Optimal Auctions for Correlated Buyers with Sampling

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    2. Characterize number of samples required
- Idea:
  - Samples as randomization device, not learning

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1 **item,** n **buyers** (quasilinear, finite type space)

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Special Case: $\mathcal{F} = \{f\}$ is CM model (samples useless)

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Implications:

- $2 \leq rank(\mathcal{F}) \leq m$: At most $m - 1$ samples; At least 1 sample
- With linearly independent distributions, 1 sample is enough
  - With 2 distributions, 1 sample is enough

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CM Auction:

- **Second price & side payments**

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For each bidder i, find $P(v_{-i})$ such that

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Full surplus extraction if rows of conditional probability matrix linearly independent.

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Compare: only 1 sample the more sophisticated way

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To prove Theorem 2: Conditions of Theorem 2 $\Rightarrow$ independent rows

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$$
\begin{array}{c} \\ (0,H) \\ (1,H) \\ (0,L) \\ (1,L) \end{array}
\begin{array}{c} \begin{array}{cccc} (0,M) & (1,M) & (0,N) & (1,N) \end{array} \\
\begin{pmatrix} \frac{9}{16} & \frac{3}{16} & \frac{3}{16} & \frac{1}{16} \\ \frac{3}{16} & \frac{9}{16} & \frac{1}{16} & \frac{3}{16} \\ \frac{1}{16} & \frac{3}{16} & \frac{3}{16} & \frac{9}{16} \\ \frac{3}{16} & \frac{1}{16} & \frac{9}{16} & \frac{3}{16} \end{pmatrix} \end{array}
\cdot \begin{pmatrix} P(0,M) \\ P(1,M) \\ P(0,N) \\ P(1,N) \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{4} \\ 0 \\ \frac{3}{4} \end{pmatrix}
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ex-post IR

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