1. The Hilbert Matrix is defined
\[ H \equiv \{ h_{ij} \} \equiv \left\{ \frac{1}{i + j - 1} \right\}. \]

Use Gaussian Elimination, Jacobi, Gauss Seidel and the SOR method to solve the linear system
\[ Hx = b. \]

Use the exact solution \( x = [1, 1, \cdots, 1]^T \) to choose \( b \).

2. In the proof of convergence of conjugate gradient method for minimizing the quadratic functions, show the following
\[
\inf_{\tilde{P}_{k+1}(A)} \| \tilde{P}_{k+1}(A)(x_0 - x^*) \|^2_A \leq \max_{\lambda \in \sigma(A)} \min_{\tilde{P}_{k+1}(A)} |\tilde{P}_{k+1}(\lambda)|^2 \|x_0 - x^*\|^2_A, \tag{1}
\]

where \( \sigma(A) \) is the set of all eigenvalues of \( A \).