Net-of-tax returns on real and financial assets

Suppose there is a riskless financial asset that costs one dollar at the beginning of the period and pays $1 + r$ dollars at the end of every period. The difference, $r$, is taxed at rate $t$. Thus, it is possible to borrow and lend at the after-tax rate of $r(1 - t)$ per period. There is also a real asset costing $x > 0$. The asset produces a riskless pretax cash flow of $k$ in perpetuity at the end of each period. For tax purposes, the original cost of the asset may be depreciated straight line at rate $0 \leq d \leq 1$. Taxable income for any period is the pretax cash flow less the depreciation. Tax is paid at rate $t$, so that at the end of the first period, the net-of-tax cash flow is

$$k - t(k - dx) = k(1 - t) + dtx.$$ 

The cash outflow to acquire the asset is not tax deductible. Suppose further that other income is available to offset tax depreciation in excess of cash flows from the asset. Then the net present value of investing in the real asset is

$$-x + \frac{1}{d} \sum_{n=1}^{1/d} \frac{k(1-t) + dtx}{[1 + (1-t)r]^n} + \sum_{n=1+1/d}^{\infty} \frac{k(1-t)}{[1 + (1-t)r]^n} = -x + \frac{k}{r} \left(1 - \frac{dtx}{(1-t)r} \left[1 + (1-t)r\right]^{-\frac{1}{d}} \right).$$

In (1),

- the first term is the cost of the asset,
- the second term is the net-of-tax cash flows over the depreciable life of the asset discounted back to time zero at the after-tax rate of return, and
- the final term is the net-of-tax cash flows from the asset after the asset is fully depreciated.
This quantity can be rewritten as (2). Each of the three components of this expression has a straightforward interpretation.

- The first term is the cost of the asset.
- The second term is the present value of the perpetual pre-tax cash flow from the asset, \( k \), capitalized at the pre-tax rate, \( r \). Note that this is the same as the after tax cash flow, \( k(1 - t) \), capitalized at the after tax discount rate, \( r(1 - t) \).
- The final term is the present value of the reduction in tax payments afforded by the depreciation deduction (often called the tax shield).\(^1\) This is the only term where tax factors \( d \) and \( t \) play a role.
  - If either the depreciation rate or the tax rate is zero, then the before tax and net-of-tax present values of the asset are the same.
  - If \( d \) and \( t \) are both positive, then the tax shield is also positive. Its value increases with both \( d \) and \( t \).
  - When tax depreciation is immediate, i.e., \( d = 1 \), then the tax shield is
    \[
    
    \frac{x}{1 + (1 - t)r} 
    
    \]
    This value can be substantial. Consider an investment that may be deducted fully from taxes in the year it is made, such as advertising. For \( t = 30\% \) and \( r = 10\% \), the tax shield is 28\% of the cost of the asset.

Figure 1 illustrates the relationship between the depreciation rate and the value of the tax shield. The importance of the tax shield to project valuation is highlighted by the three calculations in table 1.

\(^1\) Notice that this term has the same form as the present value of an annuity of \( dtx \) for \( 1/d \) periods discounted at rate \( r(1 - t) \).
Figure 1. Illustration of the dependence of the value of the tax shield (vertical axis) on the depreciation rate (horizontal axis). Assumptions: $r = 10\%$ and $t = 30\%$. 
Figure 2. Illustration of the dependence of the value of the tax shield (vertical axis) on the tax rate (horizontal axis). Assumptions: $r = 10\%$ and $d = 30\%$. 
### Table 1. Illustration of the dependence of the net present value of a real project on the income tax rate and depreciation tax shield.
In each case, the pretax discount rate is $r = 10\%$, the cost of the asset is $x = \$1,000$, and the perpetual future cash flow is $k = \$100$.
In the first panel, there are no taxes, so $t = 0$ and $d = 0$. In the second panel, the tax rate is $t = 30\%$ and the asset can be depreciated straight line over ten years. In the third panel, the tax rate is $t = 50\%$ and the asset can be depreciated straight line over two years.
1. Questions:

One surprising conclusion that might be drawn from this analysis is that the net-of-tax present value of an investment is increasing in $t$. That is, the higher the tax rate, the more attractive is the investment!

(1) The statement above is a striking one. What must be held constant (and probably is not in real life) for it to be true?

(2) What condition defines equilibrium prices and returns on financial assets relative to real assets?

(3) What opportunities would you be able to exploit if you observed that prices and returns were not in equilibrium?

(4) What effect would your actions tend to have on prices and returns?

(5) **Tricky question.** What happens to the value of the value of the depreciation tax shield as the tax rate, $t$ approaches 100%? What is the intuition behind this result?