Differential Equations in Nuclear Fusion

Mark Wychock, Tyler Gregory,
Meagan Pandolfelli, Chris Moran

Advisors: Dr. Frantisek Marko
Dr. David J. Starling

Abstract

Nuclear fusion is a thermonuclear reaction in which two or more light nuclei collide together to form a larger nucleus, releasing a great amount of binding energy the in the process. Fusion and fission are natural processes that occur in stars. Fission is the process in which an unstable nucleus splits into two nuclei over a period of time or by induced fission of a neutron bombarding a radioactive atomic nucleus. In stars, it is understood that the fusion-fission process provides a near constant source of energy from proton-proton chain reactions. Although a fusion reaction generates more energy than a fission reaction, modern nuclear power plants utilize fission processes due to the stability of the fission reaction, convenience, and cost of production. If nuclear fusion could be produced in a commercial setting, it could provide 3-4 times the energy a fission reaction generates. Fusion material such as deuterium, a key component in thermonuclear reactions, can be distilled from seawater providing a virtually infinite and promising source of energy in the future. The understanding and development of thermonuclear reactions and reactors is accomplished by the aid of differential equations. Engineers and scientists are able to observe behaviors, such as mass conservation, hydrostatic equilibrium states, and energy generation, of induced nuclear reactions from differential equations when developing fusion reactors. Nuclear fusion reactors are undergoing development to replace obsolete nuclear fission reactors. Two potential types of fusion reactors in development are laser ignition and magnetic confinement. In the research project, the scientific and mathematical applications behind magnetic confinement will be explored since it is more economical and efficient than laser confinement. Specifically, we will examine how the Grad-Shafranov equation plays a pivotal role in the operation of tokamak and stellarator reactors.

Natural Fusion in Stars

At radius r in a static, spherically symmetric star:

Mass conservation

- $$\frac{dm}{dr} = 4\pi r^2 \rho$$

Hydrostatic equilibrium

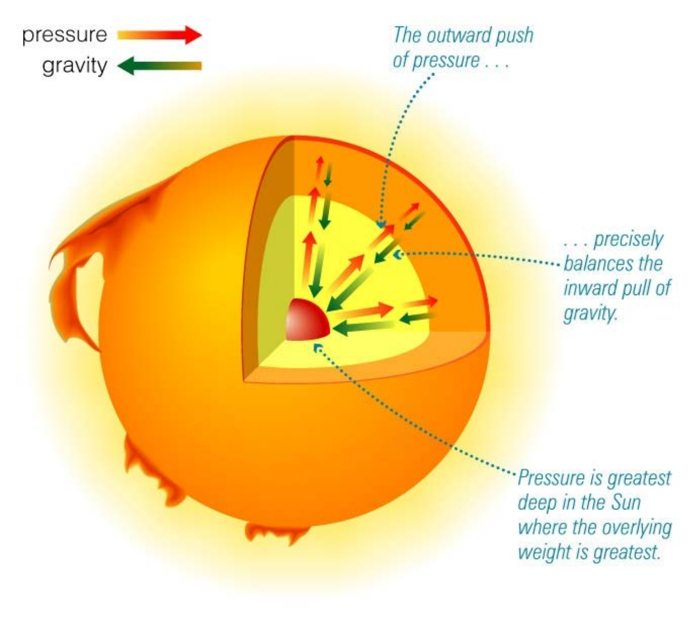
- $$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho$$

Energy transport due to radiation (only)

- $$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\kappa\rho}{T^3}\frac{L}{4\pi r^2}$$

Energy generation

- $$\frac{dL}{dr} = 4\pi r^2 \rho q$$



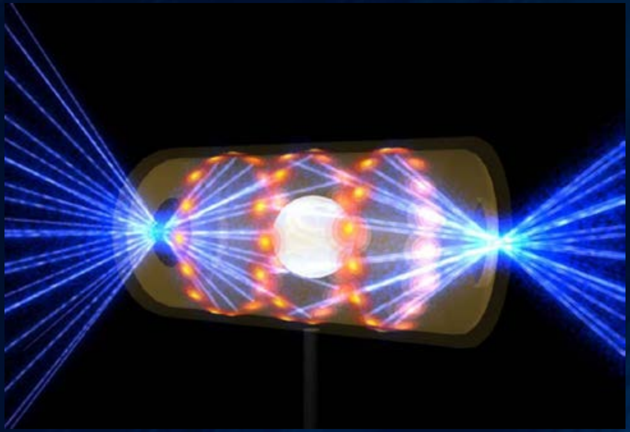
Source: http://lasp.colorado.edu/education/outerplanets/solsys_star.php

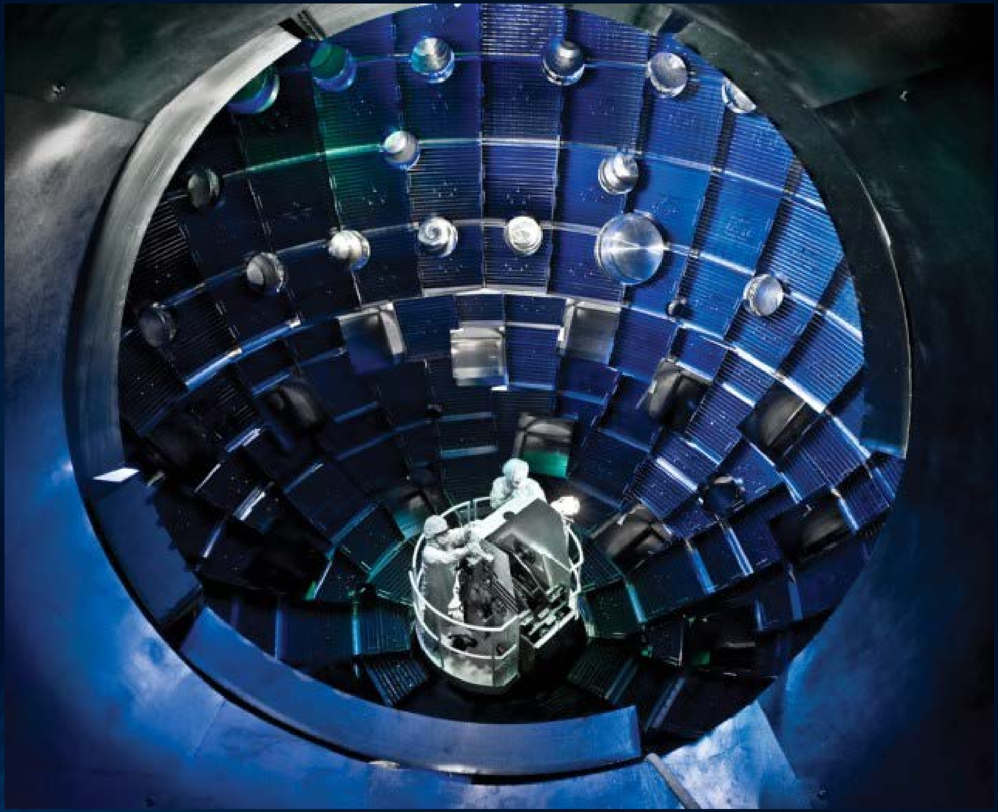
Where: m=enclosed mass; ρ=density; P=pressure; G=gravitational constant; T=temperature; κ=opacity; L=luminosity; a=radiation constant; c=speed of light in vacuum; and q=rate of energy generation per unit mass

Laser Ignition Reactors

- Lasers are fired at a pellet of deuterium-tritium, fusing it together.

Source: https://www.llnl.gov/str/pdfs/07_99.1.pdf



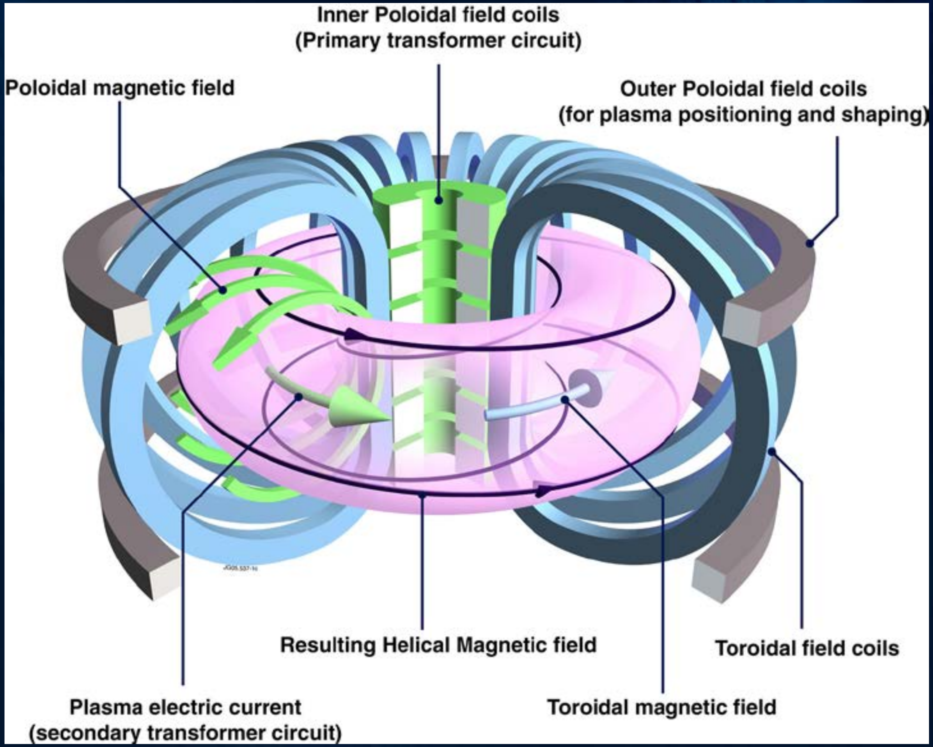


Source: http://monimega.com/blog/2013/10/10/us-fusion-lab-almost-breaks-even-takes-a-big-step-towards-clean-limitless-power/

Magnetic Confinement Reactors

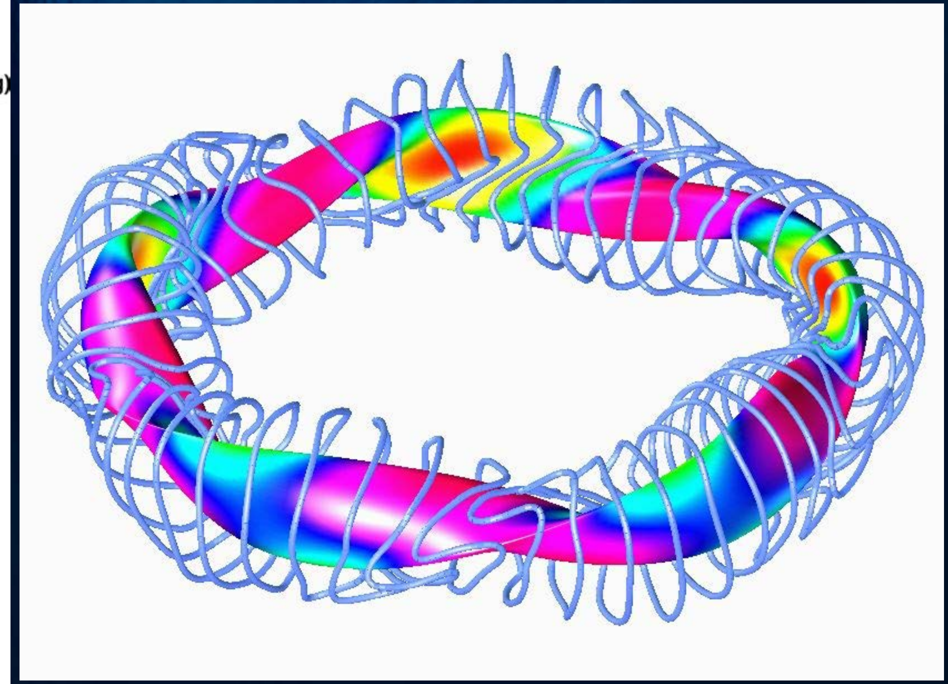
- Uses magnetic fields to confine hot fusion materials in the form of a plasma toroid.

Tokamak

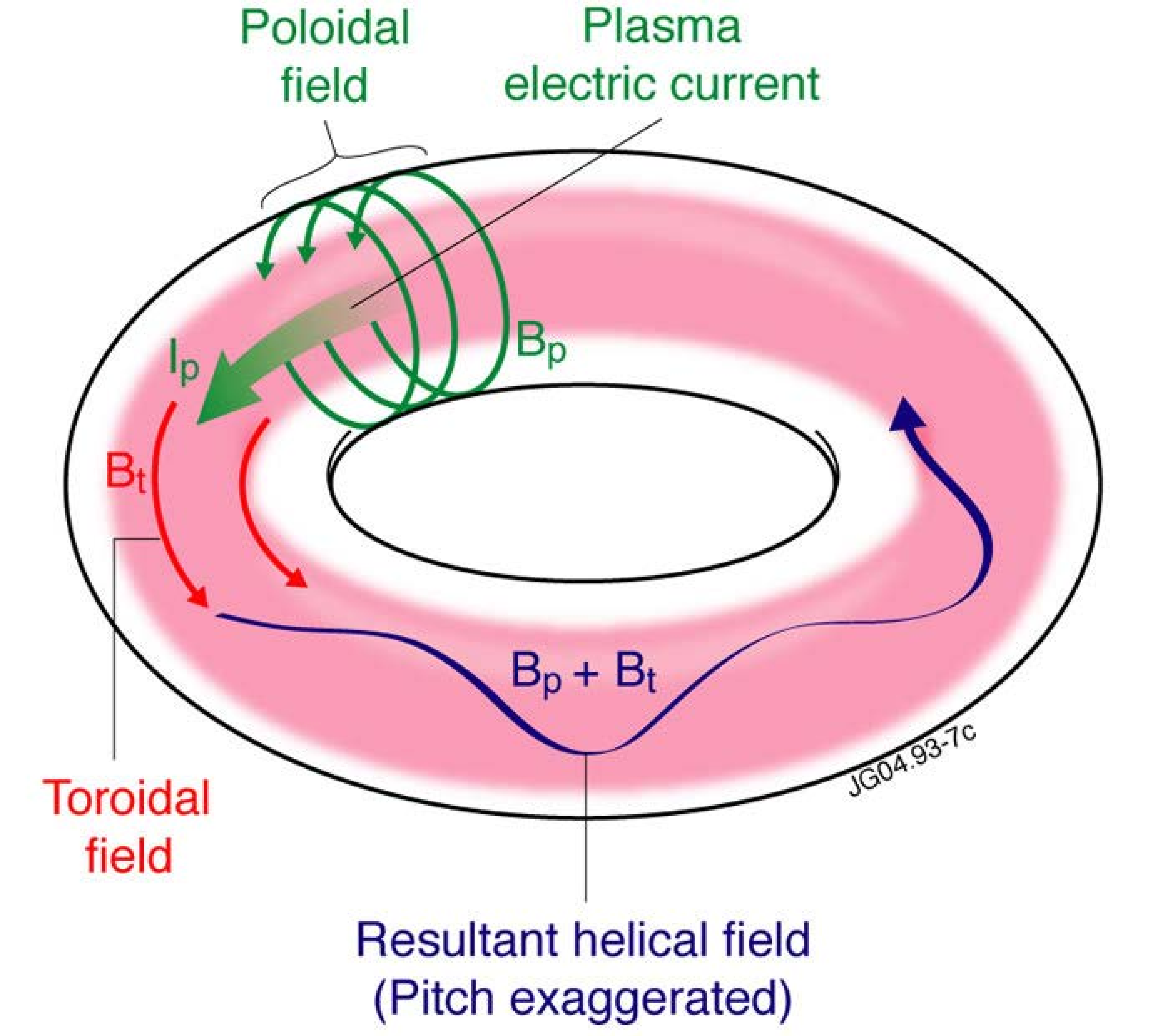


Source: http://www.efda.org/glossary/poloidal-field-coils/

Stellarator



Source: http://www.physics.ucla.edu/icnsp/Html/spong/spong.htm



Source: http://www.efda.org/fusion/focus-on/plasma-heating-current-drive/ohmic-heating/

The Grad-Shafranov Equation

$$\Delta * \psi = -\mu_0 R^2 \frac{dp}{d\psi} - \frac{1}{2}\frac{dF^2}{d\psi}$$

$$\Delta * \psi = R^2 \vec{\nabla} \cdot \left(\frac{1}{R^2} \vec{\nabla} \psi \right) = R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \psi}{\partial R} \right) + \frac{\partial^2 \psi}{\partial Z^2}$$

$F(\psi) = RB_\phi$

$$\vec{B} = \frac{1}{R} \nabla \psi \times \hat{e}_\phi + \frac{F}{R} \hat{e}_\phi$$

$\vec{B}$ = magnetic field

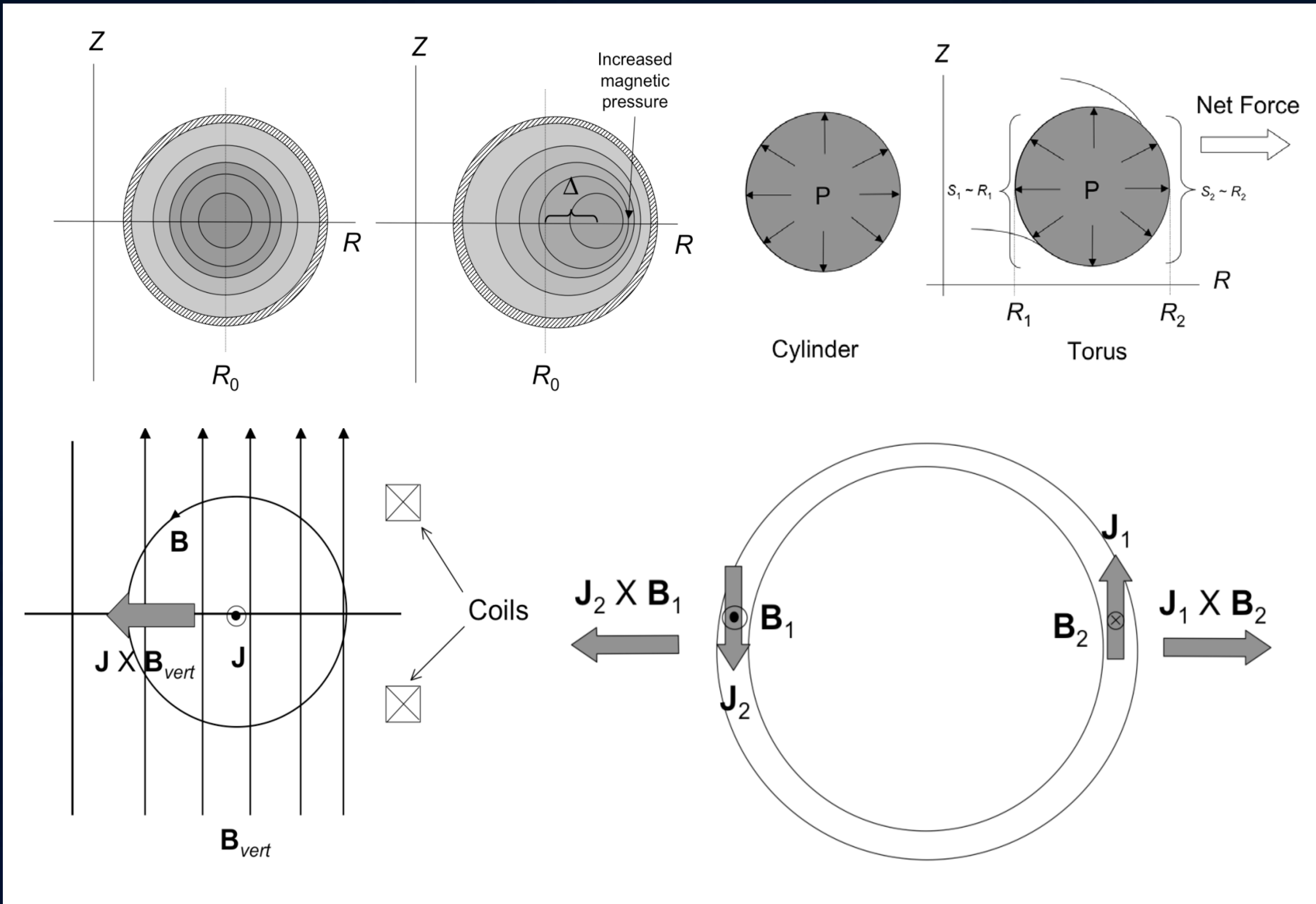
$$\mu_0 \vec{J} = \frac{1}{R} \frac{dF}{d\psi} \nabla \psi \times \hat{e}_\phi - \frac{1}{R} \Delta * \psi \hat{e}_\phi$$

$\mu_0 \vec{J}$ = current

ψ = flux

μ_0 = magnetic permeability constant

$p(\psi)$ = pressure



Source: http://www.physics.wisc.edu/grads/courses/726-f07/files/Section_18_Toroidal_Eq_02.pdf

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