Profit sharing and monitoring in partnerships

Steven Huddart\textsuperscript{a,}\textsuperscript{*}, Pierre Jinghong Liang\textsuperscript{b}

\textsuperscript{a\textit{Smeal College of Business, Pennsylvania State University, University Park, PA 16802-1912, USA}}
\textsuperscript{b\textit{Tepper School of Business, Carnegie Mellon University, Pittsburgh, PA 15213, USA}}

Available online 26 August 2005

Abstract

We consider partnerships among risk-averse professionals endowed with (i) a risky and personally-costly production technology and (ii) a personally-costly monitoring technology providing contractible noisy signals about partners’ productive efforts. Partners shirk both production and monitoring tasks because efforts are unobservable. We characterize optimal partnership size, profit shares and incentive payments when every partner performs the same tasks, and show that medium-sized partnerships are dominated by either smaller or larger partnerships. Prohibiting some partners from monitoring increases the incentives for others to monitor. We illustrate how task assignments and incentives interact, leading to improvements in partner welfare.

\textsuperscript{r}2005 Elsevier B.V. All rights reserved.

\textit{JEL classification:} C72; L25; M52

\textit{Keywords:} Incentive contracting; Monitoring; Risk aversion; Syndicates

---

\textsuperscript{☆} We thank seminar participants at the 2003 AAA meetings, Carnegie Mellon University, Columbia University, McMaster University, the 2004 Stanford Summer Camp, the Stockholm School of Economics, SUNY–Buffalo, the University of Pittsburgh, Yale University, and the UNC/Duke and Chicago/Minnesota accounting conferences for helpful discussions. We particularly thank Ulf Axelson, Stan Baiman (the referee), Qi Chen, Paul Fischer, Jennifer Francis, Jon Glover, Jack Hughes, Yuji Ijiri, John O’Brien, Mehmet Ozbilgin, Madhav Rajan, Korok Ray, Stefan Reichelstein, Jerry Zimmerman (the editor). Steven Huddart acknowledges financial support from the Smeal College Competitive Research Fund.

\textsuperscript{*}Corresponding author. Tel.: +1 814 865 3271; fax: +1 814 863 8393.

\textit{E-mail address:} huddart@psu.edu (S. Huddart).

0165-4101/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
1. Introduction

We investigate the optimal assignment of profit shares and tasks across partners in partnerships. Partnerships are composed of risk-averse individuals endowed with capacities to produce (i) output and (ii) signals that provide information about effort undertaken to produce output. Our modeling choices are guided by the tensions that we perceive exist in professional partnerships such as law, consulting, and public accounting firms. These organizations differ in important ways from corporations, which have been the subject of many principal-agent models. Unlike the usual set-up where the principal owns the productive technology operated or controlled by the agent, every partner is simultaneously an owner (who shares in the net output of the partnership) and an agent (who produces output). Ownership and control do not reside in separate persons; they are diffused among many persons. Because partners are subject to moral hazard, awarding each partner a fixed share of firm output invites undersupply of effort. This is unavoidable if aggregate firm output is the only contractible variable. In this case, reducing the size of the partnership reduces the opportunity to free ride at the cost of forgoing opportunities to share risk or capture other synergies.

Monitoring and associated incentive contracts are another way to combat shirking. In our paper, monitoring is represented as an activity that is personally costly to individual partners and which produces informative signals of partners’ efforts. The benefits of monitoring are twofold. First, for a partnership of a given size, the total certainty equivalent shared by the partners’ increases because of the improved incentives created by the signal-contingent contracts. Second, improved incentives (due to monitoring) may allow the partnership to grow in size so as to achieve better risk sharing or other synergies. This follows because the signals complement the incentive provided by a partner’s share of output. Without signals, incentives to exert effort can only be high if the number of partners is small. When monitoring is personally costly to the partner who undertakes monitoring but the benefits of monitoring are enjoyed by all partners, each individual partner shirks the monitoring task. We show how partnership size and optimal linear profit sharing

---


2Under-supply of effort in professional partnerships can take many forms. For example, partners may not be active in seeking new business, collecting fees, or training staff. Shirking can also take the form of professional negligence. In accounting firms, partners may be too accommodating to client wishes for aggressive or misleading accounting treatments. In law firms, partners may provide opinion letters that lack objectivity.
contracts depend on the extent to which monitoring activities themselves are subject to free riding.

This is an important organizational issue for professional services firms, especially public accounting firms. In 60 years, the largest accounting firms have grown from having a few dozen partners to a few thousand.\(^3\) In smaller accounting firms, a key partner is able to monitor partner performance. Wyatt (2003, p. 22) suggests that a similar degree of effective monitoring is difficult to attain in the four largest public accounting firms. In the wake of Arthur Andersen’s collapse, he suggests that these firms (or the entities that regulate them) consider whether splitting up the firm would enhance quality control in audit performance. In a like vein, Paul Volcker, a member of the Conference Board’s blue-ribbon Commission on Public Trust and Private Enterprise has questioned whether a “huge financial conglomerate is the right business model for firms providing professional audit work”.\(^4\)

To preserve a balanced budget (i.e., the output is divided among the partners with none discarded and no additional resources injected from some other source), any penalty exacted from (or reward paid to) one group must be distributed to (or funded by) other members of the partnership outside the group. Miller (1997) calls the group that collects the penalty (or pays the reward) the “sink.” In the usual setting with a risk-neutral principal, the principal serves as the “sink” who costlessly bears the risk associated with any risky compensation payment schedule. Since we consider a setting without a risk-neutral principal, imposing a risky incentive contract on one partner entails distributing the risk associated with that payment schedule across the other partners of the firm. The risk imposed by signal-contingent contracts can be reduced either by making the signal more precise or by reducing the change in the payment associated with a change in the signal.

In our model, partners commit to contract parameters that induce unobservable productive and monitoring effort choices by each partner. The optimal choice of contract parameters requires four effects to be balanced: (i) the personal cost of producing the signal, (ii) the output attributable to the signal-contingent compensation, (iii) the risk imposed on the partner who is monitored, and (iv) the risk imposed on the partners who serve as the sink.

The economics literature has considered partnerships from many varied perspectives. Alchian and Demsetz’s (1972) seminal study identifies monitoring as key to any theory of firm structure. Holmström (1982) shows that free riding cannot be eliminated if every partner lacks information about the actions of the other partners, the budget is balanced, and output varies continuously with agents’ action choices. Legros and Matthews (1993) show how, in deterministic partnerships (i.e., partnerships where output is a deterministic function of action choice), free riding can be nearly eliminated, so that the aggregate output is close to first best. Using

\(^3\)In 2002, the Big Four accounting firms, Deloitte & Touche, Ernst & Young, PricewaterhouseCoopers, and KPMG, had 2,618, 2,118, 2,027, and 1,535 partners, respectively, in their U.S. operations alone (General Accounting Office, 2003, p.17).

similar logic, Miller (1997) shows how, in deterministic partnerships of three or more, the ability of one partner to monitor another group of partners sustains efficiency with budget balance and limited liability. Like this earlier work, we assume the output of individual partners is unobservable though the aggregate output is observable and contractible. Unlike this earlier work, we assume the production technology is stochastic, and signals are publicly observable and contractible. Our analysis also is distinguished from these studies by the endogenous choice of the precision of monitoring signals, the fact that monitoring cost increases with signal precision, and the assumption that partners are risk averse.

We assume that partners’ endowment of talents are identical. That is, every partner is endowed with the same risk attitudes, personal cost of productive effort, and personal cost of monitoring effort. Given this set-up, it may seem natural to expect that partner welfare is maximized when each partner undertakes the same mix of personally costly productive and monitoring effort, which we call symmetric task assignment. In the case of a two-person partnership, we prove that a symmetric task assignment is optimal. This task assignment is induced when each partner faces the same contract parameters. As a result, the productive effort choices of the partners are equal, and monitoring effort choices of the partners are equal.

In larger partnerships, symmetric task assignment leads to shirking of the monitoring task. In this case, the per-partner certainty equivalent can be increased relative to the symmetric task assignment by inducing one partner to specialize in the monitoring task and the other partners to produce output. We identify conditions under which task specialization yields more surplus than a task assignment where every partner engages in the same mix of tasks. This task specialization, in which some partners only produce output and others mainly monitor, is a basis for hierarchy. These results are striking because partners are ex ante identical, so the gains from task specialization cannot be attributed to any comparative advantage of some subset of the partners in either task, nor are there economies of scale or scope in either the production or monitoring task that might drive the result. Instead, the gains arise from improved incentives to monitor when only a subset of partners engage in monitoring. Restricting the number of partners who monitor lessens shirking in both monitoring and production.

Our model assumes linear sharing rules, exponential utility, and normally distributed random variables. In the typology of models laid out in Lambert (2001), our model thus falls in the class of multi-action models using the LEN framework. Such models have been used in accounting (e.g., Bushman and

\[5\] Huddart and Liang (2003) consider how variation across partners in talent (or type) affects the partnership structure. In that paper, candidate partners may differ with respect to the risk of the technology they operate, their risk tolerance, and their productivity. There are greater returns to the firm to motivating (with a higher ownership stake) a talented partner than an untalented one. Also, it is less costly to motivate a partner who is risk tolerant or who operates a low-risk technology. On the other hand, the surplus that must be allocated to such a partner to keep him from defecting must be higher. Conditions exist under which partnership becomes less attractive as partners become more able. Making monitoring, incentive parameters, and firm size endogenous (as we do in this paper) when partners are of different types (as in Huddart and Liang, 2003) appears intractable.
Indjejikian, 1993, Feltham and Xie, 1994, Hemmer, 1995, and Rajan and Reichelstein, 2004) and economics (e.g., Holmström and Milgrom, 1991) to study how optimal linear incentive contracts vary with the characteristics of available signals or when agents must allocate their effort across tasks. In this paper, monitoring, unlike a generic second task, does not directly increase partnership output; instead, monitoring is undertaken by one partner to stimulate productive acts by other partners. Thus, monitoring activities arise endogenously in response to the underlying free riding problem with respect to the productive activity.

Our model, like all LEN models, admits the possibility of arbitrarily large negative outcomes. Whether such events occur in practice is open to interpretation. On one hand, the collapse of Arthur Andersen had a spectacularly negative impact on the firm’s partners. On the other hand, LLPs and similar organizational forms limit partner liability in a way that is inconsistent with our modeling choice. Note, however, that limited liability, like risk aversion, implies concave utility functions, and concave utility functions are an element of our model.

The LEN framework has also been used to study organizational form in work by Baldenius et al. (2002), who explore how the correlation structure of monitors’ signals affects the optimal assignment of monitors to agents given fixed numbers of agents and monitors, and exogenous monitoring intensities. A related paper by Ziv (2000) studies conditions under which a firm organized as a multi-layer hierarchy produces more surplus than a firm with a flat organizational structure. Ziv’s model includes a risk-neutral principal and therefore does not address how the tasks of monitoring and production are allocated across identical members of a partnership.

With risk-averse partners, Rasmusen (1987) shows that “scapegoat” and “massacre” contracts can achieve efficiency if partners are sufficiently wealthy or sufficiently risk averse. Our focus on linear contracts rules out such equilibria, which Andolfatto and Nosal (1997), among others, regard as implausible. Hemmer (2004) identifies a different set of logical difficulties that can arise when contracts are constrained to be linear. We avoid these issues by assuming each partner continuously observes the output of his own effort and performance measures are serially independent.

In Section 2, we present the basic model. Section 3 presents an analysis of symmetric task assignments. Section 4 presents an analysis of asymmetric task assignments. That section shows conditions under which an asymmetric task assignment can yield a higher certainty equivalent than the best symmetric allocation. Section 5 concludes.
2. Model

Consider a set \( N = \{1, 2, \ldots, n\} \) of partners in a firm. Each partner \( i \in N \) chooses a level of productive effort, \( p_i \geq 0 \). Each partner may also choose a level of monitoring effort \( m_i \geq 0 \). Both productive and monitoring efforts are personally costly to the partner. Denote the cost to partner \( i \) as \( c(p_i, m_i) \).

The output of the partnership is \( x = \sum_{i \in N} (f(n)p_i + \varepsilon_i) \), where \( \varepsilon_i \sim N(0, \sigma^2) \) and \( \varepsilon_i \) is independent of \( \varepsilon_j \) for \( i \neq j \). Thus, each partner’s contribution to output is random with a mean that increases in partner effort. Accordingly, total output, \( x \) is distributed \( N(f(n)\sum_{i \in N} p_i, n\sigma^2) \). The term \( f(n) \) in the output function captures the notion that there may be synergistic gains, besides risk sharing, from larger partnerships. It is useful to normalize \( f \) by setting \( f(1) = 1 \). If there are synergies, then \( f(n) \) is an increasing function of \( n \), otherwise \( f(n) = 1 \) for all \( n \). Since \( f(n) \) multiplies the sum of partners’ productive efforts, the synergistic gains are increasing in partners’ efforts. Since we assume throughout our analysis that \( f(n) \) is weakly increasing in \( n \), the synergistic gains also increase in the size of the firm.\(^7\) We assume partnership output is observable, so that a partner’s draw may depend on \( x \).

The monitoring technology provides \( s_i \), a public signal about partner \( i \)'s effort, defined as \( s_i = p_i + \xi_i \), where \( \xi_i \sim N(0, g_i(m_1, m_2, \ldots, m_n)) \), where \( m_i \) is chosen by partner \( i \).

Partner compensation is assumed to be a linear function of the observables, so partner \( i \) receives fraction \( \beta_i \) of firm output, \( x_i \); a payment that is a multiple \( z_i \) of the signal produced about his productive effort, \( s_i \); and a side-payment, \( \gamma_i \). Call \( z_i \) the incentive intensity. Also, partner \( i \) bears share \( z_{ij} \) of the payment to partner \( j \) based on the signal \( s_j \). Partner \( i \)'s compensation therefore is

\[
\beta_i x_i + z_i s_i - \sum_{j \in N \setminus i} z_{ij} s_j + \gamma_i,
\]

where \( N \setminus i \) denotes all elements in \( N \) except the \( i \)th element. A partner’s expected utility is therefore \( E[U(W_i(p_j, m_i)) \mid \{p_j, m_j\}_{j \in N \setminus i}] \) where

\[
W_i(p_j, m_i) = \beta_i x_i + z_i s_i - \sum_{j \in N \setminus i} z_{ij} s_j + \gamma_i - c(p_i, m_i).
\]

2.1. The partnership problem

Let \( P = \{p_i\}_{i \in N} \), \( M = \{m_i\}_{i \in N} \), \( z = \{z_i\}_{i \in N} \), \( A = \{z_{ij} \mid i, j \in N \text{ and } i \neq j\} \), \( \beta = \{\beta_i\}_{i \in N} \), and \( \gamma = \{\gamma_i\}_{i \in N} \). Optimal partnerships provide the largest certainty equivalent per partner and therefore are solutions to program (P):

\[
\max_{\{P, M, n, z, A, \beta, \gamma\}} \sum_{i \in N} E[U(W_i(p_j, m_i)) \mid \{p_j, m_j\}_{j \in N \setminus i}] / n
\]
subject to

\[ x = \sum_{i \in N} \left( \beta_i x + \alpha_i s_i - \sum_{j \in N \setminus i} \alpha_{ji} s_j + \gamma_i \right) \text{ for all realizations of } x \text{ and } \{s_j\}_{j=1}^N, \]

(3)

\[ E[U(W_i(p_i, m_{i}) | \{p_j, m_j\}_{j \in N \setminus i}) \geq E[U(W_i(\hat{p}_i, \hat{m}_i)) | \{p_j, m_j\}_{j \in N \setminus i}] \]

for all \( \hat{p}_i \) and \( \hat{m}_i \), for all \( i \in N \) and

(4)

\[ E[U(W_i(p_i, m_{i}) | \{p_j, m_j\}_{j \in N \setminus i}) \geq K \text{ for all } i \in N. \]

(5)

We assume that the partners play cooperatively in choosing the optimal size of the firm and contract parameters \((\beta, \alpha, A)\), then each partner chooses his actions, \( p_i \) and \( m_i \), privately and non-cooperatively. The incentive compatibility constraints, (4), impose that no partner deviates from the proposed equilibrium strategy (in \( P, M \)) given the strategy choices of the other partners. As a result, the action profile \((P, M)\) is a Nash equilibrium in the non-cooperative stage of the game. In subsequent sections where we employ specific functional forms of the personal costs and monitoring technology (see the analysis in Theorem 1 of the no monitoring and self monitoring cases given a separable personal cost function), the solution is also a dominant-strategy equilibrium and so satisfies a more stringent equilibrium definition. This approach seems natural in a partnership setting and offers insights about partnership structure that cannot be obtained from a purely non-cooperative approach.

Eq. (3) is the budget-balancing constraint. Since this constraint must hold state by state for every realization of the \( n + 1 \) random variables \( x \) and \( \{s_j\}_{j=1}^N \), it is equivalent to the following three constraints on the contract parameters:

\[ \alpha_i = \sum_{j \in N \setminus i} \alpha_{ji} \text{ for all } i \in N, \]

(6)

\[ 1 = \sum_{i \in N} \beta_i, \]

(7)

and

\[ 0 = \sum_{i \in N} \gamma_i. \]

(8)

Since a payment to one partner based on the signal \( s_j \) necessarily is funded by the other partners, we have (6). Further, budget balancing requires that the total output of the firm be divided among the partners, hence (7). We also assume that partners cannot use any personal wealth held outside the firm to break budget balance, hence any side-payments must net to zero, which is (8). The incentive compatibility constraint (4) requires that each partner prefers the equilibrium choices of \( p_i \) and \( m_i \).
to any other pair of effort monitoring choices, holding fixed the choices of the other partners. The participation constraint (5) always holds if one assumes that a partner’s alternative to membership in one partnership is membership in some other partnership or a sole proprietorship (i.e., a firm where \( n = 1 \)). This modeling choice reflects the fact that professional firms must strive to prevent their professionals from joining another firm.\(^8\) Since per-partner certainty equivalent is maximized over partnership size and compensation parameters on signals and aggregate output, there exist side-payments \( \gamma \) such that no partner can receive more from some other partnership than he receives from the optimal partnership.\(^9\) This observation leads us to drop further consideration of the participation constraint so that we may focus on the marginal incentives on partner effort and monitoring choices induced by \textit{ex ante} commitments to the contract parameters, \((\beta, \alpha, A)\).

Given exponential utility functions with risk aversion parameter \( r \), \( U(W) = -\exp(-rW) \), and normally distributed random variables, we make the usual transformation of expected utility into mean-variance terms. Partner \( i \)'s certainty equivalent is

\[
V_i = E(W(p_i, m_i)) - \frac{r}{2} \text{Var}(W(p_i, m_i))
= (\alpha_i + \beta_i f(n)p_i + \sum_{j \in N \setminus i} (\beta_i f(n) - \alpha_{ji})p_j + \gamma_i - c(p_i, m_i)
- \frac{r}{2} \left( \beta_i^2 n \sigma^2 + \sum_{j \in N \setminus i} \beta_{ji}^2 g_j(M) \right),
\]

where \( g_i(M) = g(m_1, m_2, \ldots, m_n) \) is the variance of signal \( s_i \) given the monitoring effort choices of the partners. The per-partner certainty equivalent, is \( \sum V_i/n \). In a symmetric equilibrium, each partner faces the same contract parameters and chooses the same effort levels.

### 2.2. First-order conditions

Contract parameters \((\beta, \alpha, A)\), and partnership size, \( n \), are chosen to maximize per-partner certainty equivalent. Next, each partner \( i \) chooses the levels of productive and monitoring effort that maximize the expected utility of (1) given these contract parameters. From (9), the first-order condition on the productive effort of partner \( i \) is

\[
0 = (\alpha_i + \beta_i f(n)) - \frac{\partial c(p_i, m_i)}{\partial p_i}.
\]


\(^9\)This follows because the solution to program (P) has the super-additivity property: if \( n^* \) is the optimal partnership size, then for any natural numbers \( n_1 \) and \( n_2 \) such that \( n^* = n_1 + n_2 \), the certainty equivalent from the optimal partnership of size \( n^* \) equals or exceeds the sum of the certainty equivalents from any partnerships of sizes \( n_1 \) and \( n_2 \). We thank Stefan Reichelstein for bringing this to our attention.
Also from (9), the first-order condition on the monitoring effort of partner $i$ is

$$0 = -\frac{\partial c(p_i, m_i)}{\partial m_i} - \frac{r}{2} \left( x_i^2 \frac{\partial g_i(M)}{\partial m_i} + \sum_{j \in N \setminus i} x_j^2 \frac{\partial g_j(M)}{\partial m_i} \right).$$

(11)

Note that the second-order conditions on $p_i$ and $m_i$ are satisfied provided functions $c$ and $g$ are convex. The partnership problem is solved by substituting the optimal productive and monitoring efforts for each partner $i$, $p_i^*$ and $m_i^*$, into (2) and then solving for optimal $\beta$, $\alpha$, $A$, and $n$. Since changes in productive effort affect the mean, but not the variance (or riskiness) of payments to the partners, the optimal productive effort choice, $p_i^*$, given by (10) does not depend directly on the partners’ aversion to risk, $r$. Since changes in monitoring effort, however, do affect the variance of payments to partners, the optimal monitoring effort choice, $m_i^*$, given by (11) depends directly on the partners’ aversion to risk. Since monitoring effort is governed by choices of $\alpha$ and $A$, which also affect productive effort, risk aversion does have an indirect effect on productive effort.

3. Symmetric task assignments

We first characterize the optimal symmetric linear contract for the partnership problem in three distinct cases to understand how the available monitoring technology affects profit-sharing and partnership size. In a symmetric contract, each partner $i$ receives the same fraction $\beta_i \equiv 1/n$ of firm output, $x$, and receives a payment that is the same multiple, $\alpha_i \equiv \alpha$, of the signal produced about his productive effort, $s_i$. Also, the share $\alpha_{ji} \equiv \bar{\alpha}/(n - 1)$ of the payment to partner $j$ born by partner $i$ is the same across partners $i \in N \setminus j$.

In professional organizations individuals’ contributions are difficult to quantify and assess, so effort must be devoted to gathering indications of individual performance. Partner monitoring effort choices combine to yield the precisions of signals of productive effort, $s_i \sim N(p_i, g_i(M))$, under one of two monitoring technologies: self monitoring, denoted $t = S$; and common monitoring, denoted $t = C$; as well as a benchmark case in which no monitoring technology is available, denoted $t = O$.

In the self monitoring case, each partner bears personally the cost of producing a signal of his own productive effort, so that $g_i^S(M) = 1/m_i$. This technology captures the notion that a professional can exert effort to inform his colleagues of his contributions to the organization. For instance, consider the evaluation of university

---

10We do not model the costs of removing potential bias from reports, whether produced by the self or common monitoring mechanism. Our analysis is valid provided reports can be debiased. Formally, this requires that the average amount of distortion in a report can be subtracted from each signal. For example, an instructor who is equally likely to give too many or too few points when grading an exam can adjust differently the final grades of students who did and who did not complain (a kind of self report) about their exam score. If signals can be debiased, then what matters is how much effort is devoted to the production of a precise signal.
faculty. Besides time devoted to the directly productive activity, such as teaching and research, a faculty member also devotes time to substantiating his accomplishments by, for example, maintaining a curriculum vitae; documenting the significance and impact of his work; detailing administrative and professional activities; seeking “visibility” in the field, etc. At many schools, preparation by the candidate of a dossier documenting his accomplishments is a key element of periodic performance reviews, which affect pay, tenure and promotion decisions. The situation is similar in professional accounting firms. Covaleski et al. (1998), who conducted an ethnographic field study of partners at Big 6 public accounting firms, indicate that partners seek to create in the eyes of fellow partners the perception of strong client service in contrast to merely giving good client service. Moreover, partners formally must report and interpret their performance to their peers: a regional managing partner says “we have monthly sales meetings with all the partners [in the region] every month, each partner has sales goals and targets. . . . Every month he sees the peer pressure because he’s got to get up, and there’s flip chart up there and an overhead, and he’s got to explain [to fellow partners] why he’s behind plan or ahead of plan” (Covaleski et al., 1998, p. 311).

As an alternative to reports by a partner about his own work, other partners in the firm may expend effort to gather information about a partner. The common monitoring technology captures the notion that the work of one partner in the firm may be assessed by other partners. In this case, the precision of each partner’s signal is a function of the sum of the monitoring efforts provided by all partners, \( g_i^C(M) = 1/\left(\sum_{j \in N} m_j/n\right) = n/\sum_{j \in N} m_j \). That is, the precision of the signal of productive effort for any partner is determined by the average monitoring effort over all partners. This choice of functional form assures that the precision of signals in the self monitoring and common monitoring cases are identical when partners’ monitoring efforts are equal: \( g_i^S(M) = g_i^C(M) \) for any \( n \) when \( m_i = m \) for all \( i \in N \). Thus, differences in productivity that arise are due to the altered incentives to monitor, not the inherent superiority of one monitoring technology over the other.

Professional accounting firms expend significant resources in performance review. For instance, working papers that support an audit opinion prepared under the direction of one partner are regularly and routinely subject to a quality review by other partners in the same firm. Analogous to low realizations of signal \( s_i \), a review that reveals departures from firm standards subjects a partner to significant financial sanctions. At the public accounting firm PricewaterhouseCoopers, fines of $5,000 to $50,000 that reduce the partner’s draw are imposed for violating firm standards.\(^{11}\)

In the model, monitoring intensity \( m_i \) can be likened to partner time devoted to the performance review process and incentive intensity \( z_i \), to the magnitude of the penalty (or reward) attached to the quality assessment resulting from that process.

\(^{11}\)Carter Pate, Vice Chair of Client Services, PricewaterhouseCoopers, private communication, February 4, 2004. Also, a partner’s pen may be taken away (i.e., a partner could lose the right sign an audit report in the name of the firm), which can also affect partner income.
Given self monitoring, the partner about whom the signal is informative internalizes the personal cost of producing the signal as well as much of the benefit (i.e., lower signal risk) from producing a precise signal. Given common monitoring, each partner personally bears only part of the cost of the monitoring system, so the scope for shirking the monitoring activity increases with the size of the partnership.

While common and self monitoring may take place simultaneously within the same firm, it is instructive to consider them separately for two reasons. First, the relative effectiveness of these two monitoring technologies is unknown. That is, the practically achievable precisions of signals derived from self monitoring may be very different from those derived from common monitoring. Second, the public good problem associated with common monitoring is highlighted by comparison to the self monitoring case. In all that follows, we assume the cost function for partner $i$’s productive and monitoring efforts has the form $c(p_i, m_i) = \frac{1}{2} p_i^2 + m_i$. This functional form is most tractable and leads to intuitive expressions for the optimal per-partner joint surplus, monitoring effort, and signal-contingent payments in Theorem 1, below.\footnote{Specifications such as $c(p_i, m_i) = \frac{1}{2} (p_i^2 + m_i^2)$ involve messier algebra, but the basic tensions are similar.}

The optimal level of productive effort is interior since (i) $\frac{\partial V_i}{\partial p_i} > 0$ at $p_i = 0$, and (ii) $\frac{\partial V_i}{\partial m_i} < 0$ as $p_i \to \infty$. Although the personal cost of monitoring is linear, for any $r$ and $x$, it is straightforward to show that $\frac{\partial V_i}{\partial m_i} < 0$ as $m_i \to \infty$, so there are no cases where arbitrarily high levels of monitoring are undertaken. Further, the variance of the signals is decreasing and convex in $m_i$, so second-order conditions on per-partner joint surplus with respect to $m_i$ are satisfied. Note that the optimal level of monitoring could be zero. This is so because it may be that $\frac{\partial V_i}{\partial m_i} < 0$ at $m_i = 0$, depending on partner risk-aversion, $r$, and the incentive intensity, $x$. In particular, (11) implies that if $x = 0$, then $m_i^* = 0$ for all $i \in N$.

The first-best effort levels are $p_i = f(n)$ and $m_i = 0$ for all $i \in N$. The marginal cost of productive effort is set equal to the marginal product, and no effort is allocated to costly monitoring. Under moral hazard, first-best is not attained for $n \geq 2$. For a given firm size and under moral hazard with respect to production and monitoring, optimal effort choices, ownership shares, incentive intensity parameters, and per-partner certainty equivalent are as follows:

*Theorem 1.* Given monitoring technology $g_i(M)$, firm size $n$, and cost function $c(p_i, m_i) = \frac{1}{2} p_i^2 + m_i$, for any $n$, $r$, and $\sigma^2$, the optimal symmetric linear solution has

\begin{align*}
p_i^* &= x_i^r(n, r) + \beta_i^r(n)f(n), \\
\beta_i^r(n) &= 1/n, \tag{12}
\end{align*}

and

\begin{align*}
\sum_{i \in N} V_i(n, r, \sigma)/n &= \frac{2n - 1}{2n^2} f(n)^2 - \frac{r\sigma^2}{2n} + \frac{x_i^r(n, r)^2}{2} \tag{13}
\end{align*}
for all \( i \in N \) and monitoring technology \( t \). Further, given:

\[ g_i^0(M) \text{ unbounded (i.e., no monitoring),} \]

\[ g_i^{S*}(n, r) = 0, \]

\[ a_i^{S*}(n, r) = 0; \]

\[ g_i^S(M) = 1/m_i \text{ (i.e., self monitoring),} \]

\[ g_i^{C*}(n, r) = \max\left(0, \frac{n - 1}{n} f(n) - \sqrt{\frac{r}{2} \frac{2n - 1}{n - 1}}\right), \]  \( (14) \)

\[ m_i^{C*}(n, r) = \sqrt[4]{\frac{r}{2(n - 1)}}. \]  \( (15) \)

\[ g_i^C(M) = n/\sum_{i \in N} m_i \text{ (i.e., common monitoring),} \]

\[ g_i^{C*}(n, r) = \max\left(0, \frac{n - 1}{n} f(n) - \sqrt{\frac{r}{2} \frac{n + 1}{\sqrt{n - 1}}}\right), \]  \( (16) \)

\[ m_i^{C*}(n, r) = \sqrt[4]{\frac{r}{2(n - 1)}}. \]  \( (17) \)

All proofs are in the appendix. Examination of (13) indicates that a regularity condition on \( f(n) \) is necessary to avoid unintuitive situations where size synergies grow so rapidly that optimal partnership size is unbounded, per-partner surplus is unbounded, and partners engage in monitoring even though the public good problem is acute. The condition \( f(n) \leq n/\sqrt{2n - 1} \) is sufficient to rule out such economically implausible outcomes. We impose it throughout the ensuing analysis. In the following sections, we discuss the intuition related to Theorem 1.

### 3.1. No monitoring

When monitoring is not possible, each agent’s effort choice is determined by his ownership share alone, \( p_i^* = \beta_i = 1/n \). Because the first-best value of effort is \( p_i = f(n) \), free riding becomes more severe as partnership size increases. The trade-off between risk sharing and other synergies, which are facilitated by large partnerships, and incentives to exert effort, which are strongest in small partnerships, is apparent in the expression for the per-partner certainty equivalent (13) in the case where no monitoring is possible:

\[ \sum_{i \in N} V_i^0(n, r, \sigma)/n = \frac{2n - 1}{2n^2} f(n)^2 - \frac{r a^2}{2n}. \]  \( (18) \)

The partnership size is optimal when the marginal synergistic benefit equals the loss in output from diminished incentives. The optimal partnership size depends on the cost function for productive effort. If the marginal product of effort differs by profession, then this model predicts the size of professional partnerships will vary by profession. Likewise, if there are economies of scale or scope that favor large
partnerships in a given profession, then partnerships will be larger than in the absence of such synergies. This basic tradeoff is unaffected by the specific form of the production function.

A related point concerns the structure of uncertainty. Our formulation assumes that the random components of each partner’s contributions to production are independent. This favors large partnerships since the potential gains from diversifying the risk of output $x$ are larger. Positive correlations among the random components of partners’ contributions reduce diversification benefits and hence reduce optimal partnership size. If the $\varepsilon_i$ are perfectly correlated, then $x \sim N(f(n)\sum_{i=1}^{n} p_i, n^2\sigma^2)$, in which case there are no gains due to risk sharing when partners share aggregate output. Intermediate degrees of correlation are possible also. Given less-than-perfect correlation in $\varepsilon_i$, there is a diversification benefit. The change in per-partner surplus is smaller the more positively correlated a new partner’s practice domain is with those of the existing partners. So, for instance, auditors that specialize in different industries stand to benefit more from diversification when they combine their practices than do auditors with clients in the same industry. Only if there are no scale synergies and no diversification benefit would the optimal firm structure be a sole proprietorship for all parameter values. Diversification considerations also apply when considering the random components of the signals produced by the monitoring technology. The monitoring technology imposes more risk on the partners when the errors in signals are positively correlated, so optimal partnerships are smaller, ceteris paribus, in this case.

### 3.2. Monitoring

Monitoring results in the production of noisy signals that are informative of effort. These signals can be used to increase individual partner’s effort, $p_i$, and, hence, improve the mean of firm output, $x$. Monitoring activities do not affect the total output variance, $n\sigma^2$. Irrespective of the amount of monitoring the partners undertake, this output risk must be born by the firm’s partners. Monitoring is costly because (i) it requires effort to produce and (ii) signal-contingent compensation imposes signal risk both on the partner whose draw increases in the value of the signal and the partners whose draw decreases in the value of the signal. Choices of $z_i^*$ are made before $m_i$ is selected, so they reflect the tradeoff between additional productive effort and the additional costs of monitoring effort and signal risk.13 Because noise in the signals is independent of output, incentive intensity,

---

13It is tempting but incorrect to conclude that the level of monitoring chosen by partners in the self-monitoring regime is socially optimal. Despite the fact that each partner bears the risk of fluctuations in his pay associated with a noisy signal about his own productive effort, the partner underinvests in monitoring effort because he does not internalize the cost of a noisy signal on the partners who bear the risk associated with funding the signal-contingent changes in the partner’s draw. If there were no moral hazard with respect to monitoring effort, then the first-best level of monitoring, $m_i^{FB}$, would be chosen to maximize $\sum_{j \in N} V_j/n$ rather than $V_i$. Diversifying signal risk associated with $s_j$ efficiently across the $n-1$ partners who may serve as the sink implies $z_{ij} = x_j/(n-1)$ for $j \in N \setminus i$. The first order conditions imply $m_i^{FB} = \sqrt{mn/(2n-2)}$, and $z_i^{FB}(n, r) = \max(0, (n-1)f(n)/n - \sqrt{2nr}/(n-1))$. Comparing these socially-
signals. The underinvestment is acute when self-monitoring there is underinvestment in monitoring effort and a lower incentive coefficient on the intensity $m^s$ in large partnerships whenever self-interested partners, optimal values of monitoring effort (ii) symmetric contract

Corollary 1. For any increasing function $f(n)$ and given self-monitoring and a symmetric contract, incentive intensity $x^s(n, r)$ is (i) weakly increasing in the size of the partnership and (ii) weakly decreasing in partners’ risk aversion. Also, monitoring intensity $m^s(n, r)$ is (i) weakly increasing in partnership size and (ii) increasing then weakly decreasing in partners’ risk aversion.

The model predicts larger bonus coefficients on an agent’s own reports of his efforts in larger firms. Intuitively, as partnership gets larger, more weight is placed on the noisy signals because a larger set of partners absorbs the signal risk at a lower cost. In turn, more weight on this signal induces higher monitoring efforts by individual partners because, with self-monitoring, the variance of the signal about a partner’s own productive effort can only be reduced by that partner. From (14), choosing $r < \frac{1}{18}$ implies $x^s > 0$ for all $n \geq 2$, so some self-monitoring activity is undertaken for any choice of $\sigma^2$ and regardless of partnership size. Also from (14), $\lim_{n \to \infty} x^s(n, r) = \max(0, 1 - \sqrt{2r})$, which implies that some self-monitoring is done in large partnerships whenever $r < \frac{1}{2}$. For a given $n$, as risk aversion increases, less weight is placed on the noisy signals because partners’ risk tolerance is lower. However, the monitoring intensity is not monotonic in $r$.

Fig. 1 illustrates the dependence of incentive intensity $x^s$ and monitoring intensity $m^s$ on partner risk aversion and firm size for both self-monitoring and common monitoring. The upper panel of Fig. 1 plots various isoquants of $x^s$ and $m^s$ for $n$ ranging from 2 to 40 and $r$ ranging from 0.0 to 0.3. To simplify the plots, variable $n$ is treated as continuous. When partners are nearly risk neutral, the risk associated with noisy signals is not costly for the partners to bear, so very noisy signals can be used to motivate high levels of productive effort (i.e., $x^s(n, r)$ can be large) and the optimal investment in costly monitoring is low. In fact, as $r$ approaches 0, $x^s(n, r)$ approaches $(1 - 1/n)f(n)$, so $x^s$ approaches $f(n)$ and productive effort is at the efficient level. As partners become more risk averse, monitoring efforts first increase then decrease. Initially, as the partners’ risk-aversion coefficient $r$ increases from 0, $x^s(n, r)$ decreases and $m^s(n, r)$ increases. Decreasing $x^s(n, r)$ and increasing $m^s(n, r)$ both reduce the signal-based risk that partners must bear. Thus, as $r$

\(\text{footnote continued}\)

optimal values of monitoring effort $m^{FB}$ and incentive coefficient $x^{FB}(n, r)$ with those that are chosen by self-interested partners, $m^s$ and $x^s(n, r)$, from Eqs. (15) and (14), respectively, it is evident that even with self-monitoring there is underinvestment in monitoring effort and a lower incentive coefficient on the signals. The underinvestment is acute when $n$ is small.
increases from zero, partners reduce risk by choosing more precise signals even as the weight of those signals in their compensation decreases.

At higher values of \( r \), \( m_i^{Sx}(n,r) \) decreases as \( x_i^{Sx}(n,r) \) continues to decrease. These decreases occur because the risk imposed by tying partner compensation to noisy signals becomes too costly for partners to bear, so \( m_i^{Sx}(n,r) \) and \( x_i^{Sx}(n,r) \) both decrease. At high levels of risk aversion, the risk created by making partner draws...
contingent on noisy signals of effort becomes so costly that partners prefer to forgo monitoring entirely, so that $m^{Si}_{S}(n, r) = 0$, $z^{Si}_{S}(n, r) = 0$ and incentives for productive effort are solely supplied by partners’ shares of firm output, $\beta^i(n) = 1/n$. In this case (13) reduces to (18), the certainty equivalent when no monitoring is possible. The upper-right panel of Fig. 1 also illustrates that monitoring effort, $m^{Si}_{S}(n, r)$, is greatest for intermediate values of risk aversion and large partnerships.

Analogous to the comparative static analysis presented in Corollary 1 for the case of self monitoring, we have in the case of common monitoring:

**Corollary 2.** For any strictly positive, increasing function, $f(n)$, and given common monitoring, incentive intensity $z^{C*}_{S}(n, r)$ is (i) weakly increasing then weakly decreasing in the size of the partnership and (ii) weakly decreasing in partners’ risk aversion. Also, monitoring intensity $m^{C*}_{S}(n, r)$ is (i) weakly increasing then weakly decreasing in partnership size and (ii) increasing then weakly decreasing in partners’ risk aversion.

With common monitoring, partners contribute effort to the common task of monitoring members of the partnership, instead of devoting effort to the production of noisy reports of their own effort. The monitoring function in this case takes the form $g_i(M) = n/\sum_{i \in N} m_i$, implying that increases in monitoring effort increase the precision of all signals, not solely the signal on the partner’s own effort as in the case of self monitoring; and a partner’s monitoring effort is a public good, hence, there is less investment in monitoring under common monitoring than under self monitoring. As a result, relative to self monitoring, $z$ is smaller and the region of the $(n, r)$ parameter space where $z = 0$ is larger. This is illustrated in the lower-left and lower-right panels of Fig. 1, which present isoquants of incentive intensity and monitoring effort, respectively, in the case of common monitoring. As in the case of self monitoring (see the corresponding upper panels of Fig. 1), incentive intensity and monitoring effort first increase in partnership size to combat free riding on the productive tasks. Different from the case of self monitoring, free riding on the monitoring task leads to decreasing incentive intensity and decreasing monitoring effort as the partnership size continues to increase in the common monitoring case. This difference across monitoring technologies in incentive intensity and monitoring effort points to the critical role that the structure of monitoring plays in determining optimal contracts within firms. In particular, the severity of the public goods problem posed by the monitoring structure is crucial. We return to this issue in Section 4.

### 3.3. Optimal partnership size

Theorem 1 specifies the optimal choices of symmetric contract parameters $z^*_i$ and $\beta^*_i$ given a partnership of size $n$. However, partnership size may itself be endogenous. Thus, it remains to determine what partnership size is optimal. Below, we characterize the optimal partnership size for each monitoring technology. For simplicity, in this section, we suppose $f(n) = 1$ for all $n$. Choosing $f(n)$ to be an increasing function of $n$ has the effect of increasing the optimal partnership size, but does not alter the basic tradeoffs identified earlier. Let $n^{*}(r, \sigma)$ be the optimal
partnership size given exogenous parameters $r$ and $\sigma$, and monitoring technology $t \in \{O, S, C\}$, denoting no monitoring, self monitoring, and common monitoring, respectively. We consider each of these technologies in turn. The no monitoring case is easily characterized by a first-order condition. The self monitoring and common monitoring cases are more complicated.

In the no monitoring case, recall that the ownership share $\beta^*(n) = 1/n$ is the sole incentive for productive effort when monitoring is not possible. There is no free riding in a sole proprietorship, $n = 1$, but the proprietor bears substantial risk. Risk from the production technology is diversified away as $n$ grows large; however, increasing the number of partners in the firm from $n$ to $n + 1$ diminishes the productive effort of each partner by $1/n - 1/(n + 1)$.

For some parameter values, the loss in productive effort can be less than the gain in risk sharing for any $n$: adding another partner is always welfare-improving to the existing partners if $r\sigma^2 > 2$. That is, if the production technology is sufficiently risky or the partners are sufficiently risk averse, then the optimal partnership size is unboundedly large. If $r\sigma^2 < 2$, then the optimal partnership size, $n^{O}\{r, \sigma\}$, is one of the integers closest to $2/(2 - r\sigma^2)$, where the reduction in productive effort is offset by improvements in risk sharing. This follows from the first-order condition on (18). In this case the per-partner certainty equivalent at the optimal partnership size is approximately $(2 - r\sigma^2)/8$, which follows from substitution of $n^{O}\{r, \sigma\}$ into (18).

More forces must be balanced to determine optimal firm size when monitoring is possible. Substituting $x_i^O(n, r)$ and $\beta^O_i(n) = 1/n$ from (14) and (12) into (13) yields the per-partner certainty equivalent as a function of partnership size, $n$, and the exogenous parameters $r$ and $\sigma$, given self monitoring. Likewise, substituting $x_i^C(n, r)$ and $\beta^C_i(n) = 1/n$ from (16) and (12) into (13) yields the per-partner certainty equivalent as a function of $n$, $r$, and $\sigma$, given common monitoring. In contrast to the straightforward analysis in the no monitoring case, the resulting expressions for the certainty equivalent are not globally concave in $n$ for all choices of $r$ and $\sigma$, so the first-order approach cannot be used to characterize optimal partnership size. The reason for the non-concavity is the benefits from high productivity (induced by high ownership shares and associated strong incentives to monitor) are greatest when partnership size is small, while the benefits from risk diversification (and other synergies) are greatest when partnerships are large. Combined, these forces need not yield a concave (or even quasi-concave) objective although each, considered separately, is monotone and concave. To identify the optimum, interior local optima must be compared with the per-partner surplus obtainable when the gains from risk sharing are maximal.

Intuitively, incentives provided by monitoring allow partnerships to grow in size. This is true for self monitoring. Self monitoring concentrates the incentives to monitor in each partner, who is therefore motivated to provide a precise signal of his own effort irrespective of partnership size. This concentration of incentives is sufficient to ensure that the optimal partnership size given self monitoring is always at least as big as the optimal partnership size given either no or common monitoring; however, because monitoring also is subject to free riding, this intuition need not
hold in the common monitoring case. The next corollary formalizes this partial ordering over the size of optimal partnerships.

**Corollary 3.** Suppose \( f(n) = 1 \) for all \( n \). For any \( r \) and \( \sigma^2 \),
\[
    n^{Os}(r, \sigma) \leq n^{Ss}(r, \sigma)
\]
and
\[
    n^{Cs}(r, \sigma) \leq n^{Ss}(r, \sigma).
\]

Whether the optimal partnership size given common monitoring is larger or smaller than the optimal partnership size given no monitoring depends on the risk of the production technology relative to partners’ risk tolerance. This is because increasing \( n \) improves risk sharing, but reduces incentives to monitor. If the gains from risk sharing by increasing partnership size do not exceed the cost of diluting the incentive to engage in monitoring, common monitoring makes smaller partnerships more attractive, i.e., \( n^{Cs}(r, \sigma) < n^{Os}(r, \sigma) \). This is true when the production technology risk, \( \sigma \), relative to partner risk aversion, \( r \), is small. For instance, \( n^{Cs}(0.10, 4.2) = 7 \) and \( n^{Os}(0.10, 4.2) = 9 \). For other parameter values, \( n^{Cs}(r, \sigma) > n^{Os}(r, \sigma) \). The opposite is true for other parameters. For instance, \( n^{Cs}(0.08, 4.2) = 5 \), but \( n^{Os}(0.08, 4.2) = 3 \). In this case, the monitoring technology serves to preserve incentives for the partners to exert effort as the partnership grows and thereby facilitates improved risk sharing.

Whether the optimal partnership size given either self, common, or no monitoring is finite also depends on the risk of the production technology relative to partners’ risk tolerance. Panels A, B, and C of Table 1 present the optimal partnership size given self, common, and no monitoring, respectively over exogenous parameters \( r \) in the range of 0.025 to 0.300 and \( \sigma^2 \) in the range of 0.750 to 9.000. It is apparent that the optimal partnership size, \( n^* \), increases as either partners become more risk averse (i.e., \( r \) increases) or the riskiness of the production technology increases (i.e., \( \sigma \) increases). In some common monitoring cases, the gain from improved risk sharing that follows from increasing partnership size, net of the reduction in monitoring, is positive for all \( n \), so the optimal partnership size is unboundedly large. It is also possible, however, that common monitoring makes a small partnership optimal given common monitoring even though an unboundedly large partnership is optimal when no monitoring is possible. Compare, for example, optimal partnership size across monitoring technologies at \( (r, \sigma) = (0.025, 9.000) \): \( n^{Ss}(0.025, 9.000) = \infty \), \( n^{Cs}(0.025, 9.000) = 9 \), and \( n^{Os}(0.025, 9.000) = \infty \). This suggests that partnership size is sensitive to variations in monitoring technology. Such variation might exist across professions or over time.

Table 1 illustrates interesting interactions between monitoring activities, technological risk, and firm size that have surfaced in public debate over the structure of the public accounting industry. A striking feature of the evolution of public accounting over the last 50 years has been the great increase in the size of the accounting firms that audit public companies while the accounting firms without publicly-traded clients remain small—see Wyatt (2003, p. 2) and General Accounting Office (GAO) (2003, p. 17). One explanation for the great increase over time in the
Table 1
Optimal partnership size, $n^*$, given partner risk aversion $r$ and production technology risk $\sigma$

<table>
<thead>
<tr>
<th>$r$</th>
<th>0.025</th>
<th>0.050</th>
<th>0.075</th>
<th>0.100</th>
<th>0.125</th>
<th>0.150</th>
<th>0.175</th>
<th>0.200</th>
<th>0.225</th>
<th>0.250</th>
<th>0.275</th>
<th>0.300</th>
</tr>
</thead>
</table>
| **Panel A: Self monitoring**
| $\sigma = 9.000$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $8.250$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $7.500$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $6.750$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $6.000$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $5.250$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $4.500$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $3.750$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $3.000$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $2.250$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $1.500$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |
| $0.750$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |

| **Panel B: Common monitoring**
| $\sigma = 9.000$ | $9$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $8.250$ | $8$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $7.500$ | $7$ | $12$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $6.750$ | $6$ | $9$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $6.000$ | $5$ | $7$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $5.250$ | $3$ | $5$ | $8$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $4.500$ | $2$ | $4$ | $5$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $3.750$ | $1$ | $2$ | $2$ | $4$ | $7$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $3.000$ | $1$ | $1$ | $2$ | $2$ | $3$ | $5$ | $10$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $2.250$ | $1$ | $1$ | $1$ | $2$ | $2$ | $2$ | $2$ | $2$ | $3$ | $3$ | $3$ | $3$ |
| $1.500$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $2$ | $2$ | $2$ | $2$ |
| $0.750$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |

| **Panel C: No monitoring**
| $9.000$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $8.250$ | $7$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $7.500$ | $3$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $6.750$ | $2$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $6.000$ | $2$ | $10$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $5.250$ | $2$ | $3$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $4.500$ | $2$ | $4$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $3.750$ | $1$ | $2$ | $2$ | $3$ | $8$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $3.000$ | $1$ | $1$ | $2$ | $2$ | $3$ | $5$ | $10$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ |
| $2.250$ | $1$ | $1$ | $1$ | $2$ | $2$ | $2$ | $2$ | $2$ | $3$ | $3$ | $3$ | $3$ |
| $1.500$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $2$ | $2$ | $2$ | $2$ |
| $0.750$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ | $1$ |

This table presents the optimal partnership size given symmetric task assignments and either self monitoring (Panel A), common monitoring (Panel B), or no monitoring (Panel C) for given exogenous parameters for partner risk aversion, $r$, and riskiness of the production technology, $\sigma$. In the calculations, $f(n) = 1$ for all $n$. 
size of accounting firms that audit public companies is that the risk associated with conducting audits of public companies has increased. The GAO study mandated by the Sarbanes—Oxley Act of 2002 reports that the risks associated with auditing public companies generally create disincentives for smaller firms to actively compete for large public company clients, with the result that small firms do not audit large public companies (GAO 2003, p. 45 and 49). In the model, a riskier audit task is interpreted as a production technology with a larger $\sigma^2$. Table 1 reveals that, regardless of monitoring technology, firm size increases with the riskiness of the production function, which suggests that small firms would choose not to audit the riskiest clients. Moreover, optimal firm size increases discontinuously as technology risk increases, so that unboundedly large firms are favored once risk crosses a threshold value.

The reason for the discontinuous jump in optimal firm size is that the globally optimal firm size is the greater of two local optima. One local optimum is an unboundedly large firm where production risk is shared efficiently. The second local optimum is a small firm in which every partner receives a large share of total output and so exerts high productive effort, which implies a large mean per-partner output, but where every partner bears substantial risk. Intermediate firm sizes are dominated by smaller or larger firms because such firms are too big to permit strong incentives for partners to exert effort and too small to diversify production risk efficiently.

A firm of unbounded size is a practical impossibility, but the underlying intuition is that medium-sized firms (as an organizational form), are dominated in our model either by smaller firms (when production risk is low) or larger firms (when production risk is high). This is consistent with the absence of medium-sized public accounting firms: The GAO (2003, p. 17) points out the striking fact that in 2002, there were 4 public accounting firms with more than 12,500 professional staff in their U.S. operations, but the 5th-largest firm had fewer than 2,400 professional staff. In summary, the forces at work in this analysis yield interesting predictions on changes in firm size over time (or across professions or segments of one profession) as either the risk of the production process or the efficacy of monitoring changes.

### 3.4. Optimality of symmetric contracts

To this point, we have assumed a symmetric contract, which implies symmetric task assignments. Given this symmetry, we characterized the optimal level of monitoring effort and intensity of signal-based incentives. The symmetric nature of the objective function and the constraints lead to the conjecture that symmetric contracts are optimal for all $n$ when all partners have the potential to monitor. The next result establishes that this is true for 2-person partnerships.

**Theorem 2.** Given $n = 2$ and either (i) no monitoring, (ii) self monitoring, or (iii) common monitoring, the optimal linear contract, $(\beta, x, A)$ is the symmetric contract characterized in Theorem 1.

The symmetric contracts characterized in Theorem 1 given common monitoring have the undesirable property that free riding on monitoring becomes severe in
large partnerships. This leads to lower incentive intensities in the common monitoring case relative to the self monitoring case, as shown earlier in Fig. 1. Thus, creating incentives to monitor in large partnerships that operate the common monitoring technology may increase the per-partner certainty equivalent. This question is especially germane to public accounting firms because of (i) the large size of some of these firms, (ii) the importance of quality to the audit process, (iii) the fact that significant professional expertise is required to assess quality, (iv) the recent spectacular failure of large firms, and (v) the current regulatory interest in the structure of this industry. As we show in the next section, incentives to monitor in large partnerships can be restored (and certainty equivalent can be increased above the level attainable in a symmetric contract) when one partner alone is charged with the monitoring task and other partners refrain from monitoring entirely.

4. Asymmetric task assignments

Given common monitoring, we explore asymmetric task assignments and provide conditions under which an asymmetric division of the productive and monitoring tasks across partners provides a greater certainty equivalent to the partners than the best symmetric solution. An interpretation of this result is that the trade-off between (i) free riding and (ii) risk sharing and other size synergies induces organizational form. In particular, in a setting where all partners are identical (in terms of abilities and attitudes toward risk), a flat organizational structure where all partners are assigned the same mix of production and monitoring tasks can be sub-optimal. In certain settings, per-partner certainty equivalent is greater when some partners primarily monitor and others solely produce output.14

As an alternative to a symmetric task assignment, consider an asymmetric assignment in which all partners exert productive effort but only partner 1 monitors. Relative to the programming problem studied earlier, these assumptions are equivalent to replacing the $n - 1$ first-order conditions on monitoring given in (11) for partners $i = 2, \ldots, n$, which derive from the incentive compatibility constraints (4), with the constraints $m_i = 0$ for $i = 2, \ldots, n$. Ensuring that the $n - 1$ non-monitoring partners do not monitor is a matter of organizational design. We assume either monitoring by these partners is forbidden or the monitoring system is designed so that only partner 1 may employ it. In addition, set $\alpha_1 = 0; \alpha_{ij} = 0$ for $i$ and $j \in N$ and $j \neq 1$; and $\alpha_{ij} = 0, \alpha_i = \alpha_j = \alpha^4$, and $\beta_j = (1 - \beta_j)/n$ for $j \in N \setminus 1$. Thus, partner 1 may monitor every other partner and funds the entire signal-contingent payment, $\alpha^4 s_i$, of every other partner $i$ in $N \setminus 1$.15 Partner 1 is not monitored by any partner. Under this arrangement, $\alpha_i$ for $i \neq 1$ and $\beta_i$ for all $i \in N$ are chosen collectively first. Then partner 1 privately chooses $m_1$ and $p_1$ and the other $n - 1$ partners choose $p_i$.

---

14We thank Jennifer Francis for out pointing that this may explain the existence of associate deans.

15Observe that side payments $\gamma$ can be used to set partner 1’s draw at any level regardless of the signal-contingent payments partner 1 makes.
for \( i = 2, \ldots, n \) privately. We call the solution to this problem, optimized over \( x^A \), the asymmetric linear solution at \( \beta^A_1 \).

It is important to observe that the cost of attaining signals of a given precision is the same as in preceding sections. We interpret the altered problem as a change in organizational structure. Partner 1 rationally anticipates that noisy signals of productive effort will be produced if partner 1 shirks the monitoring task. This creates a strong incentive for partner 1 to monitor. Choosing \( a_{ij} = 0 \) for \( i = 2, \ldots, n \) and \( j \neq 1 \) forces partner 1 alone to fund the payment (or to serve as the sink) for the incentive payments to the other partners. Thus, partner 1 alone shoulders costly risk if the signals are noisy. A benefit of this organizational choice is that incentives to monitor are strengthened for partner 1. The stronger incentives (but not a comparative advantage) may lead partner 1 alone to monitor more than all partners combined in the symmetric case. More precise signals imply higher values for \( a_i \) for \( i = 2, \ldots, n \), which in turn implies higher effort from the other partners.

This benefit must be offset against the cost of sharing signal risk inefficiently across the partners.

**Theorem 3.** For any \( n, r, \) and \( \sigma^2 \), the asymmetric linear solution at \( \beta^A_1 \) has

\[
p_1^* = \beta^A_1 f(n), \tag{19}
\]

\[
p_i^* = x(n, r, \beta^A_1) + \frac{1 - \beta^A_1}{n - 1} f(n) \quad \text{for } i \neq 1, \tag{20}
\]

\[
m_i^* = \sqrt{\frac{rn}{2}} \sum_{i \in N \setminus 1} x_i^2, \tag{21}
\]

\[
x^A(n, r, \beta^A_1) = \max \left(0, \left(1 - \frac{1 - \beta^A_1}{n - 1}\right) f(n) - 3 \sqrt{\frac{rn}{2(n-1)}}\right), \tag{22}
\]

and

\[
\sum_{i \in N} V^A_i(n, r, \sigma, \beta^A_1)/n = \left(\frac{1}{n} - \frac{\beta^A_1}{2n} - \frac{(1 - \beta^A_1)^2}{2n(n-1)}\right) f(n)^2 + \frac{n - 1}{2n} x^A(n, r, \beta^A_1)^2 - \frac{r\sigma^2}{2n} - \frac{rn\sigma^2}{2(n-1)} \left(\beta^A_1 - \frac{1}{n}\right)^2. \tag{23}
\]

\(^{16}\)To focus on the role incentives have in motivating monitoring effort, it is important that the cost of providing a given level of monitoring not depend on the task assignment. Accordingly, the \( n \) in \( \gamma^C_0(M) = n/\sum_{i \in N} m_i \) is the number of partners in the partnership, rather than the number of partners being monitored or the number of partners doing the monitoring. As a result, the superiority of asymmetric contracts is attributable to the incentive effect, not a comparative advantage or a technological gain from specialization. Defining the \( n \) in \( \gamma^C_0(M) \) to be the number of partners who are monitored increases the certainty equivalent under asymmetric task assignment.

\(^{17}\)In the symmetric case, recall that shirking the monitoring task becomes more severe as \( n \) gets larger; above a critical threshold, no monitoring at all is done given \( f(n) = 1 \).
In contrast to the argument advanced in Alchian and Demsetz (1972) that the monitor is disciplined solely by his role as residual claimant, this analysis shows that serving as sink for the signals $s_i$ provides separate, additional incentives to monitor. To see this, observe that even if the monitoring partner owns no stake in the output of the firm (i.e., $\beta_i^4 = 0$), Eq. (22) from Theorem 3 implies strictly positive and increasing incentive intensity (i.e., $x'(3, r, 0) > 0$ and $\partial x^4(n, r, 0) / \partial n > 0$), and hence strictly positive monitoring effort, $m_i^* > 0$, provided partners are sufficiently risk tolerant.\footnote{Examining (22), it is straightforward to show that $n \geq 3$ and $r < 1/9$ implies $x^4(n, r, 0) > 0$. Thus, for a broad set of parameter values $x^4(n, r, 0) > x^C(n, r)$. Optimal levels of $\beta_i^4$, however, are non-zero, as we discuss below. Choosing $\beta_i^4$ optimally increases the set of parameters for which $x^4(n, r, \beta_i^4) > x^C(n, r)$.}

The per-partner certainty equivalent in this particular asymmetric contract is denoted $\sum_{i \in N} V_i^A(n, r, \alpha, \beta_i^4)/n$. Similarly, denote by $\sum_{i \in N} V_i^C(n, r, \alpha) / n$ the per-partner certainty equivalent in an $n$-person partnership given common monitoring under a symmetric contract where $\beta_i = 1/n$, $\alpha_i = \alpha$, and $\alpha_{ij} = \alpha_i / (n - 1)$ for all $i$ and $j$ in $N$. Recall that in a symmetric contract, all partners undertake the same mix of monitoring and production tasks. Moreover, each partner bears a $1/(n - 1)$ share of the signal-contingent payment made to every other partner. Compare $\sum_{i \in N} V_i^A(n, r, \alpha, \beta_i^4) / n$ from expression (23) to the certainty equivalent given a symmetric contract from Theorem 1 (13),

$$\sum_{i \in N} V_i^C(n, r, \alpha) / n = \frac{2n - 1}{2n^2} f(n)^2 - \frac{\sigma^2}{2n^2} + \frac{x^C(n, r)^2}{2},$$

where $x^C(n, r)$ is given by (16). The difference between the per-partner certainty equivalents under the asymmetric and symmetric contracts decomposes into three factors

$$\sum_{i \in N} V_i^A(n, r, \alpha, \beta_i^4) / n - \sum_{i \in N} V_i^C(n, r, \alpha) / n$$

$$= \frac{1}{2} \left[ \frac{n - 1}{n} \left( x^A(n, r, \beta_i^4)^2 - x^C(n, r)^2 \right) \right]$$

$$- \left[ \frac{f(n)^2}{2(n - 1)} \left( \beta_i^4 - \frac{1}{n} \right)^2 \right] - \left[ \frac{\sigma^2}{2(n - 1)} \left( \beta_i^4 - \frac{1}{n} \right)^2 \right].$$

Label the three terms in square brackets the monitoring factor, the production cost factor, and the risk-sharing factor, respectively.

First consider the production cost factor. Observe that since $\sum_{i \in N} \beta_i = 1$, quadratic cost of effort for each partner $i$ of $\frac{1}{2} p_i^2$ implies the total productive effort exerted by the $n$ partners that is attributable to a stake in the output is always 1. This amount of effort is provided at lowest cost when every partner makes the same choice of effort, i.e., $p_i = 1/n$ for all $i \in N$. The production factor vanishes when $\beta_i^4 = 1/n$, and is negative when $\beta_i^4 \neq 1/n$. Next, consider the risk-sharing factor. The risk associated with output $x$ is shared inefficiently if $\beta_i^4 \neq 1/n$. The risk sharing factor also vanishes when output risk is shared efficiently, i.e., when $\beta_i^4 = 1/n$. These
two factors both reduce the attractiveness of asymmetric contracts relative to symmetric contracts.

The monitoring factor reflects differences in certainty equivalent due to differences in the monitoring under the asymmetric and symmetric contracts. Under either contract, the larger is the incentive intensity, the higher is the certainty equivalent. When asymmetric contracts induce a higher value of \( x^{A}(n, r, \beta_{1}^{A}) \) than the corresponding \( x^{C}(n, r) \), an asymmetric contract may be preferred, i.e., it is possible for the monitoring factor to dominate the production and risk-sharing factors, so that an asymmetric contract yields a higher certainty equivalent than the symmetric contract. Consider the special case of an asymmetric contract where \( \beta_{1}^{A} = 1/n \). In this case, the production and risk-sharing factors vanish. Comparing (22) from Theorem 3 with (16) from Theorem 1, is plain that
\[
\sum_{i \in N} V_{i}^{C}(n_{1}, r, \sigma) / n_{1} > \sum_{i \in N} V_{i}^{A}(n_{1}, r, \sigma, 1/n_{1}) / n_{1}
\]
and
\[
\sum_{i \in N} V_{i}^{C}(n_{2}, r, \sigma) / n_{2} < \sum_{i \in N} V_{i}^{A}(n_{2}, r, \sigma, 1/n_{2}) / n_{2}.
\]

For a firm of fixed size, Theorem 4 contrasts the per-partner certainty equivalent from the best symmetric contract with that of a one-monitor asymmetric contract. Thus, Theorem 4 confirms the intuition that as partnerships grow in size (for exogenous reasons), it is advantageous to direct a subset of partners to concentrate on monitoring activities since large partnerships with asymmetric contracts provide a higher certainty equivalent than any symmetric contract for some parameter values.

Mergers of large businesses in many industries must be approved by regulatory bodies that may block the merger because it would have adverse anti-competitive effects. In some cases, regulators have required large businesses to break up for the same reason. Recently, concern that the audit market for large public companies is an anti-competitive oligopoly led legislators to require the GAO to study the effects of audit industry concentration (GAO 2003, p. 2). These events illustrate the potential for a mandated audit firm break-up, which is tantamount to an exogenously-imposed firm size, \( n \). Theorem 4 speaks to this case. Absent such regulatory intervention, one would expect firm size to emerge endogenously. Thus, we examine whether asymmetric task assignment emerges when partnership structure is optimized over firm size, contract parameters \( [\beta, z, A] \), and alternative task assignments. Theorem 5 establishes that for a range of exogenous parameters \( r \) and \( \sigma \), an asymmetric task assignment dominates the best symmetric task assignment.
Theorem 5. There exists an interval \([a, b]\) such that \(r \in [a, b]\) and \(\sigma^2 > 6/(7r)\) implies the per-partner certainty equivalent given the optimal symmetric task assignment, optimized over firm size, is dominated by an asymmetric task assignment, i.e.,

\[
\sum_{i \in N} V^C_i(n^{C*}, r, \sigma)/n^{C*} < \sum_{i \in N} V^A_i(n^{C*}, r, \sigma, 1/n^{C*})/n^{C*}.
\]

Theorem 5 is proven assuming every partner has an equal stake in the firm and fixing the size of the partnership at the optimal size given symmetric task assignment, \(n^{C*}\). Choosing the ownership stake, \(\beta^A_i\), of the monitoring partner optimally can only increase per-partner certainty equivalent in the asymmetric task assignment case. Likewise, optimizing the size of the partnership to suit the asymmetric task assignment must (weakly) increase per-partner certainty equivalent. Numerical investigation shows that per-partner certainty equivalent can be higher when \(\beta^A_i\) and \(n\) are chosen optimally given asymmetric task assignment. Also, the \((r, \sigma)\)-parameter space where asymmetric contracts are preferred grows larger as the restrictions imposed on the asymmetric contract in are relaxed.

To illustrate, suppose \(r = 0.010\), \(\sigma = 8\) and \(f(n) = 1\). Given a symmetric contract, the maximal per-partner certainty equivalent, \(\sum_{i \in N} V^C(n^{C*}, 0.010, 8)/n^{C*} = 0.2888\), is attained in for partnership size \(n^{C*} = 5\). This choice of \(r = 0.010\) can be shown to lie outside the interval \([a, b]\) specified in Theorem 5. As a result, the per-partner certainty equivalent under asymmetric task assignment of the type characterized in Theorem 3 across 5 partners, \(\sum_{i \in N} V^A(5, 0.010, 8, \frac{1}{5})/5 = 0.2427\) is less than optimal per-partner certainty equivalent given symmetric task assignment. However, it is also true that \(\sum_{i \in N} V^A(n, 0.010, 8, 1/n)/n > \sum_{i \in N} V^C(n^{C*}, 0.010, 8)/n^{C*}\), for any \(n \geq 22\). That is, the per-partner certainty equivalent under asymmetric task assignment is greater than the maximal value given a symmetric contract (i.e., 0.2888) for any \(n \geq 22\).

From (22), observe also that \(x^A(n, r, \beta^A_i)\) is increasing in \(\beta^A_i\). This means that the monitoring factor cannot be separated entirely from the production and risk-sharing factors. As in the case of symmetric contracts, optimal contracts with a single monitor require that ownership stakes and incentive intensities be determined simultaneously. Continuing the example above, for a partnership of size \(n = 22\), \(\beta^A_{22} = 0.0923 > 1/22\) provides the highest certainty equivalent, 0.2902, over asymmetric contracts with a single monitor. In this case, the optimal ownership stake of the monitoring partner is more than twice the ownership share of any other partner under the symmetric contract. This is consistent with the practice of giving a larger ownership stake to partners that have oversight responsibilities over other partners, for example, an office managing partner in a large public accounting firm.

The principal finding in this section is that asymmetric task assignment dominates symmetric task assignment in large organizations.\(^{19}\) More structure must be assumed

\(^{19}\) Consistent with this finding, audit firms maintain staffs of specialized professionals who are primarily engaged in quality control. For instance, a pamphlet produced by KPMG’s Department of Professional Practice—Audit and Risk Advisory, “Our System of Quality Controls” (March 2004), states that quality control is “supported by the firm’s organizational structure, principally the separation of the responsibility for oversight of risk management and quality control (under the Vice Chair—Risk and Regulatory
to refine this observation. First, no claim has been made that designating a single partner as sole monitor is optimal. Appointing more than one monitor may further increase per-partner certainty equivalent. Second, there may be limits on a partner’s ability to monitor that are not captured in this model. For instance, a partner may be unable to monitor peers who work in other cities or whose area of expertise is far removed from his own. Also, a partner may face time constraints that limit the number of individuals he can monitor. Where the ability to monitor is constrained, it is nevertheless true that making the monitor serve as the sink for incentive payments to other partners, combined with asymmetric task assignment, combats shirking of the monitoring task. To illustrate, suppose that it is infeasible for partners in one office of a large multi-office partnership to monitor partners in another office. Further suppose that total profits are allocated (e.g., per capita) across offices. It remains to allocate profits across partners within the same office. Designating one (or a few) partners as monitors of the other partners in that office, requiring the designated monitor(s) to serve as sink for the incentive payments to the other partners, and paying all partners in that office only from the profit pool for that office, preserve incentives to monitor. In turn, this preserves incentives to produce.

5. Conclusions

Gains from risk sharing are a ubiquitous consideration in partnership design. They are also an example of a broader range of gains from scale and scope that favor larger partnerships. These gains, however, are offset by the costs of free riding, which also increase in partnership size. When the combined output of all the partners is the only observable, the size of the partnership is determined by equating the marginal improvement in per-partner certainty equivalent from better diversification (and other synergies) against the marginal loss of productive effort due to free riding when one more partner is added to the firm.

The availability of noisy signals of partners’ productive efforts permits signal-contingent contracts to be written. Such contracts improve the per-partner certainty equivalent by reducing free riding and allowing better diversification of partner-specific output shocks. However, when the choice of monitoring effort that

(footnote continued)

Matters, who reports directly to KPMG’s chairman) from the responsibility for managing the firm’s business activities and objectives.” A specialized group reporting to the Vice Chair—Risk and Regulatory Matters is “charged with promoting quality and controlling risk by focusing on the firm’s risk management processes. This group also monitors compliance with the firm’s policies and procedures through coordination of internal and external quality review programs for the Audit and Risk Advisory Services practice . . . .” Another specialized group has “responsibility for conducting compliance tests of firm policies in areas such as the independence of the firm’s professionals.” Moreover, “Each of [the firm’s geographic areas] . . . has a designated area risk management partner, and associated business units have a professional practice partner. These provide risk management and quality leadership in their respective areas or business units and direct adherence to firm policy and professional standards.”
determines the precision of signals is made privately, monitoring itself is subject to free riding. Free riding is more severe under common monitoring than under self monitoring. Moreover, the balanced-budget constraint imposes interdependence on partners’ draws. Thus, a compensation formula that ties one partner’s draw to a noisy signal of his effort imposes risk on that partner and also on other partners of the firm whose draws necessarily must depend on the signal. This interdependence means that the choice of signal precision and signal weight in compensation formulas is more complex than in models with a risk-neutral principal who can costlessly absorb signal risk. In firms where every partner is risk averse, the choice of a signal’s precision and the signal’s weighting in the formula that determines a partner’s draw must balance four factors: (i) the personal cost of producing the signal, (ii) the output due to the signal-contingent compensation, (iii) the risk imposed on the partner who is monitored, and (iv) the risk imposed on the partners who serve as the sink. Strikingly, changes in optimal partnership size can be discontinuous in the underlying parameter values. This finding offers one explanation why there are no medium-sized public accounting firms.

Given symmetric task assignments, when monitors internalize much of the benefit attributable to the monitoring activity, as in the case of self monitoring, monitoring activity is substantial even in the largest partnerships. When monitors internalize little of the benefit attributable to the monitoring activity, as in the case of common monitoring, monitoring ceases as partnerships grow in size. Incentives to monitor can be restored by concentrating the monitoring task in the hands of a subset of the partners (i.e., the role of sink can be concentrated in a single partner), thereby increasing partner welfare. Since the variability of this partner’s draw is very high when signal precision is low, this partner has a strong incentive to produce precise signals of the productive efforts of the other partners. The increase in per-partner certainty equivalent from concentrating the monitoring task in this fashion improves on the symmetric solution where each partner undertakes the same mix of monitoring and production tasks. This demonstrates that task specialization may be expected to emerge endogenously even when members of a firm are \textit{ex ante} identical. This observation seems important to understanding how and why specialization emerges in organizations. An interesting empirical implication of the analysis is that symmetric task assignment is optimal in the smallest partnerships, while larger partnerships are predicted to be structured so that some partners specialize in monitoring while others, who specialize in production, are effectively discouraged from monitoring.

\textbf{Appendix A}

\textbf{Proof of Theorem 1.} In the case where no monitoring is possible, $c(p_i, m_i) = c(p_i, 0) = \frac{1}{2} p_i^2$. The precision of signals in this case is presumed to be zero, so $g_i^0(M)$ is unboundedly large for any choice of $M$. Hence, $\alpha_i = 0$ and $\alpha_{ij} = 0$
for all \( i \) and \( j \) in \( N \) because of the unboundedly costly risk associated with these uninformative signals. From (1), the payoff of a partner reduces to

\[
\beta_i x + \gamma_i - \frac{1}{2} p_i^2. \tag{A.1}
\]

Since the partners are identical, \( \beta_i^* = 1/n \) for all \( i \in N \) best motivates the productive effort across the partners, which implies the second-best level of effort is \( p_i^* = f(n)/n \). Simplifying the certainty equivalent per partner, (9), so that it corresponds to (A.1) and substituting for \( p_i^* \) and \( \beta_i^* \) yields (13) in the case where \( x_i^*(n, r) = 0 \).

In the self monitoring case, partner \( i \)'s certainty equivalent (9) can be written

\[
\gamma_i + \alpha_i p_i + \beta_i f(n) \sum_{j \in N} p_j - \sum_{j \in N \setminus i} \alpha_j p_j - \frac{1}{2} p_i^2 - m_i
\]

\[
- \frac{r}{2} \left( n \beta_i^2 \sigma^2 + \frac{\alpha_i^2}{m_i} + \sum_{j \in N \setminus i} \frac{\alpha_j^2}{m_j} \right). \tag{A.2}
\]

The first-order conditions on expression (A.2) with respect to \( p_i \) and \( m_i \) imply \( p_i^* = \alpha_i + \beta_i f(n) \), and \( m_i^* = \alpha_i \sqrt{r}/2 \). Since the objective function is concave in \( p_i \) and \( m_i \), each partner’s choices of productive effort and monitoring effort are given by a first-order condition. Substituting these values for \( p_i^* \) and \( m_i^* \) into (A.2) yields this expression for partner \( i \)'s certainty equivalent:

\[
\gamma_i + \frac{\alpha_i^2 - \beta_i^2 f(n)^2}{2} + \beta_i f(n) \sum_{j \in N} (\alpha_j + \beta_j f(n)) - \sum_{j \in N \setminus i} \alpha_j (\alpha_j + \beta_j f(n)) - \alpha_i \sqrt{r}/2
\]

\[
- \frac{r}{2} \left[ n \beta_i^2 \sigma^2 + \sqrt{2/r} \left( \alpha_i + \sum_{j \in N \setminus i} \frac{\alpha_j^2}{\alpha_j} \right) \right]. \tag{A.3}
\]

To maximize the certainty equivalent, it is necessary to consider the impact of changing \( \alpha_i \) on the certainty equivalent not only of partner \( i \), but also the other \( n - 1 \) partners. In the symmetric solution, each partner has an equal stake in the output, \( \beta_i^* = 1/n \) for all \( i \), and bears an equal share of the cost of providing the signal-contingent incentive for every other partner, \( x_j^* = \alpha_j/(n-1) \) for all \( i \) and \( j \) and \( j \neq i \). Making these substitutions, we have

\[
\sum_{i \in N} V_i^S(n, r, \sigma)/n = \frac{1}{n} \sum_{i \in N} \left[ \gamma_i + \frac{(2n-1) f(n)^2}{2n^2} + \frac{\alpha_i f(n)}{n} \right.
\]

\[
+ \frac{1}{n-1} \sum_{j \in N \setminus i} \alpha_j \left( \frac{n-2}{n} f(n) - \alpha_j \right)
\]

\[
- \frac{r \sigma^2}{2n} - \sqrt{r}/2 \left( 2 \alpha_i + \frac{1}{(n-1)^2} \sum_{j \in N \setminus i} \alpha_j \right). \tag{A.4}
\]

It remains to solve for the incentive coefficient on signal \( s_i \), \( \alpha_i^{Si}(n, r) \). Taking just the terms involving \( \alpha_i \) from expression (4) gives

\[
\frac{x_i^2}{2} + \frac{f(n)}{n} \alpha_i + \left( \frac{(n-2)}{n} f(n) - \alpha_i \right) \alpha_i - 2 \alpha_i \sqrt{\frac{r}{2}} - \frac{1}{n-1} \alpha_i \sqrt{\frac{r}{2}} = \alpha_i \left( \frac{n-1}{n} f(n) - \sqrt{\frac{r}{2}} \frac{2n-1}{n-1} \right) - \frac{x_i^2}{2}.
\]

The first-order condition on this last expression with respect to \( \alpha_i \) implies (14).

Finally, substituting (14) into (A.4) yields (13).

In the common monitoring case, the certainty equivalent of partner \( i \) is

\[
\gamma_i + \alpha_i \beta_i + \beta_i f(n) \sum_{j \in N} p_j - \sum_{j \in N \setminus i} \alpha_{ji} \beta_j - \frac{1}{2} \beta_i^2 - m_i
\]

\[\begin{aligned}
- \frac{rn}{2} \left( \beta_i^2 \sigma^2 + \sum_{j \in N \setminus i} \alpha_{ji}^2 \right).
\end{aligned}\]

These conditions imply \( p_i^* = f(n) \beta_i + \alpha_i \), and

\[
\left( m_i^* + \sum_{j \in N \setminus i} m_j \right)^2 = \frac{rn}{2} \left( \alpha_i^2 + \sum_{j \in N \setminus i} \alpha_{ji}^2 \right).
\]

Following the same steps as in the case of self monitoring gives (16) and (17). \( \Box \)

**Proof of Corollary 1.** Taking the partial derivatives of \( \alpha_i^{Si} \) and \( m_i^{Si} \) with respect to \( n \) and \( r \) yields the results. First, consider the effects of increasing \( n \) and \( r \) on \( \alpha_i \). Since \( \alpha_i^{Si} \geqslant 0, n \geqslant 1, r > 0 \), and \( f(n) \) is an increasing function of \( n \),

\[
\frac{\partial \alpha_i^{Si}(n, r)}{\partial n} = f'(n)(n-1) + \frac{f(n)}{n^2} + \sqrt{\frac{r}{2n-1}} \geq 0,
\]

and

\[
\frac{\partial \alpha_i^{Si}(n, r)}{\partial r} = - \frac{2n-1}{4n} \sqrt{\frac{r}{2}} \leq 0.
\]

Next, consider the effects of increasing \( n \) and \( r \) on \( m_i \). \( \frac{\partial m_i^{Si}(n, r)}{\partial n} = \sqrt{\frac{r}{2}} \frac{2 \alpha_i^{Si}(n, r)}{2n} \geq 0. \)

Also,

\[
\frac{\partial m_i^{Si}(n, r)}{\partial r} = \left( \alpha_i^{Si} - \frac{2n-1}{n-1} \sqrt{\frac{r}{2}} \right) \frac{2}{2r} \sqrt{\frac{r}{2}}.
\]

It is clear that \( \frac{\partial m_i^{Si}(n, r)}{\partial r} \geq 0 \), when \( r \) is small, \( \frac{\partial m_i^{Si}(n, r)}{\partial r} \leq 0 \), when \( r \) is big, and \( \frac{\partial m_i^{Si}(n, r)}{\partial r} \) changes sign only once. \( \Box \)
Proof of Corollary 2. Consider first the comparative statics on $\alpha_i^C$. From (16),

$$\frac{\partial \alpha_i^{C*}(n,r)}{\partial r} = - \frac{n + 1}{2} \sqrt{\frac{2}{r(n-1)}} < 0$$

and

$$\frac{\partial \alpha_i^{C*}(n,r)}{\partial n} = f'(n)(n-1) + \frac{f(n)}{n^2} - \frac{n - 3}{2(n-1)^{3/2}} \sqrt{\frac{r}{2}}.$$ 

The sign of $\frac{\partial \alpha_i^{C*}(n,r)}{\partial n}$ depends on the size of $r$. For large $r$, it is positive, so $\alpha_i^{C*}(n,r)$ is increasing. For small $r$, it is negative, so $\alpha_i^{C*}(n,r)$ is decreasing. The value of $\alpha_i^{C*}(n,r) \geq 0$ and there are some values of $n$ and $r$ for which this minimum is attained, so the comparative statics are not strict.

Next consider the comparative statics on $m_i^C$. From (17),

$$\frac{\partial m_i^{C*}(n,r)}{\partial r} = \frac{\sqrt{n-1}}{n} f(n) - \frac{n + 1}{n - 1} \sqrt{\frac{r}{2}}$$

and

$$\frac{\partial m_i^{C*}(n,r)}{\partial n} = \left( \frac{f(n)(2-n)}{2n^2} + \frac{f'(n)(n-1)}{n} \right) + \sqrt{\frac{r}{2}} \sqrt{\frac{r}{2n(n-1)}}.$$ 

The comparative statics follow by inspection. □

Proof of Theorem 2. Since $n = 2$, we have $\alpha_{12} = \alpha_1$ and $\alpha_{21} = \alpha_2$ and $\beta_2 = 1 - \beta_1$. In the case of common monitoring, the first-order conditions (10) and (11) imply $p_i = \alpha_i + \beta_i$ for $i \in \{1, 2\}$ and $m_1 + m_2 = \sqrt{r(\alpha_1^2 + \alpha_2^2)}$. Substituting these values into the expression for the per partner certainty equivalent gives

$$\sum_{i \in \{1, 2\}} V_i^C(2, r, \sigma) / 2 = \frac{f(2)^2}{4} - \frac{\sigma^2}{2} + (\beta_1 - \beta_1^*) \left( \frac{\sigma^2 + f(2)^2}{2} \right) + \frac{f(2)}{2} \left( (1 - \beta_1)\alpha_1 + \beta_1\alpha_2 \right) - \frac{3}{2} \sqrt{r(\alpha_1 + \alpha_2)^2} - \frac{\alpha_1^2 + \alpha_2^2}{4}. \quad (A.6)$$

The first-order conditions on this expression with respect to $\beta_1$, $\alpha_1$ and $\alpha_2$ imply $\beta_1^* = \frac{1}{2}$ and $\alpha_i^* = f(2)/2 - 3\sqrt{r}/2$. The Hessian of the objective function (A.6) is

$$\begin{pmatrix}
-f(2)^2 - 2r\sigma^2 & -\frac{f(2)}{2} & \frac{f(2)}{2} \\
-\frac{f(2)}{2} & -\frac{1}{2} - \frac{3\sqrt{r}k\alpha_2^2}{2k^2} & \frac{3r^2\alpha_1\alpha_2}{2(rk)^{3/2}} \\
\frac{f(2)}{2} & \frac{3r^2\alpha_1\alpha_2}{2(rk)^{3/2}} & -\frac{1}{2} - \frac{3\sqrt{r}k\alpha_1^2}{2k^2}
\end{pmatrix},$$

where

$$f(t) = \frac{1}{t^2}, \quad k(t) = \sqrt{\frac{2}{t}}$$

and

$$\pi_1^{-} = \frac{f(2)}{2} - 3\sqrt{r}/2, \quad \pi_2^{-} = \frac{f(2)}{2} - 3\sqrt{r}/2.$$
where \( k = x_1^2 + x_2^2 \). It is straightforward to verify that this matrix is negative definite for all non-negative values of \( r, x_1, x_2 \) and \( \sigma^2 \), so the objective function (A.6) is globally concave. Hence, the first-order conditions identify the global optimum.

In the case of self monitoring, the first-order conditions (10) and (11) imply \( p_i = x_i + \beta_i f(2) \) for \( i \in \{1, 2\} \) and \( m_i = x_i \sqrt{r/2} \). Substituting these values into the expression for the per partner certainty equivalent gives

\[
\sum_{i \in [1, 2]} V^S_i (2, r, \sigma) / 2 = \frac{f(2)^2}{4} - \frac{r\sigma^2}{2} + (\beta_1 - \beta_1^2) \left( r\sigma^2 + \frac{f(2)^2}{2} \right)
+ \frac{f(2)}{2} \left( (1 - \beta_1) x_1 + \beta_1 x_2 \right)
- \frac{3}{2} \sqrt{r/2} (x_1 + x_2) - \frac{x_1^2 + x_2^2}{4}.
\]  

(A.7)

The first-order conditions on this expression with respect to \( \beta_1, x_1 \) and \( x_2 \) imply \( \beta_i^* = \frac{1}{2} \) and \( x_i^* = f(2)/2 - 3\sqrt{r}/2 \). The Hessian of the objective function (A.7) is

\[
\begin{pmatrix}
-f(2)^2 - 2r\sigma^2 & -f(2)/2 & f(2)/2 \\
-f(2)/2 & -1/2 & 0 \\
f(2)/2 & 0 & -1/2
\end{pmatrix}.
\]

It is straightforward to verify that this matrix is negative definite for all non-negative values of \( r \) and \( \sigma^2 \), so the objective function (A.7) is globally concave. Hence, the first-order conditions identify the global optimum. \( \square \)

**Proof of Corollary 3.** Recall from Theorem 1 that the per-partner certainty equivalent given monitoring technology \( t \) is

\[
\sum_{i \in N} V_i^n (n, r, \sigma) / n = \frac{2n - 1}{2n^2} - \frac{r\sigma^2}{2n} + \frac{x^*_{i^n}(n, r)^2}{2}.
\]

Define the derivative of this expression with respect to \( n \) to be \( D^n (n, r, \sigma) \). Then

\[
D^S (n, r, \sigma) = D^n (n, r, \sigma) + x^*_{i^n} \frac{\partial x^S_*}{\partial n}
\]

and

\[
D^C (n, r, \sigma) = D^n (n, r, \sigma) + x^*_{i^n} \frac{\partial x^C_*}{\partial n},
\]

where

\[
\frac{\partial x^S_*}{\partial n} = \frac{1}{n^2} + \frac{\sqrt{r}}{2} \frac{1}{(n + 1)^2} \quad \text{and} \quad \frac{\partial x^C_*}{\partial n} = \frac{1}{n^2} - \frac{\sqrt{r}}{2} \frac{3n - 1}{(n - 1)\sqrt{n - 1}}.
\]
Hence \( \partial x^S(n,r)/\partial n \geq \partial x^C(n,r)/\partial n \), for all \( n \) and \( r \). Further, from Theorem 1, for all \( n \) and \( r \), \( x^S(n,r) \geq x^C(n,r) \). This implies \( 0 = D^C(n^C, r, \sigma^2) \leq D^S(n^C, r, \sigma^2) \). That is, at the optimal partnership size given common monitoring, the derivative of per-partner joint surplus with respect to partnership size, \( n \), given self monitoring is weakly positive. In turn, this implies that the optimal partnership size given self monitoring, \( n^C \), is greater than the optimal partnership size given self monitoring, \( n^S \). □

**Proof of Theorem 3.** Under the asymmetric contract, the payoff to partner 1 is

\[
V_1^A(n, r, \sigma, \beta_1^A) = \gamma_1 + \beta_1^A x - \sum_{i \in N \setminus 1} x_i s_i - \frac{1}{2} \beta_1^2 p_1^2 - m_1,
\]

and partner 1’s certainty equivalent is

\[
q_1 = \frac{V_1^A(n, r, \sigma, \beta_1^A)(\gamma_1 + \beta_1^A x - \sum_{i \in N \setminus 1} x_i s_i - \frac{1}{2} \beta_1^2 p_1^2)}{\sum_{i \in N \setminus 1} x_i s_i}.
\]

The first-order conditions on \( V_1^A(n, r, \sigma, \beta_1^A) \) with respect to \( p_1 \) and \( m_1 \) imply (19) and (21). For partner \( i, i \neq 1 \), the payoff is

\[
V_i^A(n, r, \sigma, \beta_1^A) = \gamma_i + \beta_i x + x_i s_i - \frac{1}{2} \beta_i^2 p_i^2,
\]

and partner \( i \)'s certainty equivalent is

\[
q_i = \frac{V_i^A(n, r, \sigma, \beta_1^A)(\gamma_i + \beta_i x + x_i s_i)}{\sum_{i \in N \setminus 1} x_i s_i}.
\]

The first-order condition on \( V_i^A(n, r, \sigma, \beta_1^A) \) with respect to \( p_i \) implies (20). Combining (A.8) and (A.9), the certainty equivalent to be divided among the partners is therefore

\[
\sum_{i \in N} V_i^A(n, r, \sigma, \beta_1^A) = f(n) \sum_{i \in N} p_i - m_1 - \frac{1}{2} \sum_{i \in N} p_i^2 - \frac{rn}{2} \left( \sum_{i \in N} \beta_i^2 \sigma^2 + \sum_{i \in N \setminus 1} \frac{2x_i^2}{m_1} \right).
\]

By assumption, \( \beta_i = (1 - \beta_1^A)/n \) and \( x_i = x^A \) for \( i \in N \setminus 1 \). Substituting these relationships and (19)–(21), we have

\[
\sum_{i \in N} V_i^A(n, r, \sigma) = f(n)^2 + (n-1)x^A f(n) - \frac{\beta_1^2 f(n)^2}{2} - \frac{n-1}{2} \left( \frac{1}{n-1} f(n) + x^A \right)^2 - \frac{rn \sigma^2}{2} \left( \frac{\beta_1^2}{n-1} + \frac{(1 - \beta_1^A)^2}{n} \right) - 3x^A \sqrt{\frac{rn(n-1)}{2}}.
\]

The first-order condition on \( \sum_{i \in N} V_i^A(n, r, \sigma, \beta_1^A) \) with respect to \( x^A \) implies (22). In turn, substituting \( x^{AS}(n,r) \) into (A.10) gives the certainty equivalent under the
asymmetric contract

\[
\sum_{i \in N} V^*_i(n, r, \sigma, \beta^4_1) = \left( 1 - \frac{\beta^4_1}{2} - \frac{(1 - \beta^4_1)^2}{2n(n-1)} \right) f(n)^2 \\
+ (n-1) \alpha^4(n, r) \left( \left( 1 - \frac{1 - \beta^4_1}{n-1} \right) f(n) - 3 \sqrt{\frac{rn}{2(n-1)}} \right) \\
- n - \frac{1}{2} \alpha^4(n, r)^2 - \frac{rn\sigma^2}{2} \left( \beta^4_1 - \frac{(1 - \beta^4_1)^2}{n-1} \right),
\]

which reduces to (23). □

**Proof of Theorem 4.** Consider the case when \( n = 2 \). From (23),

\[
\sum_{i=1}^{2} V^*_i(2, r, \sigma, \beta^4_1)/2 = \left( 1 - \frac{\beta^4_1}{4} - \frac{(1 - \beta^4_1)^2}{4} \right) f(2)^2 + \frac{\alpha^4(2, r)^2}{4} - \frac{r\sigma^2}{4} \\
- r\sigma^2 \left( \beta^4_1 - \frac{1}{2} \right)^2.
\]

From (22), \( \alpha^4(2, r, \beta^4_1) = \beta^4_1 - 3\sqrt{r} \). Substituting in this value for \( \alpha^4(2, r, \beta^4_1) \), and taking the first-order condition on the per-partner certainty equivalent with respect to \( \beta^4_1 \) implies

\[
\beta^4_1 = \frac{1 + 2r\sigma^2 - 3\sqrt{r}}{1 + 4r\sigma^2},
\]

and \( \alpha^4(n, r, \beta^4_1) = \beta^4_1 - 3\sqrt{r} \). It follows that (i) \( \alpha^4(n, r, \beta^4_1) \) is decreasing in \( r \) and \( \sigma^2 \), and (ii) \( \lim_{r \to 0} \alpha^4(2, r, \beta^4_1) = \alpha^4(2, r)^2 - \alpha^4(2, r)^2 = 0 \). Therefore, (25) holds for \( n_1 = 2 \).

Suppose \( \beta^4_1 = 1/n \). From (22), \( \alpha^4(n, r, 1/n) > 0 \) for all \( n > 2 \) whenever \( r < 2^4/3^3f(3)^2 \) as long as \( f \) is a weakly increasing function of \( n \). From (16) in Theorem 1, for every \( r \) there exists an \( n(r) \) such that \( \alpha^4(n, r) = 0 \) for every \( n > n(r) \). Hence, \( \alpha^4(n, r, 1/n) > 0 \) and \( \alpha^4(n, r) = 0 \) for \( r < 2^4/3^3 \) and \( n \) big enough. In this case (24) reduces to \( (n-1)\alpha^4(n, r)/2 > 0 \), which establishes (26). □

**Proof of Theorem 5.** Setting \( \beta^4_1 = 1/n \) in (24), it is apparent that sufficient conditions for the optimal symmetric task assignment, optimized over all \( n \), to be dominated by an asymmetric task assignment are: (i) \( \alpha^4(n^c, r) = 0 \) and (ii) \( \alpha^4(n^c, r, 1/n^c) > 0 \). The proof identifies parameter values for which conditions (i) and (ii) are satisfied.

Regarding condition (i), (16) implies \( \alpha^4(n, r) = 0 \) if

\[
0 \geq \frac{n-1}{n} f(n) - \sqrt{\frac{r}{2}} \frac{n+1}{\sqrt{n-1}} \quad \text{or} \quad r \geq \frac{2f(n)^2(n-1)^3}{n^2(n+1)^2}.
\]
Regarding condition (ii), (22) implies $\mathcal{I}^a(n, r, 1/n) > 0$ if

$$0 < \left(1 - \frac{1 - 1/n}{n - 1}\right)f(n) - 3\sqrt{\frac{rn}{2(n - 1)}} \quad \text{or}$$

$$r < \frac{2f(n)^2(n - 1)^3}{9n^3}.$$  \hfill (A.12)

Choose $a$ and $b$ equal to the right-hand sides of (A.11) and (A.12), respectively, evaluated at $n^{C^*}$.

Now check that $a < b$. From inspection of (A.11) and (A.12), $a < b$ iff $9(n^{C^*})^3 < (n^{C^*})^2(n^{C^*} + 1)^2$ iff $7 \leq n^{C^*} < \infty$. Since $f(1) = 1$ and $f(n)$ is weakly increasing, $n^{C^*}_{f(n) = 1} < n^{C^*}_{f(n) = 1}$. Hence, $n^{C^*} \geq 7$ provided the solution to

$$\hat{c} \sum_{i \in \mathcal{N}} V_{i}^C(n, r, \sigma) / n|_{f(n) = 1, x^C = 0} = \frac{1 - n + nr^2}{n^3} = 0$$

is greater than or equal to 7. This reduces to $\sigma^2 > 6/(7r)$, as given in the statement of Theorem 5. Finally, observe that $n^{C^*}$ is finite when $\mathcal{I}^a(n^{C^*}, r) = 0$ by the regularity condition $f(n) \leq n/\sqrt{2n - 1}$ imposed following Theorem 1. \hfill $\Box$

References


Working paper, Stanford University.