(1) (3 pts) State the three conditions that a relation must satisfy to be a partial ordering (state what these conditions mean, not only their names).

A relation $R$ on $X$ is a partial ordering if it is reflexive ($xRx$ for all $x \in X$), weakly antisymmetric ($xRy$ and $yRx$ implies $x = y$), and transitive ($xRy$ and $yRz$ implies $xRz$).

(2) (4 pts) Verify that the relation $R$ on $\{a, b, c, d\}$ given by

$R = \{(a,a), (a,b), (a,c), (a,d), (b,b), (b,d), (c,c), (c,d), (d,d)\}$

is a partial ordering.

We must check the three conditions listed in the last problem. It is reflexive as $(a,a), (b,b), (c,c), (d,d) \in R$. It is weakly antisymmetric as we can see that we never have any pair of relations $xRy$ and $yRx$ in the set except for those listed when we checked that the relation is reflexive. To see that it is transitive, we consider pairs of elements in $R$ of the form $(x,y), (y,z)$. If $x = y$ (resp. $y = z$), there is nothing to check as this says that $xRx$ and $yRx$ (resp. $xRy$ and $yRy$), which clearly implies that $xRz$. So we only have to check pairs which do not include the four elements $(a,a), (b,b), (c,c), (d,d)$. Namely, we note that $(a,b), (b,d) \in R$, and indeed $(a,d) \in R$, while $(a,c), (c,d) \in R$ and $(a,d) \in R$. This exhausts all possibilities.

(3) (3 pts) The relation $R$ on $X = \{1, 2, 3, 4\}$ given by

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

is an equivalence relation (you don’t have to prove this). List the equivalence classes of $R$.

We find the equivalence classes partitioning $X$ by writing down the classes corresponding to different elements until we have hit everything. Starting with 1, we see that $[1]_R$, which is the set of elements related to 1, is equal to $[1]_R = \{1, 2\}$. We have already found that 2 is in this first class, and the elements 3 and 4 are only related to themselves, and hence lie in their own classes. Thus, the equivalence classes are $\{1, 2\}, \{3\}, \{4\}$. 