Solutions to graded exercises in Homework #3
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January 25, 2011

These exercises are from §§ 1.3 and 1.4 of the textbook.

§1.3 Ex. 24. (a) True.
(b) True. \( v + (u - v) = u \).
(c) False. The weights in a linear combination can be any scalars, including 0.
(d) True.
(e) True.

§1.3 Ex. 26. (a) The augmented matrix \([ A, b ]\) is
\[
\begin{bmatrix}
2 & 0 & 6 & 10 \\
-1 & 8 & 5 & 3 \\
1 & -2 & 1 & 3
\end{bmatrix}
\]
which easily reduces to a (non-unique) row echelon form, for instance
\[
\begin{bmatrix}
2 & 0 & 6 & 10 \\
0 & 8 & 8 & 8 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
In this way we see that the pivot columns are columns 1 and 2. Since the rightmost column is not a pivot column, it follows that the system \( Ax = b \) has at least one solution. In other words, \( b \) is a linear combination of the columns of \( A \).
(b) Trivially the third column (or any column) of \( A \) is a linear combination of the columns of \( A \). (This holds for any matrix.)

§1.3 Ex. 29. The center of mass is \((1.3, .9, 0)\). The student must show the work to obtain this vector as a weighted average of the given vectors.
§1.4 Ex. 10. The vector equation is
\[
\begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} x_2 = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}
\]
and the matrix equation is
\[
\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}.
\]

§1.4 Ex. 17. The student should put $A$ into row echelon form in order to find the pivot positions. After doing this, we see that the pivot positions are: row 1 column 1, row 2 column 2, and row 3 column 4. Since row 4 does not contain a pivot position, it follows by Theorem 4 in §1.4 that the system $A\mathbf{x} = \mathbf{b}$ is not consistent for all $\mathbf{b}$. 