Extracting an entanglement signature from only classical mutual information

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DARPA DSO InPho - W911NF-10-1-0404
ARO - W911NF-09-1-0385
NSF - PHY-0855701

Outline

- Shannon and von Neumann Entropy
- Mutual Information – *I* and *J*
- Summing *J* in three bases
- Results

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Shannon Entropy

- Shannon entropy is a measure of the uncertainty of a random variable
  - A random variable *A* with probability distribution *p(a)*
  - $H(A) = -\sum_{a \in A} p(a) \log p(a)$
- Measured in “bits” if log is base 2
- Evenly distributed probabilities gives higher entropy

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von Neumann Entropy

- von Neumann entropy is the quantum analog of Shannon entropy
  - A quantum state described by the density matrix ρ has von Neumann entropy
  - $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$
- Reduces to Shannon entropy upon projective measurements

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Mutual Information (1/4)

- Consider two random variables:
  - *A* with probability distribution *p(a)*
  - *B* with probability distribution *p(b)*
  - Joint probability: *p(a,b)*
- We can define the joint entropy:
  - $$H(A,B) = -\sum_{a\in A}\sum_{b\in B} p(a,b)\log p(a,b)$$
- And also the *conditional* entropy:
  - $$H(A|B) = -\sum_{a\in A, b\in B} p(a,b)\log\frac{p(a,b)}{\sum_{a\in A} p(a,b)}$$

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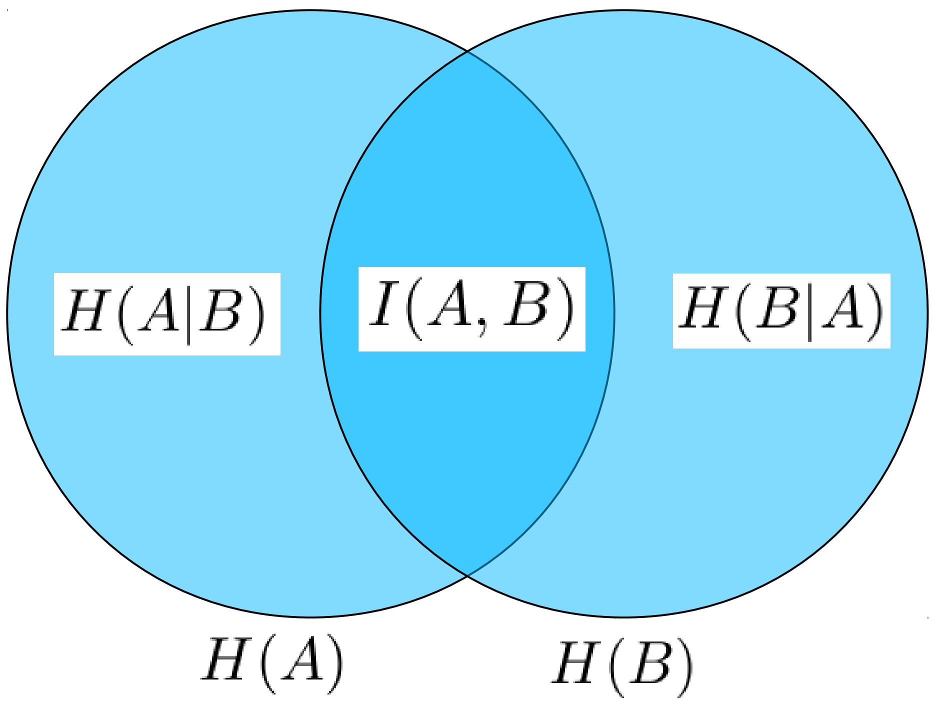
Mutual Information (2/4)

- Mutual information is a measure of how much information *A* has in common with *B*
  - $I(A,B) = H(A) + H(B) - H(A,B)$
- Pictorially:



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Mutual Information (3/4)

- Classically equivalent:
  - $J(A,B) = H(A) - H(A|B)$
- For a quantum state *ρ*:
  - $I(\rho) = S(\rho^A) + S(\rho^B) - S(\rho)$
  - *J*... is a little more complicated
    - *J* assumes knowledge after a measurement – but in what basis?
    - Assume subsystem *B* of *ρ* is projectively measured, then we have
      - $J(\rho)_{\{\Pi_b^B\}} := S(\rho^A) - S(\rho|\{\Pi_b^B\})$

Mutual Information (4/4)

- Where
  - $S(\rho|\{\Pi_b^B\}) = \sum_b p(b)S(\rho_b)$
  - $\rho_b = \frac{\Pi_b^B \rho \Pi_b^B}{\mathrm{Tr}[\rho \Pi_b^B]}$
- *I* and *J* differ in the quantum framework
- The minimized difference *I-J* is known as the *quantum discord*
- *J* represents the classical correlations in the system

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Summing *J* in three bases (1/2)

- Example:
  - *J* is maximal (1) for the singlet state in *any* basis
  - *J* is maximal for the maximally correlated mixed state in a singlet basis
- What if we sum *J* in three mutually unbiased bases? (e.g. HV, AD, RL)
  - $M_J = J(\rho)_{\{\Pi_b^B\}} + J(\rho)_{\{\Pi_{b'}^B\}} + J(\rho)_{\{\Pi_{b''}^B\}}$
  - $M_{J_C} = J_C(\rho)_{\{a,b\}} + J_C(\rho)_{\{a',b'\}} + J_C(\rho)_{\{a'',b''\}}$
  - These quantities have some interesting properties

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Summing *J* in three bases (2/2)

- They
  - are bounded by 1 for separable states (based upon simulations)
  - reach 3 for maximally entangled states
  - take fewer measurements than a CHSH type test
  - are a measure of how much information two parties can share in multiple bases

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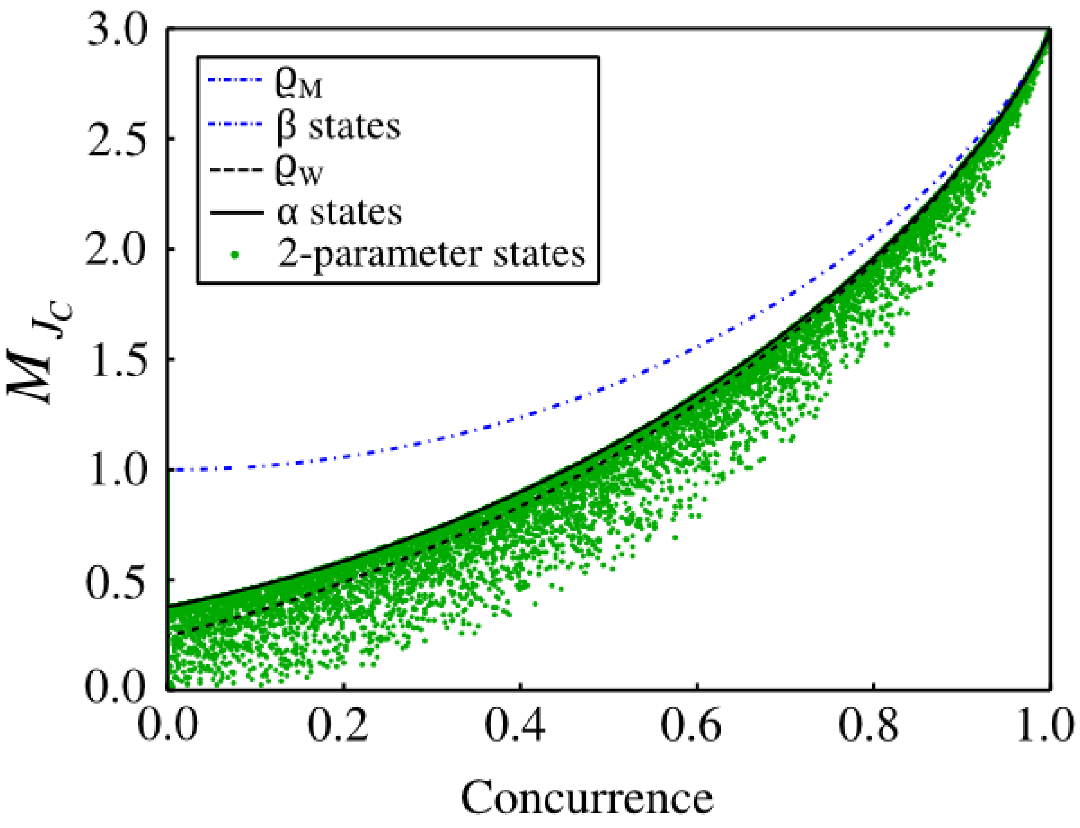
Results (1/2)

- Simulation



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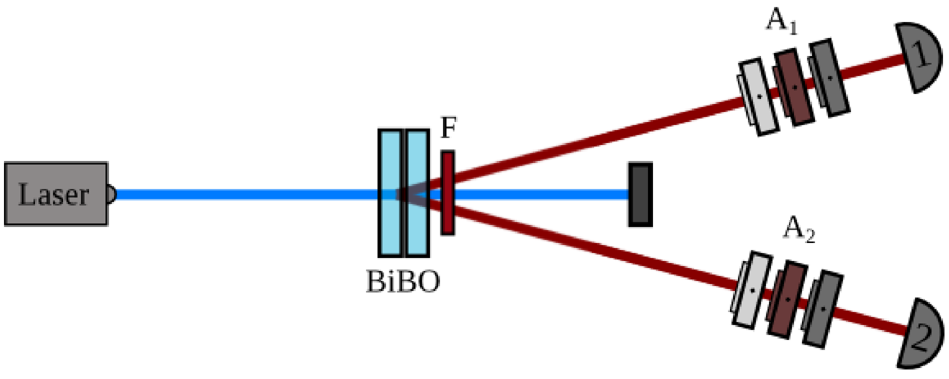
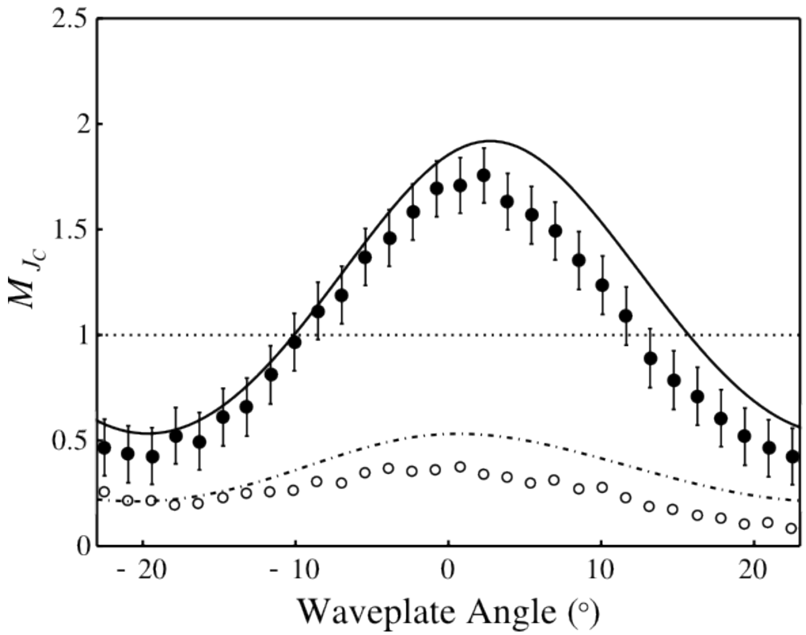
Results (2/2)

- Experimental Setup



- Singlet state (solid)
- Maximally correlated mixed state (hollow)



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Thanks for listening!

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- References

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