NUMERICAL PREDICTION OF THE NOISE PRODUCED BY A
PERFECTLY EXPANDED RECTANGULAR JET

Thomas S. Chyczewski* and Lyle N. Long†
Department of Aerospace Engineering
The Pennsylvania State University, University Park, PA 16802

Abstract

Supersonic rectangular jet flow and far field noise predictions are made by solving the governing equations using advanced numerical techniques on parallel processors. The computational domain begins at the jet nozzle exit and contains the jet plume and a small region of the acoustic near field. The equations solved for the interior grid points are the full 3D Navier Stokes equations. The far field boundary points are determined by unsteady, nonlinear characteristic based nonreflecting conditions. To model the jet nozzle exit flow, a set of equations are developed to simulate many features of this flow that have been experimentally observed to influence the jet and its radiated noise. A Kirchhoff method is used to determine the far field noise from information extracted from the finite computational domain. Each set of governing equations is spatially discretized by a sixth order central difference scheme and advanced in time using fourth order Runge-Kutta integration. Spurious high wave number fluctuations are damped by a nonlinear dissipation algorithm that has a minimal effect on the acoustic solution. The code has been efficiently implemented on the CM5 using CMFortran (essentially HPF) and should be easily ported to platforms running HPF (such as the SP2). Numerical results indicate that the algorithm, which contains no model constants (aside from the nozzle exit conditions), is capable of reproducing many experimentally observed rectangular jet flow and noise features.

1 Introduction

Jet noise analysis and reduction have been topics of research since the introduction of the jet engine as a propulsion device for aircraft during World War II. Many tools have been developed and incorporated into the design process to reduce the annoyance of jet noise. The Federal Aviation Administration began placing strict regulations on the noise produced by aircraft and this has caused a renewed interest in noise prediction in the scientific community. This is particularly true in light of the national interest to develop the High Speed Civil Transport (HSCT). Noise reduction is considered to be a crucial technology required for a viable design.

The prediction strategy that is currently receiving the most attention is direct simulation. Jet noise research has included many different implementations of this approach. The implementation under investigation in this paper solves the 3D full Navier Stokes equations in a domain that includes the noise source region and a small portion of the acoustic field. The acoustic near field solution is then used as an input to a Kirchhoff method to determine the far field noise.

Consistent with experimental evidence, the noise source is assumed to be dominated by the evolution of large scale coherent structures in the jet shear layer and thus only these scales are resolved. The influence of the small scales is assumed to be represented by numerical dissipation. Thus, no turbulence model is employed and consequently, there are no adjustable model constants used in the interior domain.

The computational domain in which the Navier Stokes equations are solved begins at the jet nozzle exit plane. A model is therefore required for the nozzle exit flow and can have an appreciable effect on the numerical solution. The effects of nozzle exit conditions on experimental and numerically simulated jets have been studied by many investigators. Hussain and Husain found experimentally that the development of the jet depends on the nozzle boundary layer momentum thickness distribution. The azimuthal variation has been shown by them to produce noticeable effects on the spreading rate of elliptic jets. These effects are the result of the influence of the momentum thickness on the generation of coherent structures. King et al. found that nozzle imperfections as small as 0.2% of the nozzle exit diameter may have a significant effect on the development of supersonic axisymmetric jets. They used this information to develop methods of enhancing jet mixing. The initial turbulence intensity was found have a significant effect on the turbulence amplification rate.
in the near-field region of the jet by Grinstein et al.\textsuperscript{11} Quinn\textsuperscript{14} found differences in the spreading rate of two jets operating under essentially the same flow conditions and geometry. These discrepancies were attributed to facility differences.

This experimental evidence suggests that the salient features of a laboratory facility will have an appreciable effect on the development of a jet. These effects can be observed in the form of varying potential core lengths, turbulence levels and jet spreading rates. To compound this potential problem, the nozzle exit conditions of the rectangular jet simulated in this paper are not defined precisely. The measurement techniques available to Kinzie\textsuperscript{16} did not permit a comprehensive study of the nozzle exit. Due to this uncertainty, a general model for the nozzle exit flow has been developed that can turn on or off some of the features that have been observed to significantly influence the jet development. The results presented in this paper are confined to studying the effect of modal excitation. The effects of nozzle exit turbulence levels and corner vortices are discussed in Chyczewski(1996).\textsuperscript{17}

Since the computational domain is limited to just a small region of the acoustic near field, a method is required to extrapolate the solution to the far field. In this work the Kirchhoff method is employed.\textsuperscript{18} It consists of constructing a surface $S$ on which the acoustic solution can be reliably calculated. The acoustic solution at any location outside of this surface can then be determined by the Kirchhoff formula. The solution is exact for sound radiation outside of a surface $S$ if that radiation is governed by the convective wave equation. However, in the application of this method to the jet noise problem, finding such a surface is difficult. This issue has been addressed previously by some investigators who have found that the far field solution is not very sensitive to the location of the surface if some precautions are taken. Lyrintzis and Mankbadi\textsuperscript{19} found that placing the surface at least one diameter away from the jet centerline is sufficient to obtain accurate solutions. Freund et al.\textsuperscript{20} performed a study analyzing the effects of using open Kirchhoff surfaces and found that it does not introduce significant errors if the surface passes through the region between the noise source and the observer.

In the next section, the governing equations are described. This consists of discussing the specific form of the Navier Stokes equations, presenting the boundary equations, which includes the model nozzle exit conditions, and finally presenting the Kirchhoff formulation used here. In section 3 the numerical approach is outlined. Special attention is given to the artificial dissipation model. Next, in section 4, rectangular jet noise prediction results are presented and compared to experimental data. Finally, in section 5, some conclusions are drawn.

## 2 Governing Equations

A supersonic rectangular jet flow is a nonlinear, viscous, unsteady, 3D problem. As such, it is governed by the full, compressible, 3D Navier Stokes equations. A nondimensional conservative form of these equations is used in this work (see Hoffmann(1989)\textsuperscript{21}). By themselves, these equations are not sufficient to model the jet problem. Boundary conditions are required to allow flow and acoustic waves to pass through the far field boundaries of the computational domain as well as to model the flow entering the domain from the nozzle exit. This section presents these equations as well as the Kirchhoff formulation used to extrapolate the acoustic solution to the far field.

### 2.1 Nonreflecting Boundary Conditions

Several approaches to the specification of nonreflecting conditions at far field boundaries have been developed. These approaches can be classified into three categories: asymptotic solutions,\textsuperscript{22,23} Fourier decomposition\textsuperscript{24,25} and quasi one-dimensional analysis.\textsuperscript{26,27} The most recent set of conditions based on the asymptotic solution of the linearized Euler equations are due to Tam and Webb.\textsuperscript{28} These asymptotic conditions have been quite successful at reducing boundary reflections for many model problems.

Giles\textsuperscript{24} derived approximate unsteady boundary conditions for two-dimensional problems by performing a Fourier decomposition of the linearized Euler equations. They have been applied to turbomachinery problems by Giles and have been found to be effective. When implemented with a buffer zone, these conditions have been able to permit nonlinear vortical structures to leave a computational domain with little reflection.\textsuperscript{25}

A drawback of both the asymptotic and Fourier methods is that their derivation employs a set of equations that have been linearized with respect to a reference solution. In many cases, such as the rectangular jet problem under consideration here, the reference solution is not known a priori and must be developed as the equations are integrated. Experimentation with the rectangular jet problem suggests that asymptotic and Fourier methods are not capable of establishing a reasonable reference, or time averaged, solution when the initial condition is a quiescent fluid.

Given this difficulty, the quasi one-dimensional boundary procedure developed by Thompson\textsuperscript{26,27} is employed. The approach consists of decomposing the full nonlinear Euler equations into modes of definite velocity and specifying nonreflecting conditions for those modes that have a velocity directed into the computational domain. These conditions have been shown to be able to allow large amplitude disturbances to leave the domain with little reflection.\textsuperscript{26}
2.2 Nozzle Exit Conditions

A complete prescription of the nozzle exit conditions requires the specification of both steady and unsteady characteristics. They are described in the following two subsections.

2.2.1 Steady Nozzle Conditions

A nearly uniform velocity profile was found at the nozzle exit by Kinzie.\textsuperscript{16} This indicates that viscous effects are confined to locations very close to the nozzle wall. Thus the simulated jets use uniform profiles for density, axial velocity and pressure. Accurate measurements of momentum thickness variations that may exist around the nozzle lip were not performed and are thus not accounted for in the nozzle model.

The values of the exit variables are found from the experiment. The exit Mach number, $M_j$, is 1.54, the acoustic speed of the jet is $c_j = 0.82c_\infty$ and since the jet is ideally expanded, the jet exit pressure is the same as the ambient pressure ($p_j = p_\infty$). From this information, the steady exit density and velocity can be found. In this paper, the nozzle exit flow is assumed to be purely axial. The effects of lateral exit flow components induced by nozzle exit corner are considered in Chyczewski (1996).\textsuperscript{17}

2.2.2 Unsteady Nozzle Conditions

There is a very limited amount of information available in the literature that discusses the unsteady features of a supersonic nozzle exit flow. In fact, the authors have not seen any published data that characterizes the unsteady features to the extent that is required to reproduce the nozzle exit conditions completely. This is most likely due to the extreme difficulty of collecting such data.

To compensate for this lack of information, a general set of unsteady nozzle exit conditions have been developed that can specify the disturbance spatial distribution, amplitude, temporal behavior and phase relation around the nozzle lip. By controlling the phase relation, different modes (flapping or varicose) can be excited at the nozzle exit. This is similar to the artificial excitation used by many experimentalists.\textsuperscript{7,16,28-30} With this model, many different features can be investigated. In this paper, the investigation is confined to studying the modal excitation.

The velocity perturbations are calculated from the following relation:

$$u', v' \text{ or } w' = \frac{n}{2} \alpha \sum_{i=1}^{4} c_i A_i \sum_{l=1}^{2} \sin(2\pi f_l t + \phi_l + \beta_l)$$

(1)

The contributions of each of these terms is given in the following sections.

Temporal Behavior.

The inner summation in equation 1 is over contributions from two characteristic frequencies. These two frequencies are the screech tone frequencies found in the minor axis plane of the experimental jet.\textsuperscript{16} These frequencies can also be determined using the linear shock cell model and weakest link theory developed by Tam.\textsuperscript{31}

For our problem these frequencies are $f_1 = 9006$ Hz (Strouhal number = 0.31) and $f_2 = 26367$ Hz (Strouhal number = 0.84).

A random component to the excitation is supplied by $\phi^n_l$ in equation 1. It is initialized to zero at the beginning of each run. It is then updated at each timestep, $n$, by the following:

$$\phi^n_l = \phi^{n-1}_l \pm \theta$$

(2)

The amplitude of the phase shift between time steps $\theta$ is 5.4 degrees. This value was found to give broad frequency spectra with an upper band limit that is near the highest frequency that the grid and scheme can resolve.

Spatial Distribution.

The perturbation on the entire nozzle lip is determined by building it up from the contributions of the four walls. This is done in equation 1 by the outer summation. $A_i$ is the spatial amplitude function for each of the walls. It is a Gaussian function centered on the lip line of each wall. The half width of the Gaussian is one tenth of the short dimension of the nozzle. The function is tapered to zero amplitude near the corners of the nozzle. This spatial function is selected since a Gaussian is a representative distribution for wall bounded shear layer perturbations (see Kinzie (1996)\textsuperscript{16} for example).

Mode Excitation.

Two different modal excitations are considered in this paper. These are the varicose and flapping modes and have been found in the experimental jet by Kinzie.\textsuperscript{16} The varicose mode is characterized by symmetric shedding of coherent structures from the nozzle lip. In contrast, the vortex shedding is asymmetric for the flapping jet case. This is illustrated in figure 1. These different modes are excited in the jet by controlling phase differences between the walls of the nozzle. The phase difference is controlled by the angle $\beta$ in equation 1. Whether or not there is a velocity component contribution to the perturbation from a wall is determined by the parameter $c_i$. The values of these two parameters for each of the velocity components is given in tables 1 and 2 for the varicose and flapping modes.

The final parameter in equation 1 is $\alpha$. It specifies the peak amplitude of the velocity perturbation. A value of 0.02 is used and corresponds to an RMS fluctuation level near 1.36 percent of the exit velocity. The
The acoustic pressure perturbations most likely originated from acoustic disturbances upstream of the nozzle exit. The first condition was found to be reasonable for a jet under these conditions by Troutt and McLaughlin. These velocities, the pressure and density are found from the steady contribution to the perturbation. Given random nature of the excitation, RMS levels vary slightly from run to run due to the random nature of the excitation.

The instantaneous velocities are obtained by adding the steady contribution to the perturbation. Given these velocities, the pressure and density are found from conditions of constant total temperature and entropy. The first condition was found to be reasonable for a jet flow under these conditions by Troutt and McLaughlin. The assumption of isentropic excitation is justified since these perturbations most likely originated from acoustic disturbances upstream of the nozzle exit.

### 2.3 Kirchhoff Formulation

The moving surface formulation given by Farassat and Myers for a rigid surface in rectilinear motion is employed. It gives the acoustic pressure \( p' \) at location \( \vec{x} \) and time \( t \) as a function of the pressure on a suitably defined surface \( S \) (where \( \vec{x} \) is outside of \( S \)):

\[
4\pi p'(\vec{x},t) = \int_S \left[ \frac{E_1}{r(1 - Mr)} \right] \tau, \\
+ \int_S \left[ \frac{p'E_2}{r^2(1 - Mr)} \right] \tau, 
\]

where

\[
r = |\vec{r}|, \quad \vec{r} = \vec{x} - \vec{y}(\tau), \quad Mr = \vec{M} \cdot \vec{r}/r, \\
E_1 = -\vec{n} \cdot \nabla p' + (\vec{M} \cdot \vec{n})(\vec{M} \cdot \nabla p') \\
+ \left[ \frac{c\cos\theta - \vec{M} \cdot \vec{n}}{c(1 - Mr)} \right] \frac{\partial p'}{\partial \tau}, \\
E_2 = \frac{1 - M^2}{(1 - Mr)^2} (\cos\theta - \vec{M} \cdot \vec{n}) 
\]

\( M \) is the Mach number of the moving surface which for the static jet problem is zero. The vector \( \vec{r} \) is the vector difference between the observer location and the location of the Kirchhoff surface element (it varies with each location on the surface). \( \vec{n} \) is the normal vector pointing out of the Kirchhoff surface, the angle \( \theta \) is measured between the vectors \( \vec{r} \) and \( \vec{n} \), and \( c_\infty \) is the freestream sound speed. The integrands are evaluated at the Kirchhoff surface emission time \( \tau^* \) which, for a stationary surface, is given by

\[
\tau^* = t - r/c_\infty 
\]

A complete description of the coupling of the Kirchhoff method into the Navier Stokes code is given in Ozyoruk and Long and is therefore not described here.

### 3 Numerical Algorithm

The governing equations are discretized in a finite difference context using fourth order accurate Runge-Kutta time integration and sixth order accurate spatial discretization. The computational domain is illustrated in figure 2 which also shows the coordinate system. The center of the nozzle exit is located at \((x, y, z) = (0, 0, 0)\).

Details on the grid generation strategy can be found in Chyczewski and Long (1995) and Chyczewski (1996).

#### 3.1 Artificial Dissipation

A desirable feature of this numerical algorithm is the explicit control of the amount of dissipation applied to the scheme. Unlike upwind methods, Runge-Kutta - central difference techniques contain very little implicit dissipation. Instead, explicit filters are used for stability and to prevent odd-even decoupling errors. Selection of an appropriate dissipation scheme is paramount in calculations where one wishes to extract the low amplitudes and high frequencies associated with acoustics. Jameson et al. proposed a blend of split second and fourth order dissipation. While this dissipation is robust near discontinuities, it may significantly contaminate the acoustic solution.

<table>
<thead>
<tr>
<th>( u' )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>( c_4 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Values of \( c_i \) and \( \beta_i \) for flapping mode excitation.
A combination of second and sixth order dissipation is used here. This dissipation has been applied to the nonlinear acoustic benchmark problems\cite{34} and has proven capable of propagating acoustic waves in the presence of strong discontinuities. It is applied to the scheme as a correction to the residual and takes the following form:

$$D(Q)_{i,j,k} = D_\zeta(Q)_{i,j,k} + D_q(Q)_{i,j,k} + D_\zeta(Q)_{i,j,k}$$

Each term is determined as follows (using $D_\zeta$ as an example):

$$D_\zeta(Q)_{i,j,k} = \epsilon^{(2)}D_\zeta^2(Q)_{i,j,k} + \epsilon^{(6)}D_\zeta^6(Q)_{i,j,k}$$

where

$$D_\zeta^2(Q)_{i,j,k} = \frac{1}{\Delta t}(Q_{i+1,j,k} - 2Q_{i,j,k} + Q_{i-1,j,k})$$

and

$$D_\zeta^6(Q)_{i,j,k} = \frac{1}{\Delta t}(-20Q_{i,j,k} + 15(Q_{i+1,j,k} + Q_{i-1,j,k}) - 6(Q_{i+2,j,k} + Q_{i-2,j,k} + (Q_{i+3,j,k} + Q_{i-3,j,k}))$$

The coefficients are determined in a manner very similar to that used by Jameson:

$$\epsilon^{(2)} = k^{(2)}\max(\nu_{i-1}, \nu_i, \nu_{i+1})$$

and

$$\epsilon^{(6)} = \max(0, k^{(6)} - \epsilon^{(2)})$$

where the values of $k^{(2)}$ and $k^{(4)}$ used here are 1.5/4 and 1.5/256, respectively. The flow gradient sensor $\nu$ is given by

$$\nu_i = \frac{[\beta_{i-1,j,k} - 2\beta_{i,j,k} + \beta_{i+1,j,k}] / [\beta_{i-1,j,k} + 2\beta_{i,j,k} + \beta_{i+1,j,k}] }{\beta_{i-1,j,k} + 2\beta_{i,j,k} + \beta_{i+1,j,k}}$$

$\beta$ is set equal to the total pressure. The Jameson scheme\cite{35} uses the static pressure. For our jet calculations, the highest flow gradients are found near the jet exit. Since the static pressure is essentially uniform there (ideally expanded jet), using the static pressure is inappropriate.

The second order dissipation coefficient, $\epsilon^{(2)}$, is explicitly set to zero in regions where the flow is sufficiently smooth, i.e. $\nu$ is below a specified value. To minimize the size of the region with second order dissipation, this value should be raised to a maximum that results in a stable scheme. An appropriate value has been determined to be 0.0050 by numerical experimentation. The locations where the second order dissipation is necessary is found to be confined to certain locations in the jet core and should have no effect on the radiated acoustic solution. This is illustrated in figure 3 where a snapshot of the locations in the minor plane where second order dissipation is applied at a typical instant.

![Figure 2: Three dimensional view of the grid.](image1)

![Figure 3: Snapshot of the locations in the minor axis plane where second order dissipation is applied.](image2)

**4 Results**

In this section, rectangular jet simulation results are presented and compared to experimental data. The results are obtained by executing a run that consists of three phases. Since the initial condition of the simulation is a quiescent fluid, one phase is required to allow transients to leave the domain and establish the jet. When the domain is free of transients, the Kirchhoff integration is started. There is a transient convergence period required by Kirchhoff methods which is based on the furthest distance between the Kirchhoff surface and an observer location.\cite{33} The time required to converge the Kirchhoff solution constitutes the second phase. The final phase is used to sample variables in the jet and in the near and far acoustic fields. Only the data collected in the last phase has been used to generate the results presented here.

Figure 4 shows the centerline velocity distributions for the varicose and flapping excited jets and compares them to the experimental data.\cite{16} The varicose jet simulation results are shifted $-2D_{\nu}$ to match the potential
core length of the experiment. This common procedure is used so that the decay rates of the centerline velocity can be compared directly. This shift is applied to all comparisons of the varicose excitation simulation with experimental data presented throughout this paper. The flapping excited jet does not require a shift.

Figure 4: Centerline velocity distributions.

The comparison between the simulations and experiment after the end of the potential core shows that the simulation overpredicts the turbulent mixing slightly for both of the mode excitations since their centerline velocities decay at a faster rate. A distinguishing feature between the two simulation profiles is that the flapping excitation case has noticeable oscillations in the potential core region. These oscillations, also present in the experimental data, are due to a shock cell structure. They are also present to a lesser degree in the varicose excitation case profile. In the experimental jet, this structure most likely originates in the throat of the nozzle as a result of an imperfect nozzle design.

Recall that the static pressure prescribed by the steady nozzle exit conditions is set to the ambient pressure; however, superimposed on this steady condition are perturbations. These perturbations are likely responsible for the weak shock cell structure found in the simulated jet. Why the flapping mode excitation produces a stronger shock cell structure is not understood. It is interesting to note, however, that the simulation reproduces fairly accurately the amplitude, wavelength and phase of the shock cell structure (given the limited resolution of the experimental data). This may suggest that the mechanisms producing the shock cell structures in both jets are similar and that the experimental shock cell structure is not solely due to an imperfect nozzle design.

Since the evolution of large scale turbulent structures plays such an important role in supersonic jet noise generation, some of the simulated properties of these structures have been determined and compared to experimental data. In his experiment, Kinzie\textsuperscript{16} used hot wire anemometry to measure the fluctuations in the shear layer. In a compressible flow, these wires are sensitive to the mass flux that is normal to the wire. Measurements were made in both the major and minor axis planes of the jet. When performing major axis plane measurements, the hot wire was oriented parallel to the minor axis plane, i.e., it was parallel to the wall from which the shear layer was emanating. This was done to improve the resolution of the shear layer measurements. When arranged in this manner, the normal component of the mass flux consists of two velocity components, i.e., the hot wire is sensitive to:

\[
m = \sqrt{(\rho u)^2 + (\rho w)^2}
\]  

Figure 5 shows the axial development of the RMS of the variable \( m \) defined in equation 17 (normalized by the jet exit mass flux) in the major axis plane. The values plotted in the figure are the maximum RMS values through the shear layer for a given axial location.

\[
\text{Figure 5: Maximum RMS levels of the mass flux normal to the hot wire in the major axis plane.}
\]

The most apparent observation made from this figure is that the simulation over predicts the peak amplitude of the perturbations by approximately a factor of 2. Thus, there is significantly more turbulent mixing in the simulated jet compared to the experimental one. This is consistent with the centerline velocity decay profile presented earlier. A possible explanation for this discrepancy is the absence of a sub-grid-scale (SGS) turbulence model. In turbulent flow, the large scales of the turbulence are continually acted on by the finer scales. These fine scales behave as a dissipation mechanism for the larger scales. The algorithm used in this research does not address this issue explicitly. The algorithm applied here relies on the dissipation supplied by the numerical scheme to behave like the sub-grid scales of the true flow. The difference between the artificial and SGS dissipation is quantified in Chyczewski\textsuperscript{17} and will not be discussed in detail here. It will just be mentioned that a comparison of these two dissipation terms reveals that the SGS dissipation is usually larger than the artificial dissipation but the difference is not considered significant enough to account for the discrepancies found here. The high amplitude perturbation found in the simulation may also be explained by the nozzle exit
conditions. Berman et al.\textsuperscript{36} found that changing the exit conditions for their subsonic calculations can reduce the peak amplitude of the perturbations.

Aside from the peak amplitude discrepancy, a comparison of the trends in figure 5 between the simulation and the experiment is encouraging. There is a high amplitude growth rate prior to the end of the potential core, the value of which compares well between experiment and simulation. After this region, there is a saturation period and finally a gradual decay in the amplitude.

The power spectral density of the time series used to determine the RMS values discussed above for the varicose excitation in the major axis is presented in figure 6. The sample used to determine these spectra spans the final phase of the run described at the beginning of this section. This phase consists of 32,768 timesteps. In order to make handling the data less cumbersome, a sample was taken once every 16 timesteps. This procedure compromises no information since the sampling frequency is still much higher than the frequencies expected to be produced by the jet. Thus, the length of the entire sample is 2048 steps. To reduce the errors associated with using a finite (and relatively small) sample record, the 2048 sample is divided into 15 records that contain 256 steps. The intervals overlap one another by 128 elements. For example, the first interval contains samples 1 through 256, the second contains 129 through 384, and so on. The spectra presented in figure 6 are obtained by averaging the spectra obtained from each of the 15 intervals. A Hanning window is also used to reduce the errors associated with using the finite record length (see Bendat(1986)\textsuperscript{37} for example). Since the area under the power spectral density is equal to the RMS of the fluctuation, which have been previously discussed, the experimental spectra have been scaled so that they have RMS values equal to the simulation. This allows a direct comparison of the spectral distribution of energy. The spectra are presented as a function of the Strouhal number, which is the ratio of the frequency to a characteristic frequency. The characteristic frequency used in these spectra is 31.387 Hz, which is the experimental jet exit velocity divided by the equivalent diameter of the nozzle exit.\textsuperscript{10}

The comparison between the experimental and simulation spectra is favorable. In both the experiment and the simulation the energy moves to lower frequencies at the larger axial locations. This also agrees with observations made by Troutt.\textsuperscript{29} The spikes in the major axis experimental spectra near a Strouhal number of 0.5 are due to the shock cell structure found in the jet. They are the screech tones due to the phase locking of the radiated noise from the interaction of the large scale structures with the shock cell structure and the excitation of instability waves at the nozzle lip. These tones are most likely absent from the simulated jet due to its relatively weak shock cell structure.

In figure 7 sound pressure level (SPL) contours are shown in the minor axis plane of the simulated jet excited by a varicose mode. The definition of an SPL used here, consistent with Kinzie,\textsuperscript{10} is:

\[
\text{SPL} = 20\log_{10}\left(\frac{P_{\text{rms}}}{P_{\text{ref}}}\right)
\]

where

\[
P_{\text{ref}} = \left(\frac{P_{\text{ch}}}{P_{\text{atm}}}\right)(20 \times 10^{-6}) \frac{N}{m^2}
\] (19)

The pressure in the anechoic chamber, \(P_{\text{ch}}\), during his experiments was 3080 N/m\(^2\). The atmospheric pressure, \(P_{\text{atm}}\), is 1.01325 \(\times\) 10\(^5\) N/m\(^2\). The root mean square of the pressure, \(P_{\text{rms}}\), is determined in the same manner as the data used for the hot wire comparisons. Consistent with the experimental data,\textsuperscript{10} the noise appears to be generated at the end of the potential core and is directed in the downstream direction.

The remainder of this section will present and discuss far field noise predictions. For the far field noise predictions presented here the Kirchhoff surface is placed as
close to the jet as possible without intersecting regions of the jet plume where there are significant hydrodynamic fluctuations. Close proximity to the jet is desirable since the grid resolution is the finest there. Also, the closer the surface is to the jet, the less the pressure waves have to travel before reaching the surface. Thus, the numerical damping and dispersion errors are minimized. It is important not to place the surface too close to the jet since the Kirchhoff formula may interpret some hydrodynamic fluctuations as radiating ones.

The entire Kirchhoff surface is defined by four surface segments. Two surfaces are in the \((i, j)\) plane and two are in the \((i, k)\) plane \((i\) is the axial index\). The two end planes (perpendicular to the jet axis) are omitted. Since the noise propagation is predominantly in the downstream direction, the downstream end plane should not make any contributions to the noise levels at the observer locations used here (described below).

The grid used in the present study (shown in figure 2) is highly clustered near the nozzle lip region. This stretching varies in the axial direction so that a constant \(j\) or \(k\) grid line gradually moves away from the jet axis. This is very convenient in terms of deciding how to define the Kirchhoff surface. Each of the four surface segments can be placed very near the nozzle lip line at the nozzle exit plane. As the jet develops in the axial direction, and the mixing increases, the surface will gradually move away from the jet axis so that it never does intersect high mixing regions.

The Kirchhoff surface is illustrated in figure 8. The extent of the surface in the axial direction is from \(x = 1D_{eq}\) to \(20D_{eq}\). The surface is not extended to the wall so that the possibility of any effects it may have on the acoustic solution is reduced. The surface terminates at 20 diameters to reduce the effects of any reflections that may occur from the downstream boundary. The range of the domain covers the axial region where turbulent mixing noise is known to be generated, i.e., the region near the end of the potential core.

The lateral locations of the surface were determined by considering the mass flux perturbation levels. Figure 9 shows contours of the RMS of such perturbations in the major axis plane of the jet excited by a varicose mode. The grid (showing every tenth \(i\) grid line) is overlaid on the figures. The arrows (between \(x = 11D_{eq}\) and \(12D_{eq}\)) point to the grid lines that seem to be optimal for the planes of the Kirchhoff surfaces perpendicular to each figure. It is apparent that the surfaces do not intersect any region where significant fluctuations are found. These appear to be the optimal locations and are used for the calculations. Similar reasoning is used in the minor axis plane. In a study of the sensitivity of the noise predictions to the location of the Kirchhoff surface, Lyrintzis\(^9\) found that there is little difference between the results if the surface is placed at least one diameter away from the jet axis.

Figure 8: The surface used to for the far field (Kirchhoff) calculations.

Figure 9: RMS of the mass flux perturbations in the major axis plane for the jet excited by a varicose mode. The values are normalized by the jet exit mass flux.
The observer locations are selected to match the microphone stations used by Kinzie. They consist of locations on an arc 25\(D_e\) away from the center of the nozzle exit. The angular range is 15 to 50 degrees measured from the jet axis. Overall sound pressure levels in the far field are presented in figures 10 and 11 for the varicose and flapping excitation cases and compared to the experimental data. The figures are a function of the angle the observer makes with the jet axis and are commonly referred to as directivity plots.

For the locations close to the jet axis (i.e., small angle \(\beta\)), the simulation overpredicts the experimental data for both the varicose and flapping cases. This is most likely due to the higher amplitude instability waves (coherent structures) found in the simulated jet. Recall that the simulation predicts a peak instability wave amplitude that is approximately twice that of the experimental jet. If the peak amplitude in the experimental jet is scaled by a factor of two, and one assumes that the noise is dominated by turbulent mixing noise, then the radiated pressure amplitude should be scaled by a factor of four (as deduced from the acoustic analogy). This results in a twelve decibel increase in the radiated noise. If the noise is dominated by Mach wave emission, then the instability wave analysis predicts that the radiated pressure be scaled by a factor of two, which results in a 6 decibel increase in the far field noise. The peak differences between the simulation and experiment for the varicose and flapping excited jets are 7 and 12 dB, respectively. These differences suggest that the flapping jet has weak Mach wave radiation while the varicose jet noise is dominated by Mach wave radiation.

The good correlation between the discrepancy in the noise source prediction and the discrepancy in the far field noise prediction leads one to conclude that the algorithm is capable of predicting the far field noise radiation for a given source amplitude. This argument is strengthened by the good agreement in the trends of the simulated and the experimental jets. Considering first the comparison for the jet excited by a varicose mode (figure 10), the angle of peak noise radiation is correctly predicted to be near 25 degrees. The noise is more directional in the simulated jet, however, compared to the experimental one. Recall from the introduction that turbulent mixing noise is fairly directional. The shock cell structure in the experimental jet may explain why its noise is less directional. This structure leads to the additional shock associated noise generation mechanisms. Since this type of noise has upstream propagating components, it will tend to make the noise less directional when the contributions of all of the noise sources are combined. Although there is also a shock cell structure in the simulated jet excited by a varicose mode, it is comparatively weak.

The spectra of the far field noise in the major axis plane is shown in figure 12 for the simulated jet excited by a varicose mode. Like the hot wire spectra presented earlier, screech tones are apparent in the experimental data. Ignoring these tones, however, like the experimental jet, the simulated one does predict a fairly broad spectral peak centered near a Strouhal number of 0.2.

5 Conclusions

In this paper a general algorithm for the prediction of supersonic jet noise is presented. The algorithm consists of the numerical simulation of the noise sources and sound radiation to the acoustic near field. The time dependent near field solution is then passed on to a Kirchhoff formulation to determine the far field noise. The algorithm has been applied to a perfectly expanded, cold, supersonic rectangular jet problem. This geometry and set of flow conditions provide two advantages. The first is that turbulent mixing should be the sole noise generation mechanism present. This type of noise is found in the lower frequencies of the spectrum and therefore reduced the grid resolution requirements. The second advantage is that these conditions correspond to those of an experiment that has been conducted recently at Penn State. Thus, a comparison with the experimental data has been possible. This comparison has revealed that the algorithm is capable of reproducing

![Figure 10: Far field SPL levels for the varicose excitation.](image1)

![Figure 11: Far field SPL levels for the flapping excitation.](image2)
Figure 12: Far field noise spectra in the major axis plane for the flapping excitation. The numbers to the right of each graph denote angular location in degrees. The heavy lines are the experimental spectra.

many of the flow and noise features found in the experimental jet. One discrepancy is that the simulation overpredicts the amplitude of the instability waves by a factor of two. Thus, future work on this project should investigate the reasons for this discrepancy and devise methods to eliminate it.

Acknowledgment

This work was supported by the NASA Graduate Student Researchers Program and by the NASA Langley Research Center, under the grants NAG-1-1470 and NAG-1-1367. The authors would like to thank Dr. Yusuf Özyörük for his efficient parallel implementation of the Kirchhoff method and for his help in porting it to our code.

References


