

MATH 035
Penn State University
Dr. James Sellers

Handout: An Introduction to Pythagoras and the Pythagorean Theorem

History of Pythagoras

Much of the history that follows is taken from Pythagoras' biography which is found at the MacTutor History of Mathematics website.

Pythagoras of Samos (~569BC - ~475BC), a contemporary of the Greek philosophers Thales and Anaximander, is sometimes described as the first pure mathematician. He is an extremely important figure in the development of mathematics yet we know relatively little about his mathematical achievements. Unlike many later Greek mathematicians, where at least we have some of the books which they wrote, we have nothing of Pythagoras's writings. The society which he led, half religious and half scientific, followed a code of secrecy which certainly means that today Pythagoras is a mysterious figure.

Pythagoras founded a philosophical and religious school in Croton (now Crotona, on the east of the heel of southern Italy) that had many followers. Pythagoras was the head of the society with an inner circle of followers known as mathematikoi. The mathematikoi lived permanently with the Society, had no personal possessions and were vegetarians. They were taught by Pythagoras himself and obeyed strict rules. The beliefs that Pythagoras held were:

- (1) *that at its deepest level, reality is mathematical in nature,*
- (2) *that philosophy can be used for spiritual purification,*
- (3) *that the soul can rise to union with the divine,*
- (4) *that certain symbols have a mystical significance, and*
- (5) *that all brothers of the order should observe strict loyalty and secrecy.*

Both men and women were permitted to become members of the Society, in fact several later women Pythagoreans became famous philosophers. The outer circle of the Society were known as the akousmatics and they lived in their own houses, only coming to the Society during the day. They were allowed their own possessions and were not required to be vegetarians.

Of Pythagoras's actual work nothing is known. His school practiced secrecy and communalism, making it hard to distinguish between the work of Pythagoras and that of his followers. Certainly his school made outstanding contributions to mathematics, and it is possible to be

fairly certain about some of Pythagoras's mathematical contributions. First we should be clear in what sense Pythagoras and the mathematikoi were studying mathematics.

Pythagoras was interested in the principles of mathematics, the concept of number, the concept of a triangle or other mathematical figures, and the abstract idea of a proof.

Pythagoras believed that all relations could be reduced to number relations. As Aristotle wrote:-

The Pythagorean ... having been brought up in the study of mathematics, thought that things are numbers ... and that the whole cosmos is a scale and a number.

To Pythagoras numbers had personalities which we hardly recognise as mathematics today:

Each number had its own personality - masculine or feminine, perfect or incomplete, beautiful or ugly. This feeling modern mathematics has deliberately eliminated, but we still find overtones of it in fiction and poetry. Ten was the very best number: it contained in itself the first four integers - one, two, three, and four [$1 + 2 + 3 + 4 = 10$] - and these written in dot notation formed a perfect triangle.

Of course today we particularly remember Pythagoras for his famous geometry theorem. Although the theorem, now known as Pythagoras's theorem, was known to the Babylonians 1000 years earlier he may have been the first to prove it. Proclus, the last major Greek philosopher, who lived around 450 AD wrote (see [\[7\]](#)):

After [Thales, etc.] Pythagoras transformed the study of geometry into a liberal education, examining the principles of the science from the beginning and probing the theorems in an immaterial and intellectual manner: he it was who discovered the theory of irrational and the construction of the cosmic figures.

The evidence is unclear as to when and where the death of Pythagoras occurred. Certainly the Pythagorean Society expanded rapidly after 500 BC, became political in nature and also spilt into a number of factions. In 460 BC the Society:

... was violently suppressed. Its meeting houses were everywhere sacked and burned; mention is made in particular of "the house of Milo" in Croton, where 50 or 60 Pythagoreans were surprised and slain. Those who survived took refuge at Thebes and other places.

The Pythagorean Theorem

In its purest form, the Pythagorean Theorem states that, if a and b are the lengths of the two “smaller” legs of a right triangle and c is the length of the hypotenuse (the leg across from the 90° angle), then $a^2 + b^2 = c^2$.

Definition: We will call a triple of numbers (a, b, c) a Pythagorean triple if $a^2 + b^2 = c^2$.

Example: The smallest Pythagorean triple is $(3,4,5)$ because $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. This is also the only Pythagorean triple which is made up of three consecutive integers.

Question: Is $(7, 11, 13)$ a Pythagorean triple?

Question: Is $(5, 12, 13)$ a Pythagorean triple?

There are two very natural questions to ask about this material at this time:

1. How does one prove the Pythagorean Theorem?
2. How many Pythagorean triples are there?

One cheap answer to Question 2 is

That leads us to the following concept:

Definition: A Pythagorean triple (a,b,c) is called primitive if the greatest common divisor of the numbers a , b , and c is 1.

So we can now modify Question 2 above as follows:

- 2': How many **primitive** Pythagorean triples are there?

We will answer Questions 1 and 2' in future lessons.