Phys212 Test II, Spring 2004

Show your work and/or explain your answers.
You should draw a meaningful figure for ALL problems.
For maximum credit you should show:

Figure
Basic equation(s) you will use (as it appears in the book, equation sheet, etc.)
Equation(s) with numbers plugged in (if numerical)
Solution to equation (you may need to show at least some of the algebra depending on the problem.)
If you use your calculator to get results, be sure to show explicitly what your entered into the calculator, and
the results it gave you.
Make sure you answer the question and use appropriate units if needed!!!

(5 pts)
1. Suppose you have an arbitrary conductor. Describe if and how the voltage varies between different parts
   of the conductor. Why?

\[ E_{\text{conductor}} = 0 \]
\[ \Delta V = -\int \mathbf{E} \cdot d\mathbf{s} \]
\[ = 0 \quad \text{so voltage is constant.} \]

(5 pts)
2. The equation for the potential energy of a charge is usually given by \( \Delta U = q \Delta V \). Why is the equation
   for a capacitor \( U = \frac{1}{2} Q V \)?

\[ U = qV, \text{ but as the capacitor charges the voltage increases from } 0 \rightarrow V_{\text{max}} \]

\[ \text{So } U = qV_{\text{max}} \]
\[ = q \cdot \frac{1}{2} V_{\text{max}} \]
\[ = \frac{1}{2} qV \]
3. If each of the three capacitors below are identical, which will have the greatest voltage when the combination is attached to a battery?
   A: capacitor A
   B: capacitor B
   C: capacitor C
   D: capacitors B and C are greatest
   E: All capacitors will have the same voltage

   ![Capacitor Diagram]

   \[ V = \frac{Q}{C} \]
   \[ V_A > V_B, V_C \]

4. A dielectric is a substance in which
   A: Charges are free to move or migrate throughout the substance.
   B: Charges can only move along the surface of the substance.
   C: Charges can only move in one direction within the substance.
   D: Charges can only slightly move or wiggle within the substance.
   E: Charges cannot move at all.

5. A voltage is applied across A-B. What is the same about these three capacitors?
   A: Charge
   B: E-field
   C: Potential energy
   D: Voltage
   E: Voltage and E-field
   F: Voltage and potential energy

   ![Capacitance Diagram]
   \[ C_1 = C_2 \neq C_3 \]

6. In a parallel circuit, which of the following is true of each part of the parallel circuit?
   A: The charges or currents are the same for each.
   B: The stored energy is the same for each.
   C: The voltages are the same for each.
   D: Nothing is the same for each part.

7. Two conducting spheres of different size are connected by a conducting wire. Sphere 1 is larger than sphere 2. The electric potential of each sphere is \( V_1 \) and \( V_2 \), and the charge density on the surface of each sphere is \( \sigma_1 \) and \( \sigma_2 \). Which of the following is true:
   A: \( V_1 < V_2, \sigma_1 < \sigma_2 \)
   B: \( V_1 < V_2, \sigma_1 = \sigma_2 \)
   C: \( V_1 < V_2, \sigma_1 > \sigma_2 \)
   D: \( V_1 = V_2, \sigma_1 < \sigma_2 \)
   E: \( V_1 = V_2, \sigma_1 = \sigma_2 \)
   F: \( V_1 = V_2, \sigma_1 > \sigma_2 \)
   G: \( V_1 > V_2, \sigma_1 < \sigma_2 \)
   H: \( V_1 > V_2, \sigma_1 = \sigma_2 \)
   I: \( V_1 > V_2, \sigma_1 > \sigma_2 \)

   \[ V = \frac{k \varepsilon_0 A}{r} = \frac{k \sigma (4 \pi r^2)}{r} = 4 \pi k \sigma r \]

   so \( \sigma = \frac{V}{4 \pi kr} \)
8. A charge of $Q$ and mass $m$, is shot toward the center of a line charge of length $L$ and charge density $\lambda$. If the charge has a speed of $v_0$ at a distance of $y_0$ from the line, what is the distance of closest approach $y_{\text{min}}$?

\[
V = k_e \lambda \ln \left[ \frac{\sqrt{y_0^2 + L^2} + L}{\sqrt{y_0^2 + L^2} - L} \right]
\]

\[
O = \Delta U + A_k
\]

\[
O = U_f - U_i + k_f - k_i
\]

\[
O = qV_i - qV_f + O - \frac{1}{2} mv_0^2
\]

\[
O = qk_e \lambda \ln \left[ \frac{\sqrt{y_{\text{min}}^2 + L^2} + L}{\sqrt{y_{\text{min}}^2 + L^2} - L} \right] - qk_e \lambda \ln \left[ \frac{\sqrt{y_0^2 + L^2} + L}{\sqrt{y_0^2 + L^2} - L} \right] - \frac{1}{2} mv_0^2
\]

Solve for $y_{\text{min}}$. 
9. Two plates of area $A$ are separated by a distance $d$ so that they form a capacitor with capacitance $C_0$. The capacitor is now partially filled with a certain oil with a dielectric constant of 7.5 so that the capacitance of the partially filled device is $4C_0$. What fraction of the device was filled with oil?

\[
C_0 = \frac{\varepsilon_0 A}{d}
\]

\[
4C_0 = C_{top} + C_{bottom}
\]

\[
C_{top} = \frac{\varepsilon_0 A_{top}}{d} + \frac{k\varepsilon_0 A_{bottom}}{d}
\]

\[
\frac{4\varepsilon_0 A}{d} = \frac{\varepsilon_0 A (1-f)}{d} + \frac{k\varepsilon_0 A (f)}{d}
\]

\[
f + f = 1 - f + kf
\]

\[
3 = 7.5f - f = 6.5f
\]

\[
f = 0.46
\]

\[\boxed{46.8}\]
10. In the circuit below, the battery is attached to the entire combination of capacitors so that \( C_3 \) is charged with 50 mC. What is the charge on either side of \( C_1 \)?

\[ \begin{align*}
C_1 &= 10 \text{ mF} \\
C_2 &= 50 \text{ mF} \\
C_3 &= 30 \text{ mF} \\
C_4 &= 80 \text{ mF}
\end{align*} \]

\[ \begin{align*}
Q_3 &= Q_4 = 50 \text{ mC} \\
V_2 &= V_{24} \\
V_{24} &= \frac{Q_{24}}{C_{24}} \\
C_{34} &= \left( \frac{1}{C_3} + \frac{1}{C_4} \right)^{-1} = 22 \text{ mF}
\end{align*} \]

\[ \begin{align*}
V_2 &= 2.27 \text{ V} \\
V_{24} &= 2.27 \text{ V}
\end{align*} \]

\[ \begin{align*}
Q_2 &= V_2 C_2 \\
Q_2 &= 114 \text{ mC}
\end{align*} \]

\[ \begin{align*}
Q_1 &= Q_2 + Q_{24} \\
Q_1 &= 164 \text{ mC}
\end{align*} \]