IV: Measurement and Superposition

We now consider the peculiar nature of measurement and superposition of states in the quantum world. We will discuss measurement first and then the superposition of quantum states.

That's not the way it happened historically. Historically, de Broglie, Schroedinger, Heisenberg and Dirac worked out the rather abstract concepts involved in the superposition of quantum states first. Then Born, Pauli, Bohr, Heisenberg and Dirac worked out the concepts needed to understand measurement in the quantum world. But I agree with those who think that it's easier to follow if you reverse the order.

1. Measurement: Suppose we perform a precise measurement of some quantized quantity, Q, on a system in some state, $\Psi$ (Upper case Greek letters will denote quantum states).

We know that we'll obtain one of the possible values, q, of Q and the likelihood of obtaining that value is measured numerically by some probability. This probability of getting the value, q, upon making a measurement of the quantity, Q, on a system in the state, $\Psi$, is denoted by $P(q / Q, \Psi)$, and, like all probabilities, satisfies,

$$0 \leq P(q / Q, \Psi) \leq 1.$$ 

If the probability is 1, i.e. if q is the only value that would ever result from a measurement of Q on the state $\Psi$, then we say that $\Psi$ is an eigenstate for the quantity, Q, and the definite value, q, is called the eigenvalue of Q corresponding to the state, $\Psi$. 'Eigen' comes from the German word for 'proper'. In such an eigenstate the system responds to the measurement as if the quantity, Q, possessed the value, q, just as the measurement was made.

Usually the probability is less than 1 and there are many values that can result from the measurement and with different probabilities. In those usual cases there is nothing one can do to guarantee a particular result in an individual measurement. The individual outcomes appear to be a matters of pure chance (Fig. 4.1).
Some have speculated that these individual outcomes are determined (or at least influenced) by the subconscious or spiritual state of the measurer. This is one of the extreme versions of the notion that the act of measurement creates the observed value of the quantity measured. The first thing to bear in mind, however, is that most measurements in quantum physics research are executed by sophisticated, highly automated devices, monitored, if at all, by tired graduate students who may be asleep at the moment the measurement is made! The second thing to bear in mind is that if, indeed, our spiritual / subconscious state does influence the individual outcomes, then, apparently, that state is itself controlled by the probabilities that govern, rigorously, the statistical distribution of the individual outcomes.

But what happens to the system as a consequence of our measurement? Classically it was always presumed that it was possible, in principle, to perform measurements that had arbitrarily small effect on the measured
system. Real measurements weren't like that, of course, but one could get as close as desired to that ideal if one tried hard enough. But we will now see that that isn't possible for quantum systems!

For how do we know that what we've really done when we think we've performed a measurement was actually a measurement? Is there anything we could, in principle, do to test such an assumption? Well, the official view is that, however evanescent the values of quantities may be in the quantum world, surely, it must be the case that after one has just performed a measurement on a system and obtained a particular value, if one immediately repeated the same measurement on the same system, one would have to get the same value again (Fig. 4.2).

![Diagram of Jz measurement regions](image)

**Fig. 4.2:** Quick repetition of a measurement yields the same result.

This is clearly a desperate attempt to hang on to some vestige of classical causality. One is claiming that the possession of a definite value of a quantity by a physical system can not, hopefully, disappear immediately! Soon, maybe, but not immediately. Furthermore, it appeals again to an ideal concept of measurement. Even classically, most real measurements can't be repeated immediately, quite apart from their disturbing nature. One is grasping here for a notion of an ideal measurement which, disturbing though
it may be, can be immediately repeated and will not then further disturb. If
one can't have at least this, what is left of the concept of measurement?!

Desperate it may be, but it seems to work. And notice what it entails. If upon
making that immediate repetition of the measurement we are guaranteed to
get the same result, then the quantum state of the system immediately after
the first measurement must be an eigenstate corresponding to that first result.
For those are the only quantum states that allow a guaranteed result for a
measurement. So clearly our ideal measurement has usually had a dramatic
effect on the quantum state of the system, changing it into an eigenstate of
the measured quantity. If the state was such an eigenstate to begin with, as
would be the case with the repetition, then no change is required.

Consider a specific example. It seems that if one looks closely enough for an
electron and finds one within a tiny volume inside an atom, then
immediately afterwards the corresponding wave function would have to be
confined to where you found it and would subsequently evolve from there.
But such a wave function would, as a consequence of the uncertainty
principle, have such high energies associated with it that the electron would
soon escape from the atom! It seems, therefore, impossible to locate an
electron within an atom without seriously disrupting the structure of the
atom and drastically changing the subsequent behaviour of the electron.

As for measurements that can't be immediately repeated, who knows what
they do to the measured system?! Eventual repetition or follow up
measurements of a different kind can lead to some, indirect inference about
the initial disturbance. Often, one doesn't really care. But if one does care,
the measurement process must be designed very carefully!

Let's represent the general case symbolically as follows: The possible values
of a quantized quantity, Q, will be \( q_1, q_2, q_3, \ldots \). The initial state will be,
\( \Psi \), and an (ideal) measurement of Q will, with probability,
\( P_Q(q_n | \Psi) \), yield
the result, \( q_n \), and transform the state into \( \Psi_{qn} = \Psi \rightarrow \Psi_{qn} \). The probabilities
satisfy,
\[
P_Q(q_n | \Psi_{qn}) = 1, \quad \text{and} \quad P_Q(q_m | \Psi_{qn}) = 0, \quad \text{for} \quad q_m \neq q_n.
\]
And for the immediate repetition of the measurement,
\[
(\Psi_{qn})_{qn} = \Psi_{qn} \quad ..
\]
This process of changing a quantum state by an act of measurement is called **state reduction**.

**2. Incompatible measurements:** Now that the immediate repetition of a measurement has been given some importance as a test of the legitimacy of a process of measurement, we are inclined to ask further questions about it. For instance, what would happen if, immediately after one *kind* of measurement, we performed a *different* kind of measurement? Presumably each measurement would lead to a definite result and a state reduction to an eigenstate of the quantity being measured. So, suppose we measure Q and R, in that order, getting the results, \( q_n \) and \( r_m \), respectively. Then, starting with the state, \( \Psi \), we get the sequence of state reductions,

\[
\Psi \rightarrow \Psi_{qn} \rightarrow (\Psi_{qn})_{rm}.
\]

This last state, \( (\Psi_{qn})_{rm} \), is an eigenstate of R, but is it *still* an eigenstate of Q? Does Q still have a definite value? Remember, the second measurement was performed *immediately* after the first! *(Fig. 4.3)*

**Fig. 4.3:** Quick sequence of two different measurements of components, \( J_z \) and \( J_x \), of angular momentum.
To answer this question we can immediately re-perform the first measurement to see if, for any result of the second measurement, we always get back the original result of the first measurement. The answer is that for some pairs of different measurements we do always recover the original result (Fig. 4.4a) – and for some pairs we don’t (Fig. 4.4b)!

Fig. 4.4a:  
$J_x \rightarrow E$ measurement sequence performed on initial $E = E_n$, eigenstate for a system with $j = 1$, i.e., $J^2 = j(j+1)\hbar^2 = 2\hbar^2$.

Fig. 4.4b:  
$J_x \rightarrow J_z$ measurement sequence performed on initial $J_z$ eigenstate for a system with $j = 1$, i.e., $J^2 = j(j+1)\hbar^2 = 2\hbar^2$. 
Another way to put this question is to compare the first measurement sequence with the reversed sequence; a measurement of \( R \) yielding \( r_m \), immediately followed by a measurement of \( Q \) yielding \( q_n \). Here the state reductions would be,

\[
\Psi \rightarrow \Psi_{rm} \rightarrow (\Psi_{rm})_{qn}.
\]

Is \((\Psi_{rm})_{qn}\) the same state as \((\Psi_{qn})_{rm}\) ? If it is, then this last state in both sequences is a joint eigenstate of both \( Q \) and \( R \).

If this is always the case, no matter what the initial state \( \Psi \) is and no matter what values \( q_n \) and \( r_m \) result, then we say that the quantities \( Q \) and \( R \) are compatible and we would write,

\[
(\Psi_{qn})_{rm} = (\Psi_{rm})_{qn} : = \Psi_{qn}, \text{ rm}.
\]

If it is not always the case, then we say \( Q \) and \( R \) are incompatible.

In the case of compatible quantities any possible definite values for both quantities can coexist in some quantum states. For incompatible quantities this can not always happen and instead we have uncertainty relations holding for such pairs of quantities.

As examples; \( x, y \) and \( z \) components of position are compatible quantities, as are \( x, y \) and \( z \) components of momentum. But \( x \) components of position are not compatible with \( x \) components of momentum (remember \( \Delta x \Delta p_x \geq \hbar /2 \)) and similarly for \( y \) and \( z \) components. No two different components of angular momentum are compatible; and so on and so on.

In all our discussion of measurement, so far, we have not mentioned time very much. We had a tiny allusion to time when we considered immediate repetitions. But what about the whole issue of when a physical system is in a particular quantum state and/or when a particular measurement is made on a system? We will not discuss this at length in this part, but we will set up some notation for such a discussion. A general quantum state for a system at the time, \( t \), will be denoted by symbols like, \( \Psi(t) \), \( \Phi(t) \), etc. The states immediately resulting from an ideal measurement of the quantity, \( Q \), being made at the time, \( t \), on \( \Psi(t) \), \( \Phi(t) \), etc. and yielding the value, \( q_n \), will be denoted by,
\[ \Psi(t)_{qn}, \Phi(t)_{qn}, \text{etc.} \]

And finally, the quantum state at time, \( t \), evolving from the states, \( \Psi(t')_{qn}, \Phi(t')_{qn}, \text{etc.} \) resulting from the (earlier) measurement at time, \( t' \), will be denoted by

\[ \Psi_{qn,t(t)}, \Phi_{qn,t(t)}, \text{etc.} \]

3. **Superposition of quantum states:** Superposition of waves is an old story. The electromagnetic waves that are accompanied by (or associated with) Einstein's photons had been known to be superposable since the late nineteenth century. De Broglie's matter waves were required by him to be superposable so that their destructive interference would prohibit the formation of atomic states other than those permitted by Bohr's quantization conditions. And Schroedinger's complete mathematization of de Broglie's ideas incorporated superposability of quantum waves from the ground up.

Finally, Dirac's very general and abstract final formulation of a mature quantum mechanics incorporated the **superposition principle** for quantum states as the basic ingredient.

To understand quantum superposition better we’ll first look at some aspects of superposition of ordinary electromagnetic (EM) waves, in particular the polarization aspect, and see how they affect the associated photons. The top line of **Fig. 4.5** represents the electric fields of vertically and horizontally polarized EM waves oscillating back and forth, as they propagate to the right, with the same frequency and in phase with each other. The bottom line superimposes these two EM waves to obtain the resultant diagonally polarized wave. So long as one refers only to the wave aspects of light there is nothing puzzling about the superposition of vertically and horizontally polarized waves yielding diagonally polarized waves. But from the photon, particle-like perspective the situation seems bizarre.

All the photons associated with the vertically polarized wave are identical, i.e. they are in the same quantum state corresponding to vertical polarization of the wave. Similarly, all the photons associated with the horizontally polarized wave are in the same quantum state, but it’s a very different quantum state than that of the photons associated with the first wave. Now our classical intuition would tell us that when we superimpose the two
waves we should have associated photons that are either in the vertically polarized or the horizontally polarized quantum state. But that is not what happens! Instead, all the photons associated with the resultant, diagonally polarized, wave are in the same quantum state and it’s neither of the previous two states. It’s a unique quantum state corresponding to the diagonal polarization of the resultant wave. Superimposing the waves changes the quantum state of every photon associated with either of the initial waves into the unique state associated with the resultant wave. The individual photon states superimpose just like the waves!

How do we know? How do we test this idea? The empirical test involves just asking: is the photon guaranteed to pass through a vertical, horizontal, or diagonally oriented polarizing filter?

My sun glasses, while not perfect, give preferential treatment to vertically polarized photons; letting them through while tending to block horizontally polarized photons because they are the ones that come from glare intense horizontal surfaces. So my sun glasses, when on my nose, are vertical polarizing filters. But if I take them off and rotate them, they can be horizontal or diagonal or anti-diagonal (i.e. perpendicular to the original diagonal direction) polarizing filters.

Fig. 4.5: Top: Vertically and horizontally polarized waves moving, in phase. Bottom: Resultant diagonally polarized wave.

\[ D = \frac{1}{2} \sqrt{2} \left( V + H \right), \quad D' = \frac{1}{2} \sqrt{2} \left( V - H \right) \]
Of course if I only had to deal with individual photons, I wouldn't need sunglasses; I'd need more light! And when we are dealing with zillions of photons, like on a sunny day, it's more appropriate to talk about the plane of oscillation of the electric field. Nevertheless, *guaranteed passage through an appropriate polarizing filter is what we mean, experimentally, by saying that a photon is initially linearly polarized*. If the preceding vertically and horizontally polarized light waves are superimposed with sufficient care, the photons of the resulting wave are guaranteed to pass through a diagonal polarizing filter! The sufficient care in question requires that the vertical and horizontal waves be exactly in phase with each other, maxima with maxima, minima with minima.

There is an infinite variety of superimposed quantum states corresponding to the infinite variety of resultant waves that can be obtained by varying the conditions of the superposition of the waves. We can also make *circularly polarized* light by superimposing V and H waves, or D and D' waves that are a quarter cycle out of step with one another (Fig. 4.6). This results in an electric field that has a constant magnitude and a direction that steadily marches around the perimeter of a circle (Fig. 4.6, right) that is perpendicular to the propagation direction of the wave. It is customary to represent delaying the oscillations by a quarter cycle by lower case $i$ and advancing the oscillations by a quarter cycle by $-i$. So we write

$$R = (1/2)^{1/2} (V + i \ H),$$

**Fig. 4.6**: Delaying the horizontal wave by one quarter wavelength yields a resultant wave that is right circularly polarized. Advancing it by one quarter wavelength would have yielded a left circularly polarized resultant wave.
for **right** circularly polarized light and

$$L = (1/2)^{1/2}( V - i H ),$$

for **left** circularly polarized light. In the former case the unique quantum

---

**Fig. 4.7**: Relations of superposition between polarized electric fields or between quantum polarization states of individual photons. Left: relationships between the linearly polarized fields or states, V, H, D and D’. Right: relationships between linearly (V, H) and circularly (R, L) polarized fields or states. Note the use of the imaginary unit, i, to represent 1/4 cycle delay since two such delays comprise a half cycle delay which is the same as reversal, – 1.
state of each associated photon is one in which the spin of the photon points in the same direction as the motion of the photon. In the latter case the spin points in the opposite direction from the motion. None of the previous linearly polarized photon quantum states had these properties. A simple test for whether a photon is in the R or L state is whether it is guaranteed to pass through a particular kind of crystal along one or another possible paths.

To add to the counterintuitive nature of these individual photon states, it turns out that if one is “looking” for circularly polarized photons, then, regardless of the initial quantum state of the photons, every photon will be observed to be either right or left circularly polarized! The initial quantum state will determine only the probability distribution for the two results. Similarly, if one is “looking” for vertically polarized photons, then, regardless of the original quantum state, every photon will be observed to be either vertically or horizontally polarized! Again, only the probability distribution of the two results will depend on the original quantum state. If one is “looking” for diagonally polarized photons, every photon will be observed to be either diagonally or anti-diagonally polarized.

The probabilities we’re talking about are determined by the numerical coefficients that appear in the superposition equations. For instance,

\[ D = (1/2)^{1/2} (V + H) = (1/2)^{1/2} V + (1/2)^{1/2} H, \]

Tells us that if we have a diagonally polarized photon and we put a vertical (V) polarizing filter in its path, the probability that it will pass through (and become a V polarized photon) is 1/2 (the square of the coefficient), i.e., a 50-50 chance. If it doesn't pass through it either gets absorbed and ceases to exist (and further experiments with that photon are off) or it gets reflected and will be found to be an H photon! Similarly, a D filter passes photons that are then D photons or reflects what are then D’ photons.

Of course, regardless of the state of a photon prior to passing through a linear polarizing filter, the state just after such passage is always one of linear polarization in the direction of the filter.

Suppose the filter isn't vertical, inclined by 45° to the diagonal, but has some smaller angle between it and the diagonal. Then the probability to pass an initially D photon will be higher than 1/2 and will approach 1, i.e.,
certainty, as the angle of the filter to the diagonal approaches zero. In the other direction, if the filter axis is inclined by more than $45^0$ to the diagonal, the probability for passing through will be less than $1/2$ and will approach zero as the polarizing filter approaches the anti-diagonal, $D'$. Linearly polarized photons never pass through (ideal) filters oriented perpendicular to the polarization of the photon.

There is, however, an interesting way to get an *originally* $D$ photon, say, through a $D'$ filter. Interpose a $V$ filter. The photon will get through the $V$ filter with $1/2$ probability and, if it does get through, it's now a $V$ photon. So now it will get through the $D'$ filter with $1/2$ probability, becoming a $D'$ photon. So the original $D$ photon gets through the $D'$ filter with a final probability of $1/4$ (Fig. 4.8).

It gets better! Put in two more filters. The first at $22.5^0$ to the $D$ direction and before the $V$ filter, the second at $22.5^0$ to the $D'$ and placed after the $V$ filter.

**Fig. 4.8**: Probabilities for a $D$ polarized photon to get through a filter sequence. For $N$ filters with consecutive angle differences of $90^0/N$, the probability for transmission is $P_N = [\sin (90^0/N)]^{2N}$. As $N \to \infty$, $P_N \to 1$. 
but before the D’ filter. Now the probability of the D photon getting through all the filters is higher than 1/4, it’s slightly above 5/8. Keep interposing more and more filters, with equal diminishing angles between consecutive filters, and the probability for getting through all the filters gets higher and higher, approaching certainty in the limit of an infinite number of filters! Of course, in reality we’d run out of room to insert ever more filters and the poor photon would get dimmer and dimmer as it got red shifted due to energy loss from plowing through all those filters. But with infinitely thin filters that extracted no energy from passing photons the simple theory would apply and all the D photons would come through as D’ photons!

For any photon quantum polarization state, $\Psi$, whatsoever, the probabilities for measuring a polarization from the pair [H,V] or [D, D’] or [L, R] always satisfy,

$$P(H/\Psi) + P(V/\Psi) = P(D/\Psi) + P(D'/\Psi) = P(L/\Psi) + P(R/\Psi) = 1.$$  

Coupled with the general rule, mentioned earlier, that the probability is always = 1 for measuring a property value that the system already definitely possesses just before the measurement, i.e.,

$$P(H/H) = P(V/V) = P(D/D) = P(D'/D') = P(L/L) = P(R/R) = 1,$$

we then infer,

$$P(H/V) = P(V/H) = P(D/D') = P(D'/D) = P(L/R) = P(R/L) = 0,$$

which are examples of the general rule, also discussed earlier, that if a system is in a state with a certain property value then attempts to measure that property will never (prob. = 0) yield a different value.

These aspects that we’ve discussed for the superposition of light waves or photons have close analogues for the superposition of electron states or proton states or the quantum states of any kind of physical system. The details will vary from case to case, but the qualitative features are largely the same in all cases.
For example, electrons carry an internal angular momentum which never goes away (Electrons never stop spinning! There's nothing anyone can do to change the 'rate' of spin. One can only change the direction of the spin. To stop an electron from spinning you'd have to stop it from being an electron!).

The magnitude of electron spin angular momentum is

\[ |S| = \hbar \left[ \left( \frac{1}{2} \right) \left( \frac{1}{2} + 1 \right) \right]^{1/2} = \hbar \left( \frac{3}{4} \right)^{1/2}. \]

The component or projection of this spin along any fixed direction is quantized and can only have two possible precise values, \( \pm \hbar / 2 \). It then follows from the angular momentum uncertainty relations, discussed in III.4, that when one component, say the z-component, \( S_z \), is definite, i.e., precise, then any other component is indefinite and perpendicular components, like \( x \) and \( y \), have zero average value. Nevertheless, any electron spin quantum state, whatsoever, is a superposition of the \( \pm \) eigenstates for any chosen spin component, whatsoever. For example,

\[
\begin{align*}
| x, + > &= (1/2)^{1/2} ( |z, + > + |z, - > ) , & S_x &= \hbar / 2, \\
| x, - > &= (1/2)^{1/2} ( |z, + > - |z, - > ) , & S_x &= - \hbar / 2, \\
| y, + > &= (1/2)^{1/2} ( |z, + > + i |z, - > ) , & S_y &= \hbar / 2, \\
| y, - > &= (1/2)^{1/2} ( |z, + > - i |z, - > ) , & S_y &= - \hbar / 2,
\end{align*}
\]

where that \( i \) symbol is the quarter cycle advance symbol again. These superposition equations tell us, for example, that if we used a Stern-Gerlach apparatus to isolate a beam of \( S_x = \hbar / 2 \) electrons and then subjected that beam to a second Stern-Gerlach apparatus, rotated so as to separate distinct \( S_z \) values, then the original beam would split into two beams of equal intensity (because of the equal size coefficients, \( (1/2)^{1/2} \), for each term in the superposition) for the values, \( S_z = \pm \hbar / 2 \).

By contrast, if we considered a system with internal angular momentum magnitude,

\[ |S| = \hbar \left[ 1(1 + 1) \right]^{1/2} = \hbar 2^{1/2} , \]
then the possible precise component values are, $-\hbar$, 0, $+\hbar$. In this case the expansion of the $S_x = +\hbar$ state in terms of the $S_z = -\hbar$, 0, $+\hbar$ states displays varying probabilities. Thus,

$$| x, + \rangle = (1/2)| z, + \rangle + (1/2)^{1/2}| z, 0 \rangle + (1/2)| z, - \rangle.$$  

So in this case, if a Stern-Gerlach apparatus separates an $S_x = +\hbar$ beam and then that beam is subject to an $S_z$ separating apparatus, the beam will be split up into three beams with the intensities given by the probabilities,

$$P(S_z = h / S_x = h) = 1/4 = (1/2)^2 = P(S_z = -h / S_x = h),$$

$$P(S_z = 0 / S_x = h) = 1/2 = [(1/2)^{1/2}]^2.$$  

The general rule (which is a bit of a mouthful), of which the discussion of the preceding pages has provided various examples, is this.

For any possible quantum state, $\Psi$, of a physical system, and any quantized measurable property, $Q$, for that system, the state $\Psi$ is a superposition of eigenstates, $\Psi_{qn}$, of $Q$ such that the numerical coefficients in the mathematical expression of the superposition determine the probabilities that a measurement of $Q$ on the system in the state $\Psi$ will result in the eigenvalue $q_n$. In particular, the mathematical expression of the superposition can take the form,

$$|\Psi\rangle = c_1 |\Psi_{q_1}\rangle + c_2 |\Psi_{q_2}\rangle + \ldots + c_n |\Psi_{q_n}\rangle + \ldots,$$

such that the probabilities, $P( q_n / Q, \Psi)$, are given by $^3$,

$$P( q_n / Q, \Psi) = |c_n|^2.$$  

Footnotes

1. This $i$ denotes the so-called square root of minus one, i.e., $i = (-1)^{1/2}$. While such a notion makes no sense in ordinary arithmetic, it turns out to be an enormously useful extension of the concept of number in algebra and all ‘higher’ branches of mathematics. Once introduced, it gives rise to the family of complex numbers of the form, $a + i b$, where $a$ and $b$ are
ordinary, so-called ‘real’ numbers. The ordinary numbers, a and b, are then called the real and imaginary parts, respectively, of the complex number, \(a + i b\). Complex numbers are very important in quantum physics.

2. Here “looking” for a particular kind of photon means to be employing apparatus that can detect the presence or absence of photons of that kind.

3. These coefficients in quantum state superpositions are usually complex numbers. If we have, \(c_n = a_n + i b_n\), where \(a_n\) and \(b_n\) are real numbers, then,

\[
|c_n|^2 = a_n^2 + b_n^2,
\]

is called the **squared absolute value** of \(c_n\).

**Appendix A: Quantons: the denizens of the quantum world.**

By the denizens of the quantum world I mean the photons, electrons, protons, neutrons, phonons, muons, pions, kaons, positrons and all the other bosons and fermions including all the leptons, hadrons and, of course, the quarks, neutrinos and gluons. The first few of those named include the constituents of atoms, molecules and their nuclei and if we construe bosons and fermions broadly enough we should include atoms, molecules and nuclei among the denizens. Historically they were among the first to be studied.

Most people call these entities **particles**. The main reason is that when spatial dimensions can be associated with them, those dimensions are tiny! But as one studies QM, one learns these denizens do not always behave like particles. As often as not they behave more like waves and this ambivalence is expressed by the notion of **wave-particle duality**, introduced by Louis de Broglie in 1923. But in fact, as Richard Feynman once put it, they, the denizens, are really “like neither!” waves or particles.
Many physicists and philosophers of physics recognize that the denizens are not particles and are not waves, but believe that it doesn’t do any harm to call them particles. Some workers do believe the denizens really are particles and a smaller number believe instead, that they really are waves. Erwin Schroedinger, one of the original discoverers of QM, spent a lot of effort trying to convince the physics community and himself that they really were waves, but he didn’t succeed. On the other hand he gave some really strong arguments for not regarding them as particles. Briefly, in effect, he argued that they’re not even individuals! The most detailed argument for the denizens being particles was mounted by the physicist, David Bohm, but outside the camp of the committed Bohmians, his argument hasn’t carried the day either. Finally, some workers don’t believe the denizens are anything, because they don’t believe they’re even real! It is something of a scandal that 85 years after QM was fully formulated, i.e., in 1927, such a range of virulent disagreement still flourishes.

In 1988 the French physicist, Jean-Marc Levy-Leblond published a note entitled, “Neither Particles nor Waves, but Quantons”. In that note and a later paper he argued that the widespread use of the ‘particle’ terminology had a debilitating influence on the clarity and precision of thought brought to bear on the problems of quantum physics. He proposed “quantons” as an alternative. At the time I was much less sensitive to such issues than now and like almost everyone else, I ignored him. Since then I believe I have seen many instances of the bad influence of the term, “particles”, claimed by Levy-Leblond, especially in the arguments offered for or against various interpretations of QM.

One, perhaps superficial, but publicly ubiquitous instance of the muddling influence of the pervasive particle terminology, which everyone is familiar with, is the solar system model of the atom with tiny electron particles zooming around a central nucleus in elliptical orbits. This image of atomic structure is frequently seen in books, on TV screens and on the web. But its origin is from the pre-QM, Bohr-Sommerfeld period (1913 – 24), and it ceased being thought accurate with the discovery of QM in 1925-27! To be sure the electrons in an atom are around the nucleus, but they envelope the nucleus more like a cloud or a fog rather than orbiting it like a satellite.

Much earlier than Levy-Leblond, in the historically early days of QM, someone suggested the word, “wavicles”, to generically characterize the denizens of the quantum world. More recently the philosopher, Nick
Huggett has argued that the denizens are a species of what he calls, “quarticles”\(^6\) and the physicist, Hans von Bayer has supported Levy-Leblond’s “quantons”\(^7\). Personally, I prefer “quantons” to the other suggestions because of its etymological connection with the prevailing naming trends (see the opening lines of this Appendix) for the various types of denizens and because it sounds less committal than the other two proposals as to the ultimate nature of the denizens. Nevertheless, I would be happy if any of these terms would displace the general use of “particles” in this context.

A natural thought which has often been expressed to me is: Why not just use the term, “quanta”? But “quanta” has an already firmly established usage that is much broader than that proposed here for “quantons”. Within QM, “quanta” refers to the minimal changes that any quantity or entity is capable of if it is not permitted continuous variation. Thus the smallest possible changes in the binding energies or the angular momenta of a quantum system are the quanta of those quantities and photons are quanta of light.\(^8\) But while photons are also quantons the quanta of angular momentum or of energy, per se, are not.

In any case, from now on in this course the generic term for the quantum denizens will be “quantons”. Hence the title of this Appendix.

**Footnotes:**

8. In any light beam of arbitrary composition from a range of frequencies or wavelengths, the minimal changes that can be made in the intensity of the frequency components of the beam consist in adding or removing single photons to or from, respectively, the frequency components of the beam, even when (as we will eventually see can occur) the number of photons in the beam is indefinite.