Section 4.4 (Present Value of Annuities)

• What’s different about the following 2 scenarios:

  – Felix plans on depositing $400 per month for the next 3 years into an account that earns 3% interest. After 3 years he plans to use the money he’s saved up to buy a new car.

  – Oscar got a car loan with 3 percent interest. He’ll be making monthly payments of $400 for the next 3 years to pay off his loan.

• Present Value Annuity Factor

  – For a given interest rate, payment frequency, and number of payments, the present value annuity factor is the present value if each payment is $1.

  – We use $a_{ni}$ to denote the present value annuity factor,

    $n$ is the number of payments and

    $i$ is the interest rate per time period (as a decimal)

  – we pronounce the symbol ‘annie’

• Present Value of an Ordinary Annuity

  \[ PV = PMT \cdot a_{ni} \]

  $PV$ represents the present value of the annuity

  $PMT$ represents the amount of each payment

  and $a_{ni}$ is the present value annuity factor (as defined above)

• Present Value of an Annuity Due

  \[ PV = PMT \cdot a_{ni} (1 + i) \]
• A look at how paying down loans works.

Mackenzie took out a a $5000 loan for 5 years at 5% annual interest.

– The annuity value for \( n = 5 \) and \( i = 0.05 \) is \( a_{5|0.05} = 4.329476671 \)

– We find the PMT using Present Value of an Annuity Formula

\[
PV = PMT \cdot a_{i|n}
\]

\[
\frac{5000}{4.329476671} = PMT
\]

\[
1154.87 = PMT
\]

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<th>Year</th>
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<th>Interest Charged</th>
<th>Payment</th>
<th>Ending Balance</th>
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</table>
• Finding Present Value Annuity Factors ($a_{\overline{n}|i}$)
  
  – *With a Table:* [http://accountinginfo.com/study/pv/table-pv-a-01.pdf](http://accountinginfo.com/study/pv/table-pv-a-01.pdf)
  
  – *With a Formula*

  * **Present Value Annuity Factor** (Two Formulas)

  \[
  a_{\overline{n}|i} = \frac{1 - (1 + i)^{-n}}{i}
  \]

  \[
  a_{\overline{n}|i} = \frac{s_{\overline{n}|i}}{(1 + i)^n}
  \]

  Where $i =$ interest rate per payment period
  
  $n =$ number of payment periods

• **Example:** Find $a_{\overline{10}|0.048}$

• **Four Step Method for entering $a_{\overline{n}|i}$ into your calculator**

  (Based on $a_{\overline{n}|i} = \frac{s_{\overline{n}|i}}{(1 + i)^n}$ version of the formula)

  Step 1:

  $(1 + i)^n =$

  Step 2:

  $-1 =$

  Step 3:

  $/i =$

  Step 4

  $/(1+i)^n =$
1. Jeremiah is at a used car lot and can afford monthly payments of $225 a month for 3 years. His credit qualifies him for an interest rate of 5.8%. How much can Jeremiah afford to pay for his used car?

2. Suppose Jeremiah picks out a used car that is $7010. He still qualifies for an interest rate of an interest rate of $5.8$, and will take out a three year loan as planned. Find Jeremiah’s monthly payments.

3. A customer at a car dealership can afford to pay $280 a month for a 4 year car loan. Assuming they qualify for 4.85% interest, and their first payment will be due immediately, how much can they afford to borrow? If they have a $2000 down payment saved up, how much can they afford to spend on a car?

4. Ferguson needs to take out a $210,000 mortgage to buy a house. He will take out a 30 year mortgage, and his interest rate will be 3.95% (compounded monthly). What will his monthly payments be? How much in interest will Ferguson pay over the course of these 30 years?

5. Ferguson changed his mind. He still needs to take out a $210,000 mortgage to buy a house, but he decides to take out a 15 year mortgage instead. If his interest rate is still 3.95% (compounded monthly). What will his monthly payments be? How much in interest will Ferguson pay over the course of these 15 years?

6. Hiro just retired, and he has $720,000 in his 401(k) retirement account. He plans on using this account to make monthly payments (to himself) to cover his expenses. He assumes he will pay himself over the next 20 years, and assumes that this account will continue to earn 6.11% interest (for the next 20 years). Under these assumptions, how much can Hiro afford to pay himself each month? What happens if Hiro is still alive after 20 years?

7. (Optional) Clarissa is planning on buying a house. She has determined that the most she can afford each month is $675. She is planning on taking out a 30 year mortgage, and she qualifies for an interest rate of 4.1%. Assuming there are no additional closing costs or fees, what is the most money she can afford to borrow? If Clarissa has saved up $15,000 as a down payment, what is the most expensive house she can afford to buy?