Mikhail A. Kagan
List of refereed publications on Quantum Gravity and Cosmology

(The order of authors is alphabetical)


   While the standard (introductory physics) way of computing the equivalent resistance of nontrivial electrical circuits is based on Kirchhoff’s rules, there is a mathematically and conceptually simpler approach, called the method of nodal potentials, whose basic variables are the values of the electric potential at the circuit’s nodes. In this paper, we review the method of nodal potentials and illustrate it using the Wheatstone bridge as an example. We then derive a closed-form expression for the equivalent resistance of a generic circuit, which we apply to a few sample circuits. The result unveils a curious interplay between electrical circuits, matrix algebra, and graph theory and its applications to computer science. The paper is written at a level accessible by undergraduate students who are familiar with matrix arithmetic. Additional proofs and technical details are provided in appendices.


   Quantum matter in quantum space-time is discussed using general properties of energy-conservation laws. As a rather radical conclusion, it is found that standard methods of differential geometry and quantum field theory on curved space-time are inapplicable in canonical quantum gravity, even at the level of effective equations.


   As we typically teach in an introductory mechanics course, choosing a “good” coordinate frame with convenient axes may present a major simplification to a problem. Additionally, knowing some conserved quantities provides an extremely powerful problem-solving tool. While the former idea is typically discussed in the context of Newton’s laws, the latter starts with introducing conservation of energy even later. This work presents an elegant example of implementing both aforementioned ideas in the kinematical context, thus providing a “warm-up” introduction to the standard tools used later on in dynamics. Both the choice of the (non-orthogonal) coordinate frame and the conserved quantities are rather nonstandard, yet at the same time quite intuitive to the problem at hand. Two such problems are discussed in detail with two alternative approaches. The first approach does not even require knowledge of calculus. In an online Appendix 1 I also present the brute-force solution involving a coupled system of differential equations. In addition, a few exercises and another similar problem for students’ “homework” are provided in the appendix.


   As has been widely discussed recently, our planet may become a target for asteroids. We consider several scenarios proposed to prevent asteroid collisions with Earth. The asteroid 99942 Apophis is considered as a typical representative. Among others, the recent “gravitational tractor” scenario² is discussed. For a simplistic toy-model we obtain estimates for both the mass of the tractor and the amount of fuel required to tow a potentially dangerous asteroid off-course so as to avoid a collision with the Earth. In addition, we analyze two more scenarios titled “sling-shot” and “bumping”, and comment on their relative efficiency compared to the “towing” scenario. Based on the analysis, the bumping scenario looks most promising.

A consistent implementation of quantum gravity is expected to change the familiar notions of space, time and the propagation of matter in drastic ways. This will have consequences on very small scales, but also gives rise to correction terms in evolution equations of modes relevant for observations. In particular, the evolution of inhomogeneities in the very early universe should be affected. In this paper consistent evolution equations for gauge-invariant perturbations in the presence of inverse triad corrections of loop quantum gravity are derived. Some immediate effects are pointed out, for instance concerning conservation of power on large scales and non-adiabaticity. It is also emphasized that several critical corrections can only be seen to arise in a fully consistent treatment where the gauge freedom of canonical gravity is not fixed before implementing quantum corrections. In particular, metric modes must be allowed to be inhomogeneous: it is not consistent to assume only matter inhomogeneities on a quantum corrected homogeneous background geometry. In this way, stringent consistency conditions arise for possible quantization ambiguities which will eventually be further constrained observationally.


A fully consistent linear perturbation theory for cosmology is derived in the presence of quantum corrections as they are suggested by properties of inverse volume operators in loop quantum gravity. The underlying constraints present a consistent deformation of the classical system, which shows that the discreteness in loop quantum gravity can be implemented in effective equations without spoiling space-time covariance. Nevertheless, non-trivial quantum corrections do arise in the constraint algebra. Since correction terms must appear in tightly controlled forms to avoid anomalies, detailed insights for the correct implementation of constraint operators can be gained. The procedures of this article thus provide a clear link between fundamental quantum gravity and phenomenology.


Within a perturbative cosmological regime of loop quantum gravity corrections to effective constraints are computed. This takes into account all inhomogeneous degrees of freedom relevant for scalar metric modes around flat space and results in explicit expressions for modified coefficients and of higher order terms. It also illustrates the role of different scales determining the relative magnitude of corrections. Our results demonstrate that loop quantum gravity has the correct classical limit, at least in its sector of cosmological perturbations around flat space, in the sense of perturbative effective theory.


Inhomogeneous cosmological perturbation equations are derived in loop quantum gravity, taking into account corrections, in particular, in gravitational parts. This provides a framework for calculating the evolution of modes in structure formation scenarios related to inflationary or bouncing models. Applications here are corrections to the Newton potential and to the evolution of large scale modes which imply non-conservation of curvature perturbations possibly noticeable in a running spectral index. These effects are sensitive to quantization procedures and test the characteristic behavior of correction terms derived from quantum gravity.

Cosmological perturbation equations are derived systematically in a canonical scheme based on Ashtekar variables. A comparison with the covariant derivation and various subtleties in the calculation and choice of gauges are pointed out. Nevertheless, the treatment is more systematic when correction terms of canonical quantum gravity are to be included. This is done throughout the paper for one example of characteristic modifications expected from loop quantum gravity.


Non-minimal actions with matter represented by a scalar field coupled to gravity are considered in the context of a homogeneous and isotropic universe. The coupling is of the form -(1/2)x_i phi^2R. The possibility of successful inflation is investigated taking into account features of loop cosmology. For that end a conformal transformation is performed. That brings the theory into the standard minimally coupled form (Einstein frame) with some effective field and its potential. Both analytical and numerical estimates show that a negative coupling constant is preferable for successful inflation. Moreover, provided fixed initial conditions, larger |x_i| leads to a greater number of e-folds. The latter is obtained for a reasonable range of initial conditions and the coupling parameter and indicates a possibility for successful inflation.


Non-minimally coupling a scalar field to gravity introduces an additional curvature term into the action which can change the general behavior in strong curvature regimes, in particular close to classical singularities. While one can conformally transform any non-minimal model to a minimally coupled one, that transformation can itself become singular. It is thus not guaranteed that all qualitative properties are shared by minimal and non-minimal models. This paper addresses the classical singularity issue in isotropic models and extends singularity removal in quantum gravity to non-minimal models.


In this paper we review a model based on loop quantum cosmology that arises from a symmetry reduction of the self-dual Plebanski action. In this formulation the symmetry reduction leads to a very simple Hamiltonian constraint that can be quantized explicitly in the framework of loop quantum cosmology. We investigate the phenomenological implications of this model in the semi-classical regime and compare those with the known results of the standard Loop Quantum Cosmology.

**PUBLICATIONS IN PREPARATION (the order of authors is alphabetical)**