Magnetic Fields due to Currents

"Water, fire, air and dirt, [freaking] magnets, how do they work?"
- *Insane Clown Posse*

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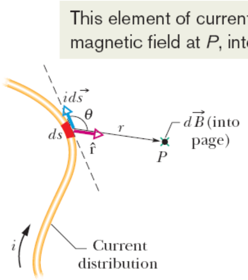
PHYS 212

Magnetic Field from a Current

Moving charges are affected by magnetic fields:

- $\vec{F}_B = q\vec{v} \times \vec{B}$
- But there is a symmetry to this force/field relationship
- Moving charges also *create* magnetic fields
- An empirical result:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2} \text{ (Biot-Savart Law)}$$

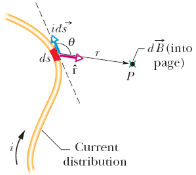


Magnetic Field from a Current

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

- μ_0: **permeability of free space** or **magnetic constant**
- $\mu_0 = 1.26 \times 10^{-6}$ m kg/s^{2}A^2
- r is distance between current and point of interest
- $\hat{r}$ points from current to point of interest
- $d\vec{s}$ points along current direction

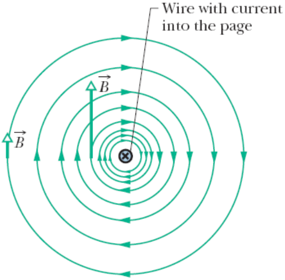
This element of current creates a magnetic field at *P*, into the page.

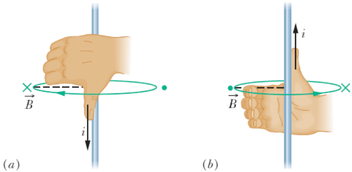


Magnetic Field from a Current

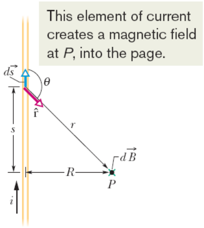
A long wire:

- What is the field from a long wire with current i?





Magnetic Field from a Current



Let's do the math:

- Start with Biot-Savart law and fill in the gaps:

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin(\theta)}{r^2}$$

- $r = \sqrt{R^2 + s^2}$
- $\sin(\theta) = \sin(\pi - \theta) = O/H = R/r = R/\sqrt{R^2 + s^2}$

$$dB = \frac{\mu_0}{4\pi}\frac{iR\,ds}{(s^2 + R^2)^{3/2}}$$

Magnetic Field from a Current

This field is from a small part of the wire:

$$dB = \frac{\mu_0}{4\pi} \frac{iR\,ds}{(s^2 + R^2)^{3/2}}$$

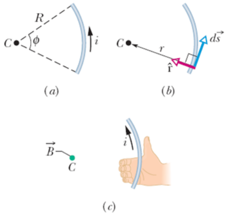
But *this* is the total magnetic field:

$$\begin{aligned} B &= \frac{\mu_0 iR}{4\pi} \int_{s=-\infty}^{s=\infty} \frac{ds}{(s^2 + R^2)^{3/2}} \\ &= \frac{\mu_0 i}{4\pi R} \left[\frac{s}{(s^2 + R^2)^{1/2}} \right]_{-\infty}^{\infty} \\ &= \frac{\mu_0 i}{4\pi R} [1 + 1] \\ &= \frac{\mu_0 i}{2\pi R} \end{aligned}$$

The direction is given by the right-hand-rule.

Magnetic Field from a Current

What about an arc of wire?

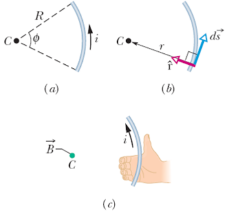


The total magnetic field:

$$
\begin{aligned}
B &= \frac{\mu_0}{4\pi}\int \frac{i\,ds\sin(\theta)}{r^2} \\
ds &= R\,d\phi \\
\sin(\theta) &= \sin(\pi/2) = 1 \\
R &= \text{constant} \\
i &= \text{constant}
\end{aligned}
$$

Magnetic Field from a Current

What about an arc of wire?

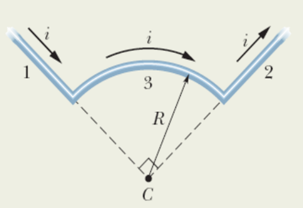


The total magnetic field:

$$\begin{aligned} B &= \frac{\mu_0 i}{4\pi}\int_0^{\phi}\frac{R\,d\phi\sin(\pi/2)}{R^2} \\ &= \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi \\ &= \frac{\mu_0 i\phi}{4\pi R} \end{aligned}$$

Magnetic Field from a Current

Lecture Question 12.1:
How does the result of the previous calculation change if we include current in sections 1 and 2 in the figure below?



(a) The magnetic field is larger.

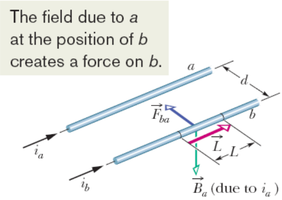
(b) The magnetic field is smaller.

(c) The magnetic field is the same.

(d) The magnetic field changes direction.

Force Between Currents

How do two currents affect each other?



- The first current creates a magnetic field:

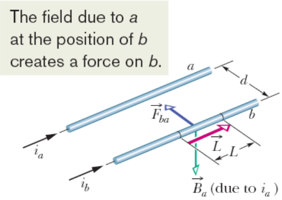
$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

- The second current feels a force from this magnetic field:

$$F_{ba} = i_b L B_a = \frac{\mu_0 L i_a i_b}{2\pi d}$$

Force Between Currents

How do two currents affect each other?



$$F_{ba} = i_b L B_a = \frac{\mu_0 L i_a i_b}{2\pi d}$$

- Parallel currents attract
- Antiparallel currents repel

Force Between Currents

Lecture Question 12.2:

Two parallel wires have currents that have the same direction, but differing magnitude. The current in wire A is i; and the current in wire B is 2i. Which one of the following statements concerning this situation is true?

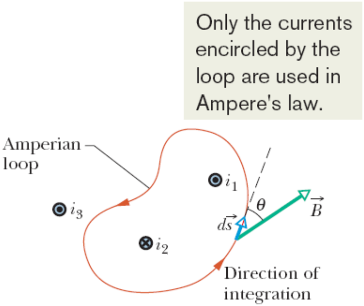
- (a) Wire A attracts wire B with **half** the force that wire B attracts wire A.
- (b) Wire A attracts wire B with **twice** the force that wire B attracts wire A.
- (c) Both wires attract each other with the **same** amount of force.
- (d) Wire A repels wire B with **half** the force that wire B attracts wire A.
- (e) Wire A repels wire B with **twice** the force that wire B attracts wire A.

Ampere's Law

Ampere's Law

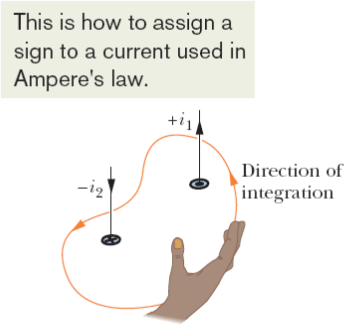
- Similar to Gauss' Law, we can find the "enclosed current"

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \tag{1}$$



Ampere's Law

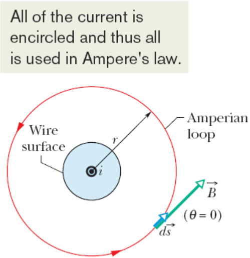
- The right hand rule tells us if the current is positive or negative



- Although *general*, this law is often applied to problems with symmetry

Ampere's Law

Long straight wire:



Let's apply Ampere's Law:

$$\oint \vec{B} \cdot d\vec{s} = \oint B\cos(\theta) ds$$

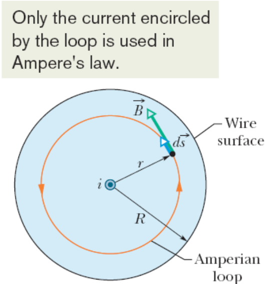
$$= B \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i_{enc}$$

$$B = \frac{\mu_0 i}{2\pi r}$$

Ampere's Law

Long thick wire:



Let's apply Ampere's Law again:

$$\oint \vec{B} \cdot d\vec{s} = \oint B\cos(\theta)ds$$

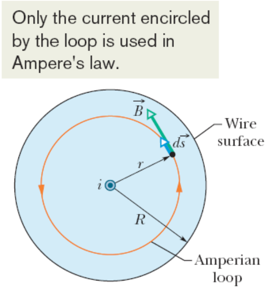
$$= B\oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i_{enc}$$

$$B = \frac{\mu_0 i_{enc}}{2\pi r}$$

Ampere's Law

Long thick wire:



But what is i_{enc}?

- For uniformly distributed current, we use a ratio:

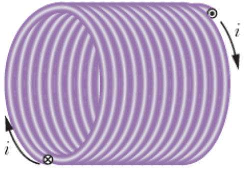
$$i_{enc} = i \times \frac{\pi r^2}{\pi R^2} = i(r/R)^2$$

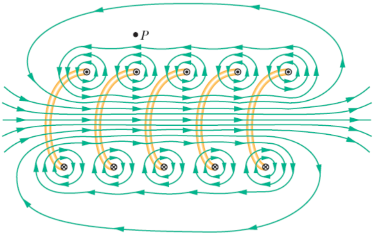
- Therefore,

$$B = \frac{\mu_0 i}{2\pi R^2} r$$

Solenoid and Toroid

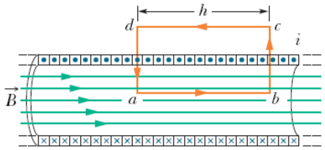
Solenoid:





Solenoid and Toroid

Infinite Solenoid:

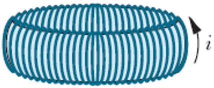


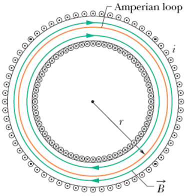
For this loop, there are four sections

$$
\begin{aligned}
\oint \vec{B} \cdot d\vec{s} &= \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s} \\
&= \int_a^b \vec{B} \cdot d\vec{s} = Bh \\
&= \mu_0 i_{enc} \\
&= \mu_0 N i \\
&= \mu_0 (nh) i \\
B &= \mu_0 n i \qquad \text{(ideal solenoid)}
\end{aligned}
$$

Solenoid and Toroid

Toroid:





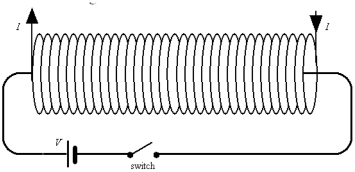
This loop is just a simple circle

$$\begin{aligned} \oint \vec{B} \cdot d\vec{s} &= B(2\pi r) \\ &= \mu_0 i_{enc} \\ &= \mu_0 N i \\ B &= \frac{\mu_0 N i}{2\pi} \frac{1}{r} \end{aligned}$$

Solenoid and Toroid

Lecture Question 12.4:

When the switch below is closed, what happens to the portion of the wire that runs inside of the solenoid?



(a) There is no effect on the wire.

(b) The wire is pushed downward.

(c) The wire is pushed upward.

(d) The wire is pushed toward the left.

(e) The wire is pushed toward the right.