1. Prove that for all $n \in \mathbb{N}$ we have $n(n + 2) \leq 2^{n+1}$.

2. Prove that for all $n \geq 7$ we have $3^n \leq n!$.

3. Let $\mathcal{U} = \left\{ \frac{2n+1}{n+1} : n \in \mathbb{N} \right\}$.
   
   (i) Prove that $\mathcal{U}$ is non-empty and bounded above by 2.
   
   (ii) Prove that if $a$ is a real number with $a < 2$, then there is an $n \in \mathbb{N}$ such that $a < \frac{2n+1}{n+1}$.
   
   (iii) Prove that $\sup \mathcal{U} = 2$. 

Return by Monday 11th March