**IMPROPER INTEGRALS**

**MA 141**

**Evaluate:** \( \int_{0}^{3} \frac{dx}{x - 1} \) if possible.

**Solution:** Observe that the line \( x = 1 \) is a vertical asymptote of the integrand. Since it occurs in the middle of the interval \([0, 3]\), we must re-write the integral as

\[
\int_{0}^{3} \frac{dx}{x - 1} = \int_{0}^{1} \frac{dx}{x - 1} + \int_{1}^{3} \frac{dx}{x - 1}
\]

where

\[
\int_{0}^{1} \frac{dx}{x - 1} = \lim_{t \to 1^-} \int_{0}^{t} \frac{dx}{x - 1} = \lim_{t \to 1^-} \ln|t - 1| - \ln|1| = \lim_{t \to 1^-} (\ln|t - 1| - 0) = \lim_{t \to 1^-} \ln(1 - t) = -\infty
\]

since \( 1 - t \to 0^+ \) as \( t \to 1^- \). Thus, \( \int_{0}^{1} \frac{dx}{x - 1} \) is divergent. This implies that \( \int_{0}^{3} \frac{dx}{x - 1} \) is divergent. [We do not need to evaluate \( \int_{1}^{3} \frac{dx}{x - 1} \).]