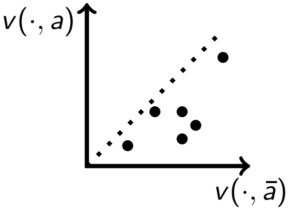

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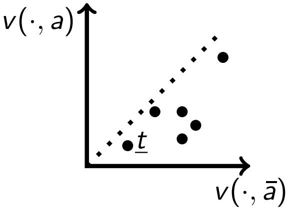

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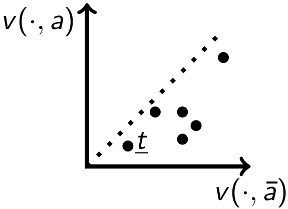
   - optimal to sell $\bar{a}$ at price $v(\underline{t}, \bar{a})$



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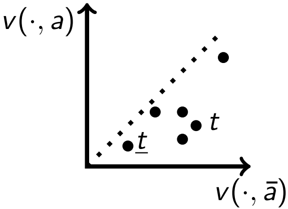

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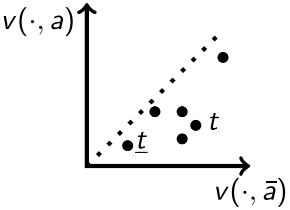
Intuition: allocation of $\underline{t}$ distorted to extract rents from t



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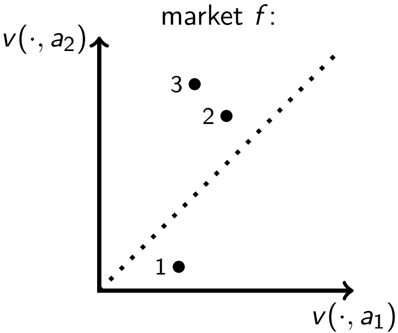

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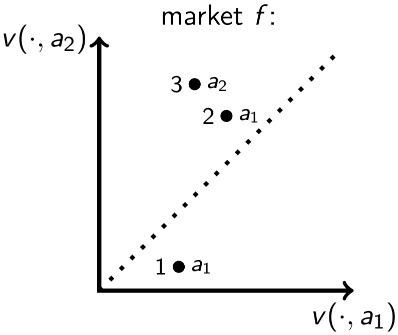

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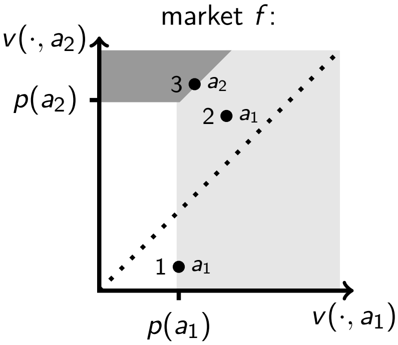

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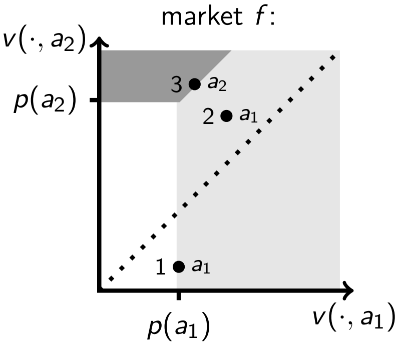

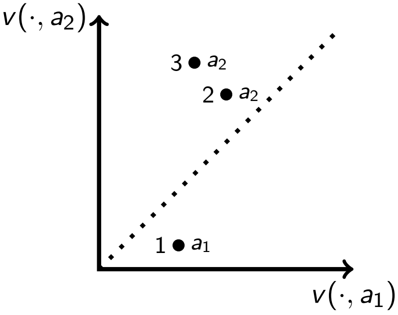

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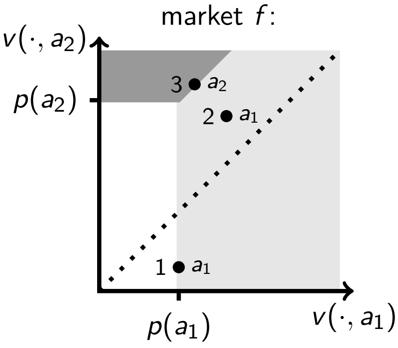

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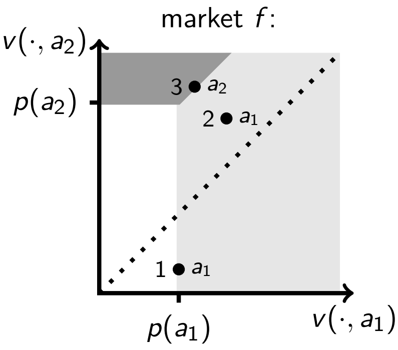

In addition to types 2 and 3

- include all other types in the "chain of information rents"

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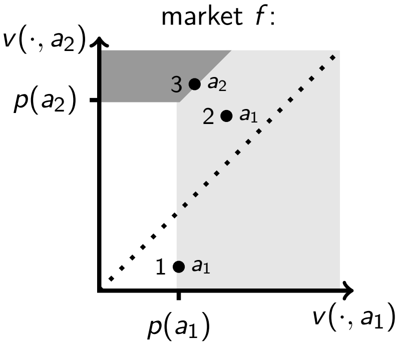

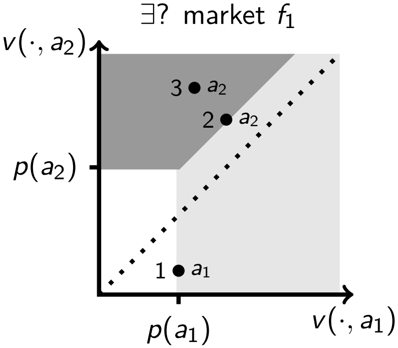

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Final step: ensure mechanism is optimal for some market f_1

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$f = (1 - \epsilon) f_1 + \epsilon f_2$

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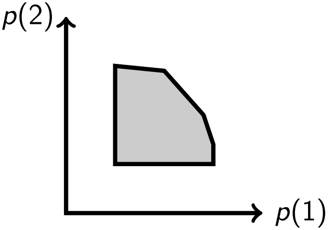

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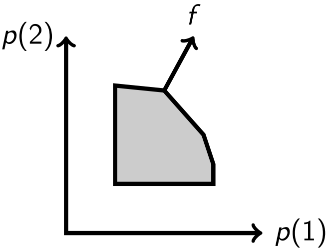

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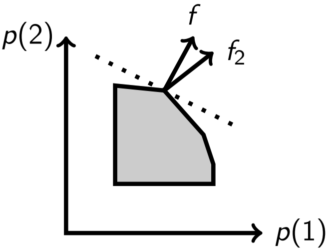

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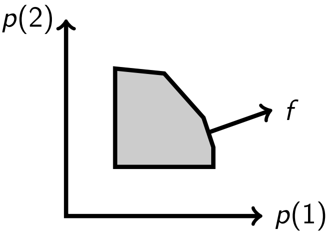

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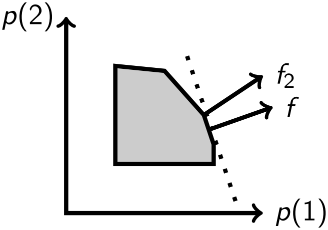

Recall Proof Outline

Given inefficient market f

1. $\exists f_1$ s.t. $f_1 \succ_{CS} f$ ✓
   - $\forall t \in Supp(f_1) : CS(t, f') \geq CS(t, f), (\exists t, >)$

Define f_2 s.t. $f = \epsilon f_1 + (1 - \epsilon) f_2$

2. Small ϵ, generic f: OptMech(f) = OptMech(f_2) ✓

Conclusions

There is scope for intervention:

- PI segmentations generically exist
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Thanks!