A large body of supply chain literature addresses the issue of supply chain coordination via contracts. The bulk of this research provides a static analysis of what contract types can coordinate the supply chain under various demand and industry structure scenarios. The issue of what contracts will actually be implemented, and what the parameters of those contracts will be, has received considerably less attention.

This paper will examine the practicality of implementing the coordinating buyback contract in a single period newsvendor scenario. We will accept the models of contract coordination and instead focus on the issue of contract formation, specifically issues that may prevent the formation of a coordinating contract. Our analysis addresses considerations of transaction costs and risk aversion. We find that for moderate levels of transaction cost and risk aversion, the coordinating contract may become Pareto inferior to the wholesale contract.

1. Introduction

This paper examines limitations and impediments to the implementation of a coordinating contract in a single period newsvendor scenario. Specifically we examine the buyback contract when transaction costs are non-zero and suppliers are risk averse. In the remainder of section 1 we provide a brief overview of the coordination problem. In section 2 we summarize the coordination literature from an economics, marketing and supply chain perspective.

Section 3 develops provides a more detailed development of the analytical models used to evaluate supply chain coordination. Following the established coordination literature we show that a buyback contract can coordinate the supply chain and that under a common set of assumptions this contract Pareto dominates the wholesale contract. We go on to calculate the gains from coordination for the supply chain as a whole and the
range of potential gains for each party. We further show that the range of profit allocation parameters is limited for Pareto dominant contracts, and use a simple bargaining model to determine an equilibrium division of gains.

Section 4 evaluates impediments to implementing the coordinating contract, specifically transaction costs and supplier risk aversion. We show how these issues reduce the set of feasible contracts. Section 5 provides a numerical illustration of the of the coordination problem. We assume uniform demand and then calculate the parameters for a wholesale and buyback contract scenario demonstrating an expansion of output and supply chain profits. We then go on to calculate the range of transaction costs and risk aversion parameters that render this coordinating contract Pareto inferior to the wholesale contract. Section 6 provides summary conclusions and topics for future research.

**The Supply Chain Coordination Problem**

Supply Chain Coordination literature addresses an issue associated with a supply chain composed of independent firms; the fact that optimal decision making at the firm level sub-optimizes the profit of the entire supply chain. This issue has been widely addressed in the economics literature under the general heading of vertical control, and in the marketing literature under the heading of channel coordination.

The problem arises because the retailer’s cost includes the profit of the supplier. Since her unit costs are higher then that would be faced by an integrated firm, the retailer chooses a stocking level below what would be chosen by an integrated firm.
The figure below provides an illustration. In this scenario the supplier offers a good for sale at a set wholesale price.

The supplier’s profit is monotonically increasing in the order quantity. The retailer on the other hand faces declining profits as the probability of sale declines, so the retailer chooses an order quantity that maximizes her profit. The graph clearly demonstrates the impact of double marginalization as the retailer’s optimal order point is lower than the optimal point for the supply chain as a whole.

Coordination of the supply chain concerns the design of an incentive aligning contract that motivates the supplier and retailer to operate so as to maximize the profit of the entire supply chain. The supply chain coordination literature addresses the mechanism that can be used to align incentives. In general, the literature examines the mechanisms which can be used to modify the marginal cost of the retailer so as to motivate her to make the same decisions an integrated firm would make. The literature shows that
under a wide range of circumstances, there are multiple contract forms which can coordinate the supply chain. Furthermore these contracts are shown to be Pareto efficient, that is, all parties are better off under the coordinating contract. Since each party is better off we would expect coordinating contracts to be widely implemented in practice, yet this is not the case. The purpose of this analysis will be to examine a few of the practical limitations that may prevent a coordinating contract from being implemented.

2. Literature Review

Vertical Control and the Economics Literature

The economics literature uses the term vertical control to describe how upstream producers attempt to control the actions of downstream retailers. Tirole (1988) provides an excellent overview of the issue and a summary of the literature. The primary economic issue is one of double marginalization, which occurs in situations where both the supplier and retailer have the ability to set prices above marginal cost. (Spengler 1950) The two successive markups yield a retail price above the integrated channel price. The higher price will lower demand and reduce the supplier's profit. The non-integrated industry is therefore less profitable than the integrated industry.

A secondary issue addressed in the economics literature is that of downstream moral hazard. A downstream moral hazard arises when demand for the item is a function not only of retail price, but also of the level of promotion provided at the retail level. The retailer, forced to bear all promotional costs, is not motivated to provide the optimal level of promotion, he may shirk on promotional responsibility, and/or attempt to free ride on the promotional activity of others. When total sales depends on hidden actions by both parties we have a situation known as double moral hazard. Tirole (1988) briefly address the issue of bilateral moral hazard, but for a more detailed analysis see Bhattacharyya and Lafontaine(1995) or Romano (1994).
Channel Coordination and the Marketing Literature

Closely related to supply chain coordination is the topic of channel coordination, which has been widely analyzed in the marketing literature. Channel coordination can be defined as the setting of all manufacturer and retailer controlled variables at the levels that maximize channel profits. (Jeuland and Shugan, 1983) A series of papers address the issue of channel coordination under various market and coordination scenarios. An influential paper on this topic is Jeuland and Shugan, 1983. They develop a single manufacturer, single retailer model and demonstrate that in a non-coordinated environment that each actor makes self optimizing decisions that sub-optimize the entire channel. Specifically the manufacturer set prices too high, quality too low and promotional efforts too low. The retailer makes similar sub-optimizing decisions. Jeuland and Shugan discuss several mechanisms for coordination but focus primarily on the use of quantity discounts as a coordinating mechanism. They demonstrate that a quantity discount schedule can be developed that causes the retailer’s marginal cost and marginal revenue to cross at the optimal quantity, thus inducing the optimal order quantity.

Following up on Jeuland and Shugan’s paper several other papers were published that examined coordination under various scenarios. Lal (1990) extends the analysis by examining the use of a franchise contract to coordinate the channel. Lal examines both the double marginalization and double moral hazard problems, as well as the issue of horizontal free riding by franchisees. Gerstner and Hess (1995) examine channel coordination, not through contracts, but through the use of price promotions. They develop a model with a segmented customer base, where each segment has a different willingness to pay. They demonstrate how double marginalization can cause the distributed channel to exclude the low value segment whereas the integrated channel would serve both segments. Ingene and Parry (1995) extend the coordination analysis to address a scenario with multiple competing retailers. The model in this paper includes a single manufacturer, a single product and multiple retailers. The paper analyzes a linear quantity discount schedule and a two-part tariff. Ingene and Parry’s model allows for non-price competition between retailers based on promotion,
advertising, or pre-sales support. Weng (1995) examines the issue of quantity discounts as a coordinating mechanism.

**The Supply Chain Coordination Perspective**

While the general problem of coordination has been examined in the economics and marketing literature, the supply chain literature examines the problem from a slightly different perspective. The majority of the economics and marketing literature is based on models that assume deterministic and price dependent demand functions, while the supply chain coordination literature tends to treat demand as stochastic, and independent of price. The operations literature also has a strong emphasis on analyzing the problem in terms of order quantities and the resulting impact on inventory. As one would expect the bulk of the marketing and economic literature is focused on price as the primary choice variable and little attention is given to inventory effects.

Cachon (2003) provides an extensive review of coordinating literature and a detailed tutorial on the supply chain coordination problem. He examines multiple demand scenarios and evaluates how various contract types can coordinate the supply chain in each scenario. He provides an extensive bibliography of coordination papers for each scenario. Cachon’s analysis of the buyback contract in the newsvendor scenario forms the basis on which the analysis in this paper is developed. Tsay, Nahmias, and Agrawal (1999) provide another detailed review of supply chain contracting, focusing more on a conceptual framework for classifying supply chain coordination literature.

Pasternack (1985) is an influential and detailed analysis of the buy back contract. Pasternack demonstrates that either a no return policy or a full return policy is system sub-optimal. He goes on to show that a policy of unlimited returns at partial credit can be system optimal and channel coordinating. Many other papers have built upon Pasternack’s analysis to evaluate buyback contracts in more complex settings. Padmanabhan and Png (1997) examine a supply chain with competing retailers and evaluate how a buy back contract can manipulate competition. Emmons and Gilbert (1998) examine buybacks in the context where the news vendor can set retail price.

3. The Basic Model – Supply Chain Contracts

We will restrict our analysis in this paper to the simple news vendor model and address other more complex demand scenarios in future research. We also restrict the analysis to the baseline wholesale contract and the buyback contract, which is the simplest coordinating contract in this scenario. The newsvendor model posits a seller who faces a single selling period with uncertain demand. We shall refer to the upstream participant as the supplier and the downstream participant as the retailer.

The following diagram shows the basic interactions within the supply chain.\(^1\)

---

\(^1\) Adapted from Tsay, Nahmias, and Agrawal with slight changes in notation.
Market demand is described by the function $F$. For our analysis we assume $F$ is stochastic and price independent and that it satisfies the usual conditions; it is continuous, strictly increasing, and twice differentiable, with $F(0) = 0$.

The function $S(q)$ gives the expected sales for any order quantity. $S(q)$ is the minimum of the order quantity and the realized demand. If $q$ exceeds demand there will be leftover items, whereas if demand exceeds the order quantity, the excess demand remains unfilled. Direct costs for production and distribution are given by $c_s$ and $c_r$. Goodwill costs, the implicit cost of a stock out, are given for the supplier and retailer as $g_s$ and $g_r$.

For an arbitrary (price independent) demand distribution, expected sales will be given by\textsuperscript{2}:

$$S(q) = q - \int_0^q F(y) \, dy$$

The total expected revenue can then be expressed as

$$TR = pS(q)$$

The corresponding marginal revenue can be expressed as:

$$MR = pS'(q)$$

$$MR = p - pF(q)$$

Total revenue will therefore be increasing in $q$, while marginal revenue is decreasing. The retailer will then make the normal optimization decision, equating marginal revenue and marginal cost.

The retail price $p$ is fixed and established exogenously to the model. The supplier has a constant production cost of $c_s$. The retailer has a constant handling cost per unit of $c_r$, so that the retailer’s total unit cost is $c_r + c_s < p$. The retailer faces the standard newsvendor conditions, unsold inventory has an end-of-season salvage value of $v$, and unsatisfied

\textsuperscript{2} Note that $S$ represents expected sales, and as such will never exceed the mean of the demand distribution $F$. 

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demand creates a goodwill penalty \( g \). For simplicity, in our analysis, we assume \( v \) and \( g \) are both zero, so the retailer must scrap unsold inventory and the only stock-out penalty is foregone profits. The retailer’s total profit therefore includes her margin on those items which are sold, less the full cost of the unsold inventory\(^3\).

The Wholesale Contract

The simplest scenario is the wholesale contract where the supplier charges a constant wholesale price to the retailer. In this scenario the retailer bears all the risk of demand uncertainty.

The profit functions for the supplier, retailer and the overall supply chain are then:

\[
\pi_{sw}(q_w, \text{ww}) = \text{ww} \cdot q_w - c_s \cdot q_w \\
\pi_{rw}(q_w, \text{ww}) = p \cdot S(q_w) - (\text{ww} + c_c) \cdot q_w \\
\pi_{SC}(q_w, \text{ww}) = p \cdot S(q_w) - (c_s + c_c) \cdot q_w
\]

We assume the standard approach where the supplier announces a wholesale price and the retailer then selects an order quantity. Cachon shows that the retailer’s optimal order quantity is the well known solution to the newsvendor problem\(^4\):

\[
q_w = F^{-1} \left( \frac{p - \text{ww} - c_c}{p} \right)
\]

The supplier then selects a wholesale price to maximize his profits given the expected response of the retailer. The wholesale price is found by differentiating the supplier’s profit function and solving for the optimal price which can then be used to determine the retailer’s order quantity. Because the retailer faces an issue of double marginalization, the quantity she selects \( q_w \), is strictly less than the supply chain optimal quantity; the quantity that would be selected under an integrated supply chain. We should also observe that the supplier’s profit is increasing in \( q \) and independent of the final

\(^3\) The newsvendor model is widely used in the operations literature. Cachon (2003) provides an overview. For more details see for example Petruzzi and Dada 1999.  
\(^4\) Cachon’s formula includes terms for salvage value and goodwill penalty which we have assumed to be zero in this analysis.
realization of demand; the supplier bears none of the demand risk under the wholesale contract.

The Buyback Contract

Under a buyback contract the supplier agrees to purchase back or provide a refund to the supplier for items that remain unsold at the end of the selling season. A buyback contract has two financial parameters, the wholesale price $w$ and the buyback price $b$. The payment from retailer to supplier is $w$ per unit, less $b$ per unit unsold.

$$T_w(q,w, b) = wq - b[q - S(q)]$$

We assume that $b \leq w$, i.e., items are bought back at less then or equal to the price at which they are sold. The profits of the supplier and retailer are given by:

$$\pi_r (q,w,b) = (p - b) S(q) - (w - b + c_r )q$$

$$\pi_s (q,w,b) = bS(q) + (w - b - c_s )q$$

$$\Pi_{SC} (q,w,b) = pS(q) - ( c_r + c_s )q$$

Furthermore, it can be shown (Cachon 2003) that if we set $w$ and $b$ such that the following condition is satisfied, then the buyback contract can coordinate the supply chain.

$$\lambda = \frac{p - b}{p} = \frac{w - b + c_r}{c_r + c_s}$$

With $\lambda$ defined as above we can now show that:

$$\pi_r = \lambda \Pi(q)$$

$$\pi_s = (1-\lambda) \Pi(q)$$

This simplified model shows that the parameter $\lambda$ represents the division of profits between retailer and supplier. Specifically, $\lambda$ represents the percentage of total supply chain profits allocated to the retailer. The critical result is that under a contract that allows profits to be divided arbitrarily between supplier and retailer the supply chain will
be coordinated\textsuperscript{5}. The intuition behind this statement should be clear - if each firm shares a linear portion of the total supply chain profits, then in the margin they will set their choice variables to the level that optimizes the supply chain profit, the definition of supply chain coordination. The buyback contract also differs from the wholesale contract in terms of risk allocation. Under the buyback contract a portion of the risk is transferred back to the supplier.

Under the coordinating wholesale contract we can find the optimal order quantity by maximizing any of the 3 profit functions, yielding:

\[
q_B = F^{-1}\left(\frac{P - c_f - c_s}{P}\right)
\]

With this contract the marginal cost to the retailer is no longer constant. Each additional unit purchased has a decreasing probability of sale, and therefore an increasing probability of buyback. A straightforward calculation of marginal cost demonstrates that under the buyback contract it decreases with order quantity:

\[
TC = wq - b[q - S(q)] = bS(q) + (w - b) S(q)
\]

\[
MC = bS'(q) - (w - b)
\]

We have shown previously that marginal revenue is also decreasing in order quantity. The following graph shows how the buyback contract achieves coordination; marginal cost falls faster then marginal revenue.

\textsuperscript{5} Cachon (2003) states that "in general, a contract coordinates the retailer's and supplier's actions whenever each firm's profit is an affine function of the supply chain's profit." (p.245) He also states that "if a coordinating contract can allocate rents arbitrarily, then there always exists a coordinating contract that Pareto dominates a non-coordinating contract." (p. 230)
The retailer's marginal revenue also declines with q as the probability of sale decreases. Properly designed, the buyback contract cause the retailer's marginal revenue and cost to intersect with the supplier’s marginal cost at the supply chain optimal quantity.

As a result the supplier and retailer have profit curves that are optimized at the same order quantity, as shown below. Incentives are aligned and the supply chain is coordinated. The specific division of profits between supplier and retailer is established by the selection of the contract parameter $\lambda$, an issue we return to shortly.
Coordination Gains

To evaluate the practicality of a coordinating contract we need to first look at the potential gain from implementing a coordinating contract. Clearly neither firm will accept a new contract that is inferior to the baseline contract, and each firm wishes to maximize its own share of the gain from coordination. The pie to be divided is not the total supply chain profit, but the incremental profit achieved by coordination.

The coordinating contract leads to expansion of output and in increase in total supply chain profits.

\[ \Delta q = q_0 b - q_0 w \]

\[ \Delta q = F^{-1} \left( \frac{p - c_S - c_r}{p} \right) - F^{-1} \left( \frac{p - w_w - c_r}{p} \right) \]

Since the supplier’s wholesale price will always be strictly greater than her cost, \( \Delta q \) will always be strictly positive.
The profits of the total supply chain are therefore increased by:

\[ \gamma_{SC} = p \left( S(q_B) - S(q_W) \right) - (c_c + c_s) \Delta q \]

**Supplier’s Potential Gain**

The gain from coordination for the supplier is found by comparing the supplier profit under the wholesale and buyback contracts.

\[ \pi_{sw} = (w_w - c_s) q_w \]
\[ \pi_{SB} = (1 - \lambda) \left( p S(q_B) - (c_c + c_s) q_B \right) \]
\[ \gamma_S = (1 - \lambda) \left( p S(q_B) - (c_c + c_s) q_B \right) - (w_w - c_s) q_w \]

While under the wholesale contract the supplier’s profit was independent of the final realization of demand, under a buyback contract this is no longer the case. This is a key point, the supplier achieves a strictly higher expected profit in return for taking on a portion of the demand risk. There is a non-zero probability the realized profit under the buyback contract will be less than the wholesale contract profit. To determine this probability we calculate the demand realization where buyback profits equal wholesale profits. Denote the realization of demand as \( d \), and the realization of profits as \( y \):

\[ y_{sb} = y_{sw} \]
\[ b \text{Min}[ d, q_B] + (w - b - c_s) q_B = w_w q_w - c_s q_w \]

Which occurs when demand is realized as:

\[ \text{Min}[ d, q_B] = \frac{w_w q_w - c_s q_w - (w - b - c_s) q_B}{b} \]

The probability of lower profits under a buyback contract can therefore be expressed as:

\[ P(y_{sb} \leq y_{sw}) = F \left( \frac{w_w q_w - c_s q_w - (w - b - c_s) q_B}{b} \right) \]

Another factor that will be important when we evaluate risk aversion is the probability that the supplier’s profit is less than zero, which by similar logic we calculate as:

\[ P(y_{sb} \leq 0) = F \left( \frac{(c_s + b - w) q_B}{b} \right) \]
Retailer’s Potential Gain

We can perform a similar calculation for the retailer.

\[
\pi_{rw} = pS(q_w) - (w_w + c_c) q_w
\]

\[
\pi_{rb} = \lambda (pS(q_b) - (c_c + c_s) q_b)
\]

\[
\gamma_R = [\lambda (pS(q_b) - (c_c + c_s) q_b)] - [(pS(q_w) - (w_w + c_c) ) q_w]
\]

Division of Profits

An issue generally unaddressed in the coordination literature is the final selection of the contract parameter \(\lambda\), that is the percentage of profits allocated to each party. At a minimum the contract must be feasible, that is it must provide each party with a positive gain from coordination. Within this range the final realization of \(\lambda\) is a result of the negotiation between supplier and retailer. In general this issue is not addressed in detail in the coordination literature\(^6\). As a baseline we assume all information is complete and symmetric, no outside options exist, and bargaining is costless. Under these conditions an accepted model for is the Nash bargaining solution, (Nash (1950), Rubenstein and Osborne (1990)).

The Nash solution maximizes the Nash product:

\[
f_N = \arg\max_{(f_s, f_d) \in \{u_s, u_d\}} (u_s - d_s) (u_R - d_R)
\]

Where \(u\) represents the utility from agreement and \(d\) represents the utility from a failure to reach an agreement. We assume that if the parties fail to negotiate a coordinating contract, they continue with the wholesale contract, so the Nash product can be characterized as:

\[
f_N = \arg\max \ (\gamma_S) \ (\gamma_R)
\]

Summary

In summary when faced with newsvendor demand, a problem of double marginalization, and a wholesale contract, the optimal decision for a retailer is to order a quantity that is

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\(^6\) Jeuland and Shugan (1983) briefly address this issue and make reference to the Nash solution.
sub-optimal from a supply chain perspective. Several alternative contract mechanisms, of which a buy back contract is the most straight-forward, exist which change the marginal cost faced by the retailer and her to order the supply chain optimal quantity.

The resulting coordination creates a strictly positive increase in output and expected supply chain profits. The coordinating contract Pareto dominates the wholesale contract, that is each party can be made strictly better off. Since the contract Pareto dominate we might expect to find it widely implemented. In the next section we evaluate two factors that can help explain why coordinating contracts are not more widely adopted.

4. Inefficiencies and Impediments

Transaction Costs
The coordinating buyback contract increases the overall supply chain profits, so a Pareto optimal contract can be constructed making each player’s profit strictly larger. However, the buyback and other coordinating contracts are more difficult and costly to administer. Costs of implementation include monitoring and compliance costs, systems update, and if the gods are physically returned shipping, handling and restocking costs.

Denote by $\Omega$ the total overhead costs associated with administering the buyback contract per selling season, independent of who bears the cost. We denote the per unit cost, the fixed cost amortized over order volume, as $\omega$.

Because we assume the costs are fixed, they do not impact marginal costs and therefore volumes are unaffected. The upper bound on overhead costs is clearly the level at which overhead costs fully offset the gains from coordination. That is a buyback contract is only feasible if can be administered as a level that is below the gain from coordination.

$$\Omega < \gamma_{SC}$$
Risk Aversion

The operations literature on supply chain coordination typically assumes both supplier and retailer are risk neutral. As we have shown previously, to coordinate the supply chain with a buyback contract the supplier must absorb some of the retailer’s risk. If we assume that the supplier is risk averse, the level of supply chain profit she is willing agree to under a coordinating contract will change.

Let us assume the supplier has a utility function of the form:

\[ u(\pi) = \pi^\alpha \]

where \( 0 \leq \alpha < 1 \) represents a measure of the supplier’s risk aversion. Specifically we define the supplier’s utility function as follows\(^7\):

\[ u(y) = \begin{cases} 
 y^\alpha & y \geq 0 \\
 y & y < 0 
\end{cases} \]

Under a wholesale contract the risk averse supplier receives utility defined by:

\[ u_{sw} = [ (w_w - c_s) q] ^\alpha \]

Calculating the utility received under a buyback contract is more complicated. The supplier’s profit function has two distinct forms based on the realization of demand. If demand is realized above the order quantity the supplier’s profit is simply the difference between the retail price and cost. If however, demand is less than the order quantity, then profits are reduced by the buyback factor times the unsold quantity.

\[
\begin{align*}
  y (d > q) &= w_b q_b - (q_b - d) b - c_s q_b \\
  y (d < q) &= w_b q_b - c_s q_b
\end{align*}
\]

Expected profits can then be expressed as

\[
E[y] = \int_0^{q_b} (w_b q_b - (q_b - x) b - c_s q_b) f(x) \, dx + \int_{q_b}^\infty (w_b q_b - c_s q_b) f(x) \, dx
\]

\(^7\) Because there is a non zero probability that profits are negative we must define a utility function that is defined and real when y is less then zero.
Utility on the other hand has three distinct regions, when demand causes negative profits, when demand yields positive profits but is less than order quantity, and when demand exceeds order quantity. Expected utility for the risk averse supplier is then found by the following 3-part integral:

\[
E[u_{sb}] = \int_0^{q_{be}} (w_b q_b - (q_b - x) b - c_s q_b) f(x) \, dx + \int_{q_{be}}^\Phi (w_b q_b - (q_b - x) b - c_s q_b)^2 f(x) \, dx \\
+ \int_{q_{be}}^\infty (w_b q_b - c_s q_b)^2 f(x) \, dx
\]

where \(q_{be}\) is the supplier's breakeven quantity. To better illustrate the coordination process we now develop a relatively simple numerical example.

5. Numerical Example

As a baseline we assume demand is normally distributed on \([0,1]\). This relatively simple distribution allows us to generate relatively tractable closed form solutions. The distributions for demand and expected sales are:

\[
f(q) = 1 \\
F(q) = q \\
S(q) = q - \frac{q^2}{2}
\]

We assume some basic, normalized cost figures; price is 10, manufacturing costs are 4 and distribution costs are 2. Total and marginal revenue are:

\[
TR = ps(q) = pq - \frac{pq^2}{2} = 10q - 5q^2 \\
MR = TR' = p - pq = 10 - 5q
\]

**Wholesale Contract**

The retailer's order quantity is

\[
q_w = \phi^{-1}\left(\frac{p - w_w - c_r}{p}\right) = \frac{p - w_w - c_r}{p} \\
q_w = \frac{10 - w_w - 2}{10} = \frac{8 - w_w}{10}
\]
The supplier's profit function is then:

\[ \pi_s = w_w q_w - c_s q_w = (w_w - c_s) \left( \frac{p - w_w - c_r}{p} \right) \]

\[ \pi_s = (w_w - 4) \frac{8 - w_w}{10} \]

Maximizing supplier's profit yields:

\[ w_w = 6 \quad \pi_s = .4 \quad q_w = .2 \quad S(q) = .18 \]

Plugging back into the retailer's profit function yields

\[ \pi_{rw} = p S(q_w) - (w_w + c_r) q_w \]

\[ \pi_{rw} = 10 (.18) - (8) .2 = .2 \]

**Buyback Contract**

With a buy back contract the order quantity is determined based on total supply chain cost.

\[ q_B = F^{-1} \left( \frac{p - c_x - c_S}{p} \right) = \frac{p - c_x - c_S}{p} \]

\[ q_B = \frac{10 - 2 - 4}{10} = .4 \]

**Gains**

The change in output is:

\[ \Delta q = q_{DB} - q_{DW} = .2 \]

And the increase in supply chain profit is:

\[ \gamma_{SC} = p \left( S(q_{DB}) - S(q_{DW}) \right) - (c_r + c_s) \Delta q \]

\[ \gamma_{SC} = 10 \left( \left( .4 - .4^2 \right) - \left( .2 - .2^2 \right) \right) - (6) (.2) \]

\[ \gamma_{SC} = .2 \]

The supplier's gain as a function of \( \lambda \) is

\[ \gamma_S = (1 - \lambda) \left( p S(q_{DB}) - (c_r + c_s) q_{DB} \right) - (w_w - c_s) q_{DW} \]

\[ \gamma_S = (1 - \lambda) \left( 10 S(.4) - (6) .4 \right) - (6 - 4) .2 \]

\[ \gamma_S = 0.4 - 0.8 \lambda \]
And the retailer’s gain is

\[ \gamma_R = [\lambda (pS(q_B) - (c_x + c_s) q_B) - [pS(q_B) - (w_w + c_x) q_B)] \]
\[ \gamma_R = 0.8 \lambda - .2 \]

Supplier and retailer gain clearly sum to supply chain gain, independent of \( \lambda \). The range of \( \lambda \) that makes the buyback contract feasible for each party is \([.25,.5]\). The expected value of \( \lambda \) can be found from the Nash product.

\[ f_N = \arg\max (\gamma_S) (\gamma_R) \]
\[ f_N = \arg\max (0.4 - 0.8 \lambda) (0.8 \lambda - .2) \]
\[ f_N = .375 \]

Given this value of \( \lambda \), we can calculate the parameters of the buyback contract that coordinate the supply chain.

\[ \lambda = \frac{p - b}{p} \]
\[ b = (1 - \lambda) p = (1 - .375) 10 = 6.25 \]
\[ \lambda = \frac{w_b - b + c_x}{c_x + c_s} = \frac{w_b - 6.25 + 2}{6} \]
\[ w_w = 6.5 \]

Hence, in order to coordinate the supply chain the supplier must offer a buyback contract that sells items at 6.5, but offers to buy them back at 6.25. In other words the supplier must offer to compensate the retailer at 95% of original cost for unsold items. With these terms, profit, as a function of realized demand, can be expressed are as follows:

\[ y_{SB} = (1 - \lambda) (pS(q_B) - (c_x + c_s) q_B) \]
\[ y_{SB} = 0.625 (10 \min[d,.4] - (6) .4) \]
\[ y_{SB} = 6.25 \min[d,.4] - 1.5 \]

The following graph shows the profit level for each party under each contract as a function of realized demand. While the supplier has strictly increased profits by offering the buy back contract she has also taken on risk.
We can calculate the probability the supplier will achieve a lower profit under the buyback contract as follows:

\[
P(H_{ysb} < H_{ysw}) = \Phi\left(\frac{(q_b - c_b) q_w - (w_b - b - c_b) q_B}{b}\right) = \frac{(6.5 - 4) \cdot .2 - (6.5 - 6.25 - 4) \cdot .4}{6.25} = .32
\]

The probability of a financial loss is:

\[
P(H_{ysb} \leq 0) = \Phi\left(\frac{(c_s + b - w_b) q_B}{b}\right) = \frac{(4 + 6.25 - 6.5) \cdot .4}{6.25} = .24
\]

So while the supplier’s expected profit will rise by 25%, there is a 38% chance that profits will be below the wholesale level, and a 24% chance of a financial loss, assuming no transaction costs. We now turn to the issue of impediments and evaluate how transaction costs and supplier risk aversion may impact the desirability of the buyback contract.

**Transaction Costs and Risk Aversion**

In section 3 we argued that transaction costs are fixed, and that the buyback contract is only feasible if the administration cost is less than the gain from coordination, therefore:
\[ \omega_{\text{max}} = 0.2 \]
\[ \omega_{\text{max}} = \frac{\Omega_{\text{max}}}{q_b} = 0.5 \]

The maximum per unit administration cost is .5, which represents 5% of the retail price. If the cost of administering the buyback contract is less than 5% of sales, positive gains are still available from coordination, but from the supplier’s perspective those gains are bought with increased risk. We now examine the issue of supplier risk aversion.

We previously wrote the supplier’s expected profit function in terms of an integral over expected demand, if we substitute the values from this example we get the following expression

\[ E[y] = \int_0^{0.4} (6.25x - 1.5) \, dx + \int_{0.4}^{1} \, dx = 0.5 \]

Similarly, if we substitute into the expression for the supplier’s utility we get:

\[ E[u_{sb}] = \int_0^{0.24} (6.25x - 1.5) \, dx + \int_{0.24}^{0.4} (6.25x - 1.5) \, dx + \int_{0.4}^{1} \, dx \]

\[ E[u_{sb}] = 0.42 + \frac{1^a}{6.25 + 6.25 \alpha} \]

The total gain in utility from a coordinating contract is then:

\[ \Gamma_{usb} = \int_0^{0.24} (6.25x - 1.5) \, dx + \int_{0.24}^{0.4} (6.25x - 1.5) \, dx + \int_{0.4}^{1} \, dx - 0.4^a \]

The critical value for the risk aversion coefficient in this case is approximately .73, below this level the increased risk offsets the expected gain for the supplier.

**Transaction Costs and Risk Aversion**

The above analysis shows that if we have either transaction costs, or supplier risk aversion, the buyback contract may be Pareto inferior to the wholesale contract. We now evaluate the attractiveness of the buyback contract when both conditions exist simultaneously.
Again we assume that the administrative costs associated with the buyback contract are fixed, but we now make the additional assumption that these costs are split equally between the supplier and retailer.

The following graph shows the expected utility gain the supplier receives as a function of total transaction costs and the supplier's risk aversion coefficient.

![Graph showing expected utility gain vs. administrative cost and risk aversion coefficient.]

This graph demonstrates that with moderate transaction costs and risk aversion the buyback contract quickly become unattractive to the supplier.

6. Conclusions

The majority of the literature on supply chain coordination addresses the issues of what contracts can coordinate the supply chain under various demand scenarios. The literature shows that coordination is possible under a wide range of conditions. The purpose of this paper has been to explore why coordinating contracts are not more
common. We concentrate on two issues, transaction costs and risk aversion and show that in the presence of one or both of these conditions the buy back contract may no longer be Pareto dominate. We believe both these issues are significant practical considerations.

We have shown that moderate transaction costs, less then 5% of sales price in this example, the buyback contract no longer increases the profit of the supply chain as a whole. We have also shown that the supplier’s gain comes at the expense of risk, and that a risk averse supplier may find this trade off undesirable. Finally we have shown that when transaction costs and risk aversion are both present the contract can easily be Pareto inferior, with costs in the range of 1.5% 3.0% of sales the contract can become undesirable for a risk averse supplier.

This paper presents a preliminary analysis of 2 factors that may prevent the implementation of coordinating contracts. Many other factors may influence this decision and additional research is required. Our theoretical analysis is itself limited, and further investigation is required. Assessment of the coordination under different demand distributions and different cost structures is needed. In particular, an assessment of the impact of fixed cost of production is warranted.

Empirical validation of all aspects of the coordination problem is also required. To validate this analysis, empirical validation of the range of transaction costs is required. A more detailed assessment of supplier’s preference for risk is also warranted. Unfortunately, access to relevant data is difficult to come by and empirical research into the contracting problem has to date been limited.
7. References


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