Selling to a Group

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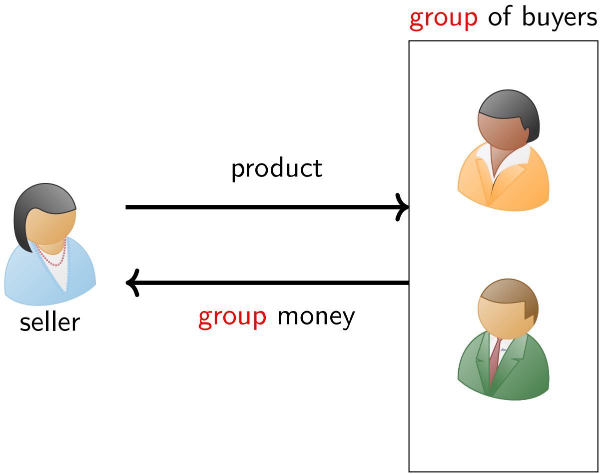
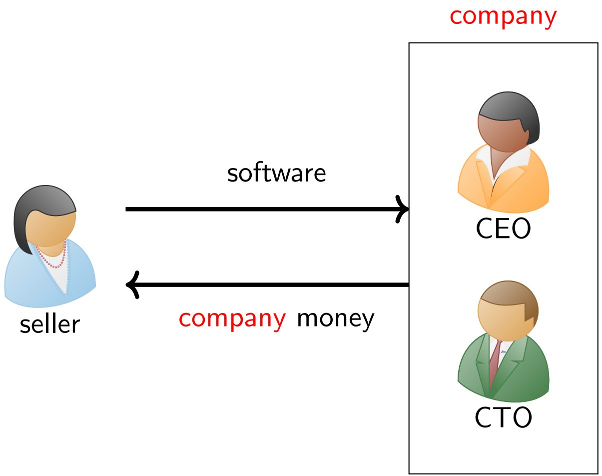
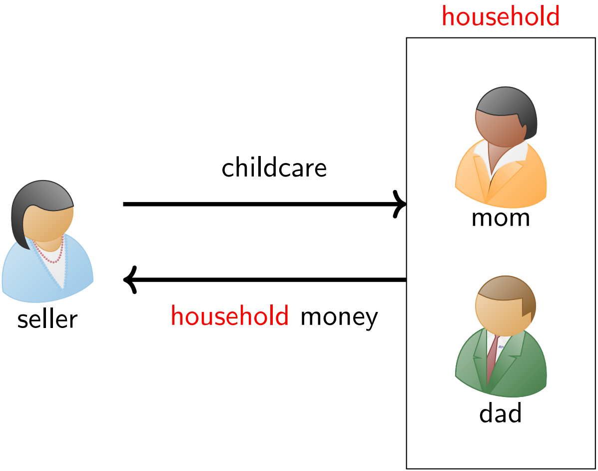
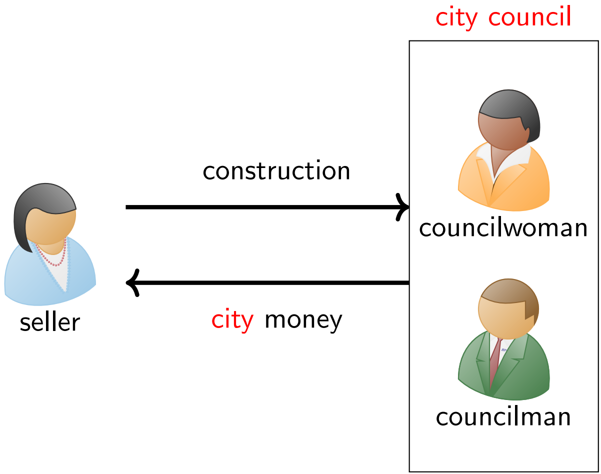
with Aditya Kuvalekar (Essex) and Elliot Lipnowski (Columbia)

ACM EC 2021

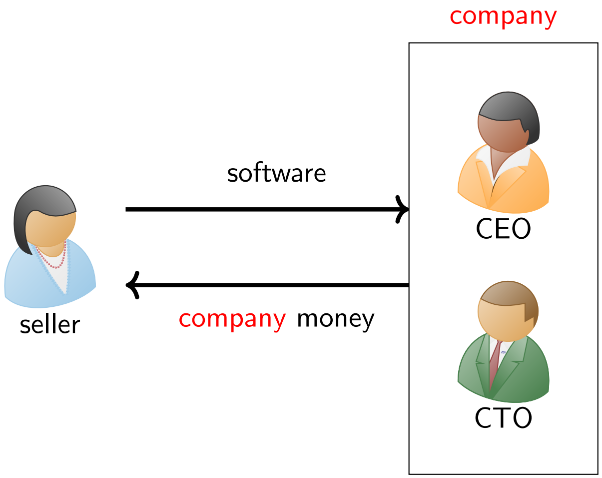

group of buyers
product
seller
group money

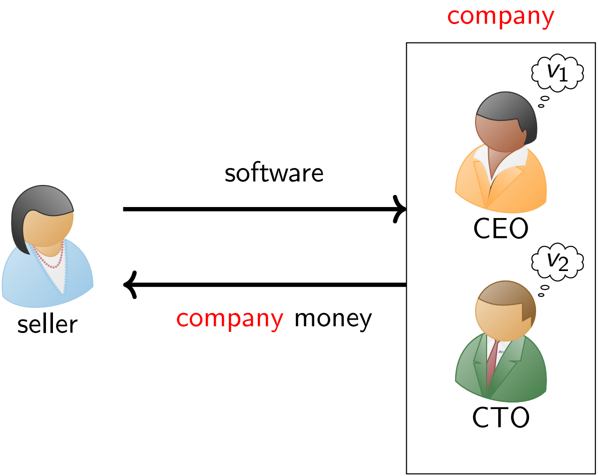

company
software
seller
CEO
CTO
company money

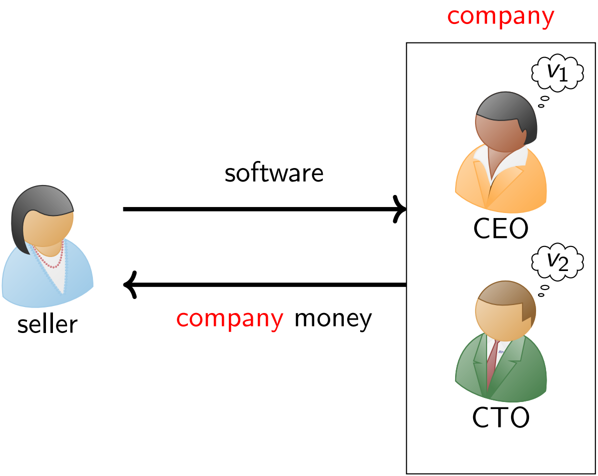

household
childcare
mom
seller
household money
dad



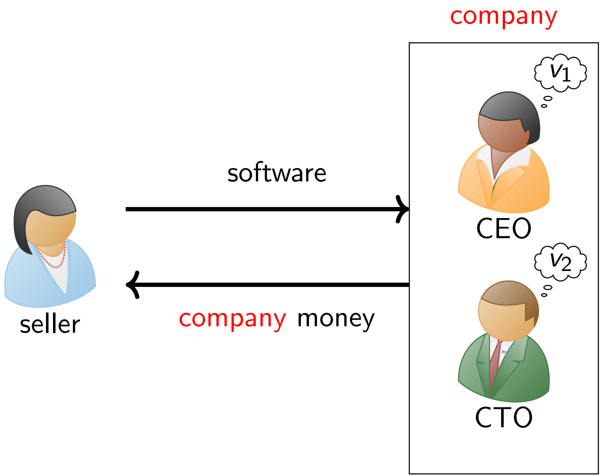
city council
construction
councilwoman
seller
city money
councilman


company
software
CEO
seller
company money
CTO

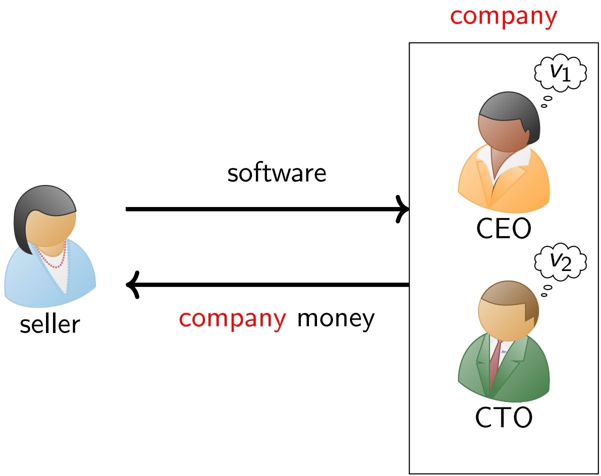

company
v_1
software
CEO
v_2
seller
company money
CTO



company pays v_1 for software $\approx$ company doesn't buy software

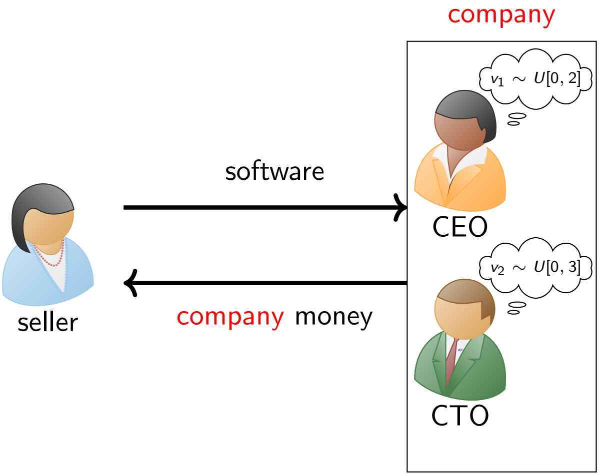


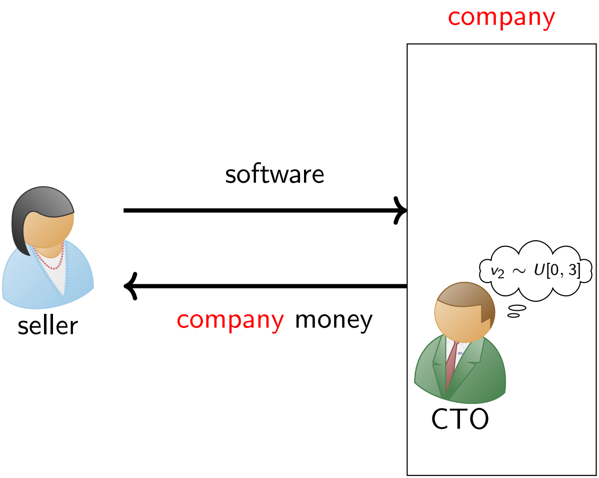
What mechanism is optimal (maximizes seller's revenue)?

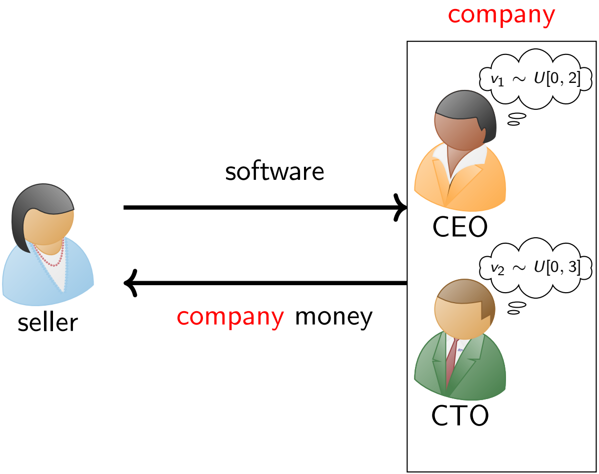


What mechanism is optimal (maximizes seller's revenue)?

(s.t. Incentive compatibility & Individual rationality)


company
$v_1 \sim U[0, 2]$
software
CEO
$v_2 \sim U[0, 3]$
seller
company money
CTO


company
software
$v_2 \sim U[0, 3]$
seller
company money
CTO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software
CEO
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company money
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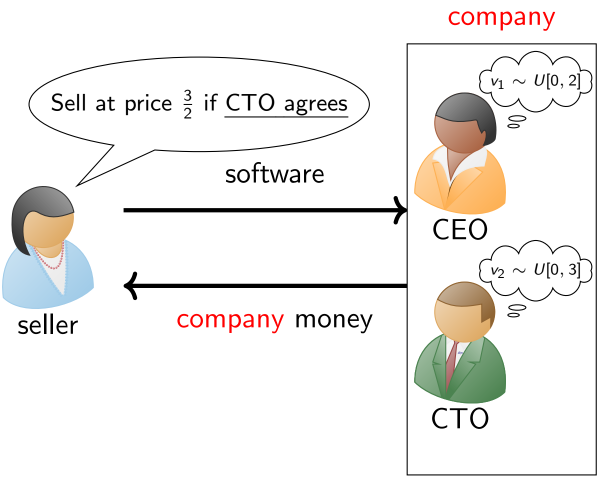
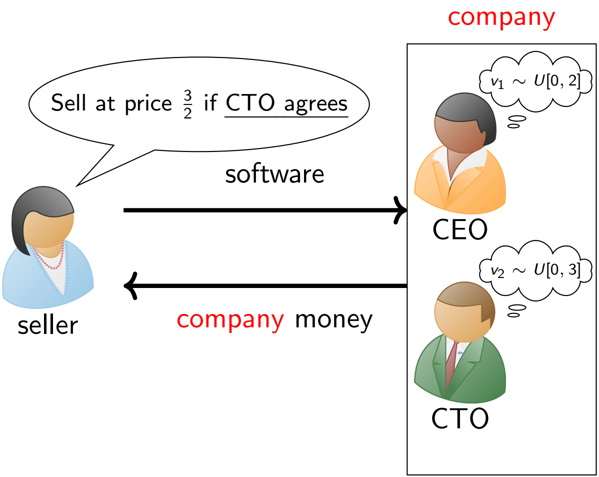
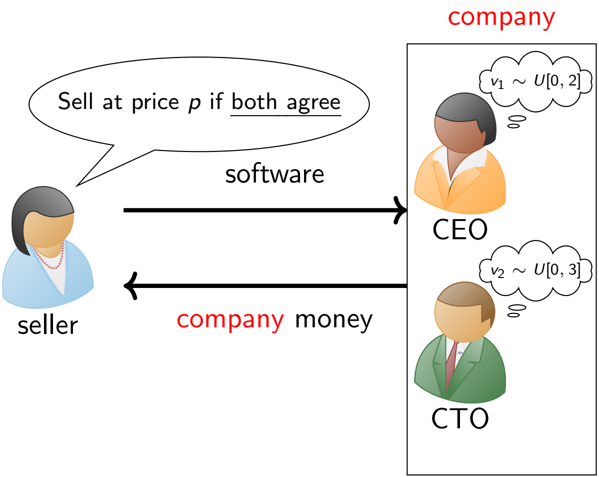
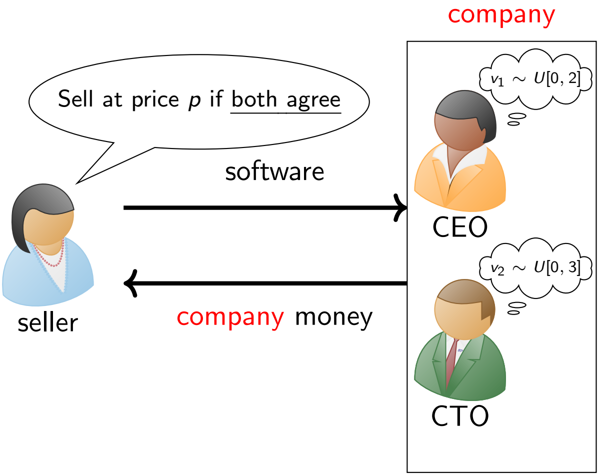

company
Sell at price $\frac{3}{2}$ if CTO agrees
software
$v_1 \sim U[0,2]$
CEO
$v_2 \sim U[0,3]$
CTO
seller
company money

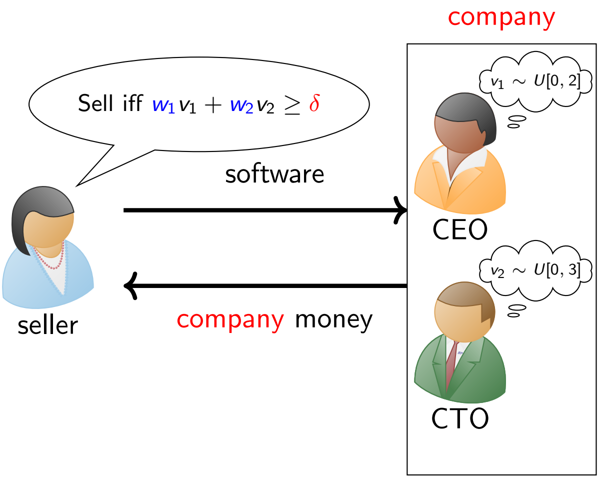


Not IR: $v_1 = 0$ then Utility of CEO $= -\frac{3}{2} \cdot \frac{1}{2}$


company
Sell at price p if both agree
software
$v_1 \sim U[0,2]$
CEO
$v_2 \sim U[0,3]$
CTO
seller
company money



$\max_p p\mathbb{P}[v_1 \geq p]\mathbb{P}[v_2 \geq p] \approx 0.35$ at $p \approx 0.78$



$$w_1 = \sqrt{\frac{3}{7}},\ w_2 = 1 - w_1,\ \delta = 1 + \frac{w_2}{2}$$

Theorem

The following mechanism is optimal:

1. *Allocation maximizes weighted sum of virtual values*

2. *Weights minimize weighted virtual surplus*

3. *Transfer rule is "defined appropriately"*

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$$Allocate \Leftrightarrow \sum_i w_i^* \phi_i(v_i) \geq 0$$
$$where\ \phi_i(v_i) = v_i - \frac{1-F_i(v_i)}{f_i(v_i)}$$

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$$w^* \in \arg\min_w \mathbb{E}[\max(\sum_i w_i \phi_i(v_i)), 0)]$$

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so that payment identity is satisfied

Proof step 1: revenue as virtual surplus

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Lemma

Over all mechanisms with interim allocations $X_1, \ldots, X_n$

$$\textit{optimal revenue} \;= \min_i \mathbb{E}[X_i(v_i)\phi_i(v_i)]$$

Proof step 1: revenue as virtual surplus

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Compare to if individual transfers were allowed

$$\text{optimal revenue} = \sum_i \mathbb{E}[X_i(v_i)\phi_i(v_i)]$$

Proof step 2: duality

$$\max_{\text{allocation}} \min_{i} \qquad \mathbb{E}[X_i(v_i)\phi_i(v_i)]$$

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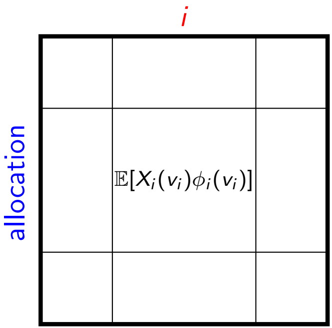

Proof step 2: duality

$$\max_{\text{allocation}} \min_{i} \quad \mathbb{E}[X_i(v_i)\phi_i(v_i)]$$

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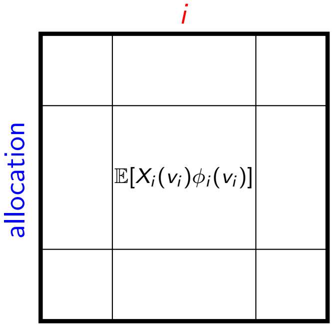

Proof step 2: duality

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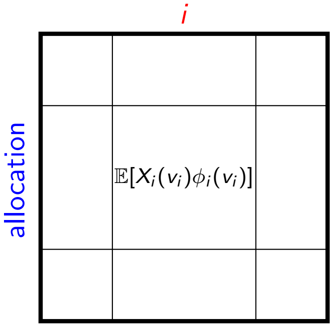

Which agent has a higher weight?

Which agent has a higher weight? the “weaker” one

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If ϕ_1 is smaller than ϕ_2 in the “hazard rate” order, then $w_1^* \geq w_2^*$.

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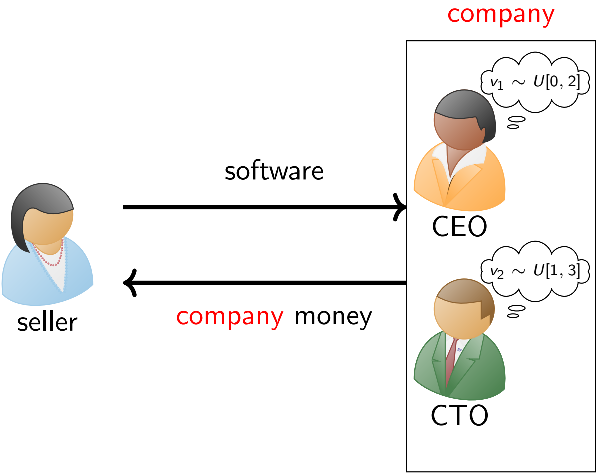

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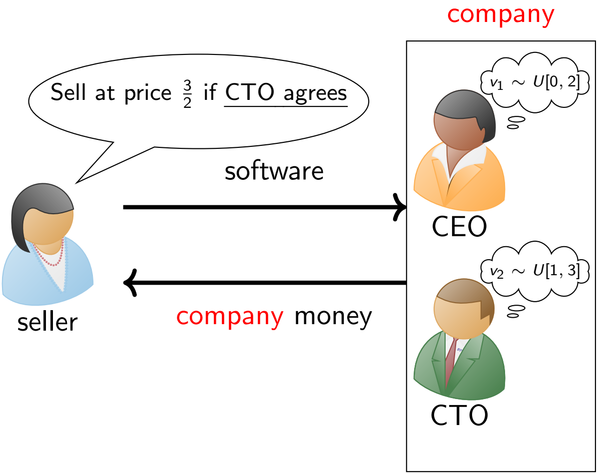

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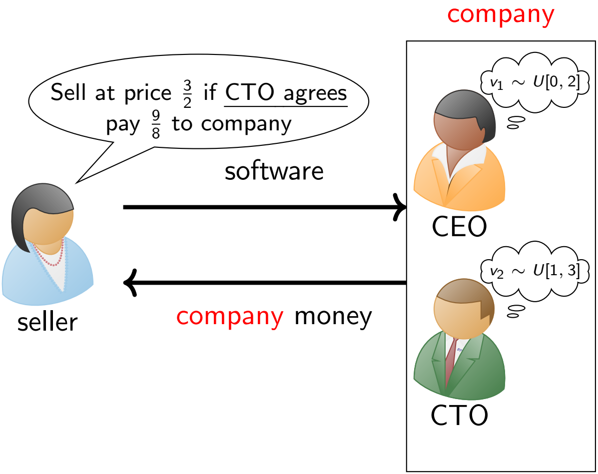

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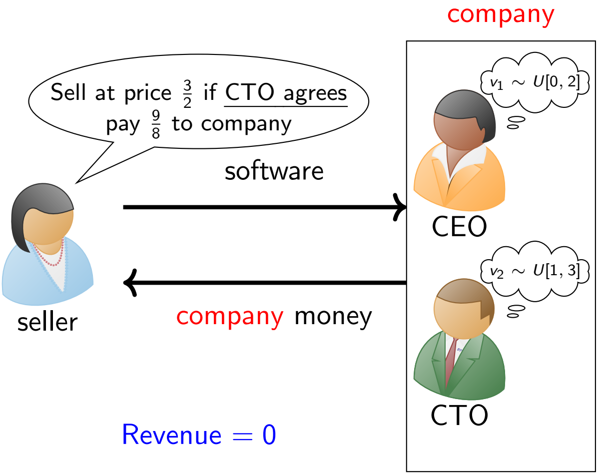

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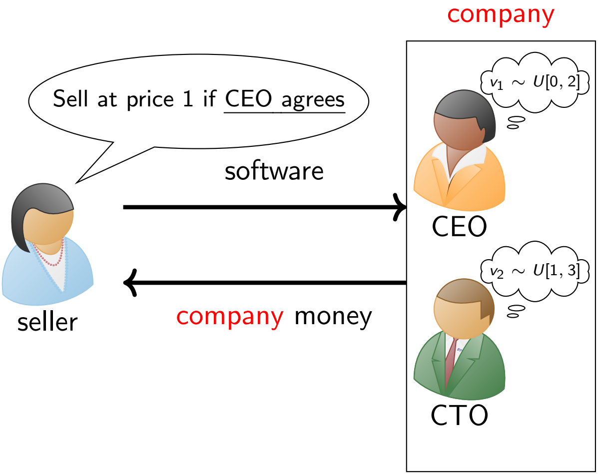

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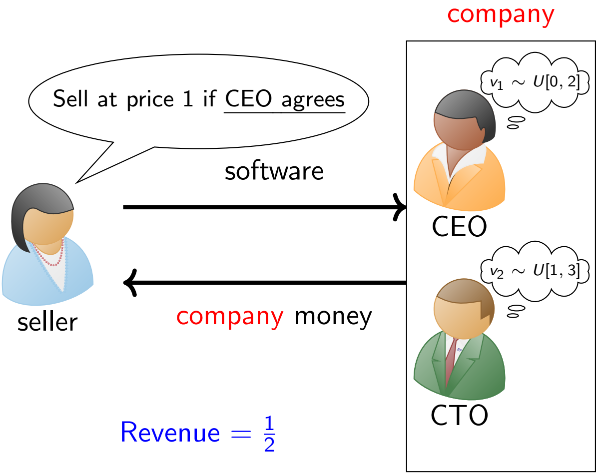

Revenue $= \frac{1}{2}$

Talk to us about these extensions and directions

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Posted pricing is not even approximately optimal

Pareto efficient mechanisms

What if only one agent has to agree to mechanism (or half of them)?

What if agents pay out of pocket, but in fixed shares?

Correlated/interdependent values?

Single seller, single product, “single” buyer

- Posting a price is not optimal

Single seller, single product, “single” buyer

- Posting a price is not optimal
- Pay more attention to “weaker” agents

Single seller, single product, “single” buyer

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