Progress in time-domain calculations of ducted fan noise - Multigrid acceleration of a high-resolution CAA scheme

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The numerical simulation of an aeroacoustics problem using the full time-dependent Euler or Navier-Stokes equations requires both the mean and unsteady flow fields. Sometimes the mean flow is equivalent to the steady state flow and used to start the unsteady solution process. In this case, both the steady and unsteady flow fields have to be obtained through the same numerical method, so that the restart process is smooth and no spurious waves are generated. However, due to its low dissipation and consequently slow convergence, a high-resolution computational aeroacoustics scheme is not suitable for computing the steady flow field. In this paper, such a scheme is accelerated to convergence, without altering its residual, by using a full approximation storage multigrid method for the prediction of ducted fan noise. It is demonstrated that significant convergence improvements are obtained using the multigrid method, making it possible to attain steady state solutions on extremely fine meshes designed for high-frequency radiation problems. Far-field noise results for a JT15D inlet are presented and compared with data at realistic flight configurations.

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Progress in Time-Domain Calculations of Ducted Fan Noise: Multigrid Acceleration of a High-Resolution CAA Scheme

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Abstract

The numerical simulation of an aeroacoustics problem using the full, time-dependent Euler or Navier-Stokes equations requires both the mean and unsteady flow fields. Sometimes the mean flow is equivalent to the steady state flow and used to start the unsteady solution process. In this case, both the steady and unsteady flow fields have to be obtained through the same numerical method, so that the restart process is smooth and no spurious waves are generated. However, due to its low dissipation and consequently slow convergence, a high-resolution computational aeroacoustics scheme is not suitable for computing the steady flow field. In this paper, such a scheme is accelerated to convergence, without altering its residual, by using a full approximation storage multigrid method for the prediction of ducted fan noise. It is demonstrated that significant convergence improvements are obtained using the multigrid method, making it possible to attain steady state solutions on extremely fine meshes designed for high-frequency radiation problems. Far-field noise results for a JT15D inlet are presented and compared with data at realistic flight configurations.

1 Introduction

As more powerful computers are being introduced, it is becoming possible to include more physics in the numerical simulations of aeroacoustics problems by solving the full Euler or Navier-Stokes equations by solving the full Euler or Navier-Stokes equations. In this approach, however, the acoustic field is obtained by taking the difference between the instantaneous and mean fields. If the transient part of the instantaneous pressure is associated only with acoustic waves, the mean field will be equivalent to a solution that will be obtained by applying time-invariant boundary conditions in the near-field (e.g., steady solid wall boundary conditions).

This solution is often referred to as the steady state solution. An example case where the transient flow field is driven only by acoustic waves is a turbofan inlet at steady flight.

As part of an ongoing research effort to predict ducted fan noise, a hybrid, nonlinear, time-domain code has been developed. This code solves the 3-D Euler equations to determine the near-field acoustics and uses a moving surface Kirchhoff method to predict the far-field noise. The flow solver uses an explicit, spatially and temporally fourth-order accurate, finite difference, Runge-Kutta (R-K) time-integration scheme. Due to the approach taken, the steady state solution is obtained first and then the time-accurate inlet fan-face conditions (acoustic sources) are turned on, and the solution is advanced in a time-accurate manner. This procedure inherently requires the same scheme, namely fourth-order central finite difference, for evaluating the residuals of the governing equations in both the steady and time-accurate calculations so that starting from the steady solution is smooth and no spurious waves are generated.

However, there are several difficulties associated with using a high-order explicit scheme for steady state calculations. The fourth-order scheme is really designed for time-accurate problems with low dissipation and dispersion. In other words, low-frequency modes are not damped out. Moreover, typical inlet radiation problems involve high blade passing frequencies (BPF). Hence, very fine meshes are usually needed, resulting in very small time steps and consequently slow convergence. Also, ducted fan noise predictions are often made at low free stream Mach numbers as associated with landing or take-off conditions, which are considered more critical in terms of community noise regulations. The disparity of the eigenvalues in these cases is a degrading factor in achieving sufficiently rapid steady state solutions.

There are several convergence acceleration techniques applied to explicit time-marching schemes, such as residual smoothing, and multigrid methods. Residual smoothing is most effective when applied implicitly and, therefore, not appropriate for the current approach. The most suitable technique for the current situation

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and the governing equations of conformal mappings. The orthogonal mesh system is created through a sequence of conformal mappings around an airfoil and the JT15D engine inlet are presented as well as some far-field fan noise results at realistic conditions.

2 Hybrid Ducted Fan Radiation Code

The hybrid code solves the 3-D Euler equations on a 3-D body fitted coordinate system (structured meshes) and passes the near-field acoustic pressure to a Kirchhoff method based on the formulation of Farassat and Myers to predict the far-field sound. The governing equations are solved in a relatively small domain using nonreflecting boundary conditions based on the work of Bayliss and Turkel and Tam and Webb. An orthogonal mesh system is created through a sequence of conformal mappings and the governing equations are formulated in cylindrical coordinates to effectively treat the grid singularity at the centerline. Fourth-order accurate, cell-centered finite differencing and four-stage, noncompact R-K time integration are performed to advance the solution. Jameson type artificial dissipation is used to suppress spurious waves. The acoustic source is formed using the eigensolutions of the cylindrical duct problem and the rotor-stator interaction theory of Tyler and Sofrin. The Euler solver and the Kirchhoff method are coupled such that as soon as the Euler solution becomes available, the Kirchhoff surface integrations are performed in a recursive manner to predict the far-field noise. All calculations are carried out on parallel computers using the data parallel paradigm. Özzyörük and Long describes the fourth-order flow solver with emphasis on the hybrid code's parallel aspects. Özzyörük and Long discusses the acoustic source model and the Kirchhoff coupling issues for engine noise predictions.

3 Multigrid Method

Through a multigrid method, low frequency errors of the fine grid solution are transferred onto a sequence of coarse grids and are smoothed by updating the solution there. Then the corrections to the solution obtained on the coarse grids are interpolated back to the fine grid with its own low frequency errors having been aliased into high frequency errors which can now be damped out on the fine mesh.

This procedure is applied in a systematic way to push the low frequency errors quickly out of the domain. Hence, a significant convergence improvement is gained basically for two reasons. First, the number of operations required for each iteration to update the solution is reduced significantly on coarse grids, clearly due to the fewer grid points. Second, the time step sizes on coarse grids are larger than on the fine grid. In other words, the effect of the boundary conditions that drive the flow field is felt more rapidly throughout the domain.

In multigrid methods the grids of varying coarseness are usually obtained by simply deleting every other grid line of the next finer grid. The fine grid size (number of grid points) is chosen such that the grid lines representing the boundaries are retained in this process.

3.1 FAS Scheme

The semi-discretized Euler equations and the far-field boundary conditions can all be written as

\[ \frac{dQ}{dt} = -J[F(Q) - D(Q)] \]  

where \( Q \) is the vector of dependent solution variables (conservative state variables for the interior points, perturbations of primitive variables for the outer boundaries), \( J \) is the Jacobian of the coordinate transformation, \( F(Q) \) represents the collection of the spatial derivatives and \( D(Q) \) represents artificial dissipation. Thus, \( [F(Q) - D(Q)] \) is the residual. Equation 1 is integrated using the classical (noncompact) four-stage R-K scheme, resulting in fourth-order time accuracy.

Jameson's FAS multigrid time-advancing algorithm, however, uses his and his co-workers compact R-K scheme, which is given as

\[
Q^{(0)} = Q^n,
Q^{(i)} = Q^n - \alpha s J \Delta t [F(Q^{(i-1)}) - D(Q^{(0)})],
Q^{n+1} = Q^{(4)},
\]  

where the superscript \( n \) is the time step, \( \Delta t \) is the time increment from one time step to the next, and \( \alpha = [1/4, 1/3, 1/2, 1] \).

Here we adopt this scheme directly for the steady state calculations, since any time-advancing method will not alter the steady state solution so long as the residual evaluation scheme remains the same. In other words, at the steady state \( dQ/dt = 0 \) leaving the field governed by \( F(Q) - D(Q) = 0 \).
It is useful to introduce the following notation in discussing Jameson's FAS multigrid scheme below. A transfer (restriction) operator for the state variables is denoted by $T$, for the residuals by $W$, and an interpolation (prolongation) operator is denoted by $I$. All these operators take on subscripts and superscripts that indicate the mesh spacing of the two grids that are involved in the data link, whose direction is understood to be always toward the grid of the superscript. These grids are assumed to be given by the sequence, from the mesh of the desired resolution ($h$) to the coarsest,

$$G_h, G_{2h}, \ldots, G_{r_{h/2}}, G_{r_{h}}, \ldots, G_{v_{h}}$$

where a subscript indicates the mesh spacing of the grid, and $r$ and $v$ are integers that are given by integer powers of 2.

First the solution on mesh $G_{2h}$ is initialized through the restriction of the data from mesh $G_h$ as

$$Q_{2h}^{(0)} = T_h^{2h} Q_h,$$

where $Q_h$ is the current value of the solution variable on mesh $G_h$. Then a residual forcing function $P_{2h}$ is established such that the solution on grid $G_{2h}$ is driven by the residual calculated on grid $G_h$ using the solution $Q_h$. This is achieved by setting

$$P_{2h} = W_h^{2h}(F_h(Q_h) - D_h(Q_h)) - (F_{2h}(Q^{(0)}_{2h}) - D_{2h}(Q^{(0)}_{2h}))$$

This function is added to the residuals of the R-K scheme, which become

$$Q_{2h}^{(s)} = Q_{2h}^{(0)} - \alpha_s(J\Delta t)^{2h}[F(Q_{2h}^{(s-1)}) - D(Q^{(0)}_{2h}) + P_{2h}],$$

Hence, the value of the residual at the first stage will be $W_h^{2h}(F_h(Q_h) - D_h(Q_h))$ since the other terms will cancel. Thus the solution on grid $G_{2h}$ will be driven by the residual transferred from grid $G_h$. It should be noted that Jacobian and the time step in the above R-K scheme are associated with grid $G_{2h}$, where the time step (local) will be significantly larger, consequently a faster propagation of the signals (errors) and an improved convergence rate will result.

The above procedure is continued until the coarsest mesh ($G_{v_{h}}$) is reached, where the correction to the solution will be given by

$$\Delta Q_{v_{h}} = Q_{v_{h}}^{(4)} - Q_{v_{h}}^{(0)}$$

Then, when going back from a coarse grid to the next finer mesh, the accumulated correction is given, on an intermediate grid $G_{r_{h/2}}$, by

$$\Delta Q_{r_{h/2}} = Q_{r_{h/2}}^{new} - Q_{r_{h/2}} = T_{r_{h}}^{r_{h/2}}(Q_{r_{h/2}}^{new} - Q_{r_{h}}^{(0)})$$

where $Q_{r_{h/2}}$ is the solution on mesh $G_{r_{h/2}}$ after the R-K time stepping on grid $G_{r_{h/2}}$ and before the transfer from grid $G_{r_{h}}$, and $Q_{r_{h/2}}^{new}$ is the final value of $Q_{r_{h/2}}$, resulting from both the correction calculated in the time step on grid $G_{r_{h/2}}$ and the correction transferred from grid $G_{r_{h}}$.

Finally the updated solution on the fine grid is given by

$$Q_{h}^{n+1} = Q_{h}^{new} = Q_h + \Delta Q_h$$

The restriction and prolongation operators used to generate the data connections between the grids for the second-order finite difference and finite volume multi-grid methods have been established fairly well. For example, in finite difference methods, direct injection of the data from a fine grid to the next coarser grid is one practical and extensively used method (Figure 1). In finite volume methods the transfer of the data is realized through a volume weighted average of four cells on the fine grid which make up one cell of the next coarser grid. This way the flow state variables are conserved. It is very natural to take the volume weighted average of the data in finite volume methods because of the fact that the data is assumed to reside at a cell center, which on the next coarser grid will be affected mostly by the largest of those four constituent cells.

Figure 1: Data transfer from fine mesh to the coarse mesh.

As indicated, the current algorithm uses a fourth-order accurate cell-centered finite difference method, wherein no grid points coincide with the boundaries of the domain. Only in this way does the mesh structure of our current method resemble that of the classical finite volume method. The weighted cell centers in our method, which are the grid points of its finite difference scheme, are not the geometrical cell centers. Instead, they are obtained via high-order interpolations, or the mesh formed by these points is directly produced using the grid generator described in Özyörük such that no grid points exist at the solid boundaries. This is
important in terms of the smoothness of the geometrical derivatives over the stencils of the finite difference scheme.

For the multigrid application here we generate in each of the $\xi, \eta$ and $\zeta$-directions (curvilinear coordinate directions) twice as dense a mesh as the fine mesh. We refer to this new mesh as the source mesh, which now has grid lines on the boundaries of the domain. The source mesh readily provides the weighted cell centers, or the grid points of the current finite difference method for all the meshes used in the multigrid convergence acceleration process. This point is illustrated, for clarity, in only one dimension, in Figure 2. Hence, a given mesh level is at least as smooth as the others.

3.2 Restriction (transfer) operators

In the current approach the transfer operator $T$ for the dependent variables is defined as

$$Q_{2h} = T_{h}^{2h}Q_{h} = \frac{1}{4} \sum_{cell=1}^{4}(Q_{h})_{cell},$$ (10)

where the four cells that are involved in the summation make up one cell of grid $G_{2h}$, as illustrated in Figure 1 for a finite volume method. This simple averaging is preferred over the volume weighted averaging of a finite volume method due to our grid system.

However, the transfer of the residuals is performed along the same lines as Jameson’s method. We simply sum the residuals of those four constituent cells on grid $G_{h}$ to obtain the residual of a $G_{2h}$ cell. Thus for the interior grid points we simply write

$$F_{2h}(Q_{2h}) - D_{2h}(Q_{2h}) = \nu_{h}^{2h}(F_{h}(Q_{h}) - D_{h}(Q_{h}))$$

$$= \sum_{cell=1}^{4}(F_{h}(Q_{h}) - D_{h}(Q_{h}))_{cell}$$ (11)

Since the time-accurate calculations use nonreflecting boundary conditions on the outer boundaries of the domain, steady state calculations for the engine inlet problems are also performed using nonreflecting boundary conditions. These conditions are put in the same time-dependent partial differential form as the interior equations (see Özyörük or Özyörük and Long). Then the multigrid method is applied to the entire system of equations. Therefore, the residual transfer operator is defined differently for the far-field boundary points. An averaging along the boundary and a consequent scaling by the coarse cell Jacobian is performed for the transfer of nonreflecting boundary conditions residuals.

3.3 Prolongation (interpolation) operator

The corrections are prolonged back to the next finer mesh by using interpolation. For simplicity, consider the 1-D coarse mesh and the next finer mesh shown in Figure 3. Two finer grid points fall between two coarse grid points. The variation between two coarse grid points is assumed linear and the data at the finer grid points are simply calculated by using the formulae

$$Q_{2h+1/2}^{2h+1/2} = \frac{1}{2}(Q_{2h}^{2h} + Q_{2h}^{2h+1})$$ (12)

$$Q_{h}^{2h} = T_{h}^{2h}Q_{2h}^{2h+1/2} = \frac{1}{2}(Q_{2h}^{2h} + Q_{2h}^{2h+1/2})$$ (13)

where a superscript indicates the grid point the data is associated with (see Figure 3). Special treatments, such as extrapolation, are needed at or near the boundaries.

3.4 Cycling Strategy

V-cycles with 3 mesh levels are used, although using more grid levels generally improves the efficiency of multigrid methods. After every restriction of the data from a fine grid to the next coarser grid, the solid wall and the fan-face boundary conditions are applied to prevent large jumps that might trigger large adaptive dissipation coefficients. Similarly, these boundary conditions are also applied after every prolongation operation of the data from a coarse mesh to the next finer
mesh. The V-cycles are started on the finest mesh from the first step of the pseudo-time advancement and continued until convergence is obtained.

4 Results

In this section, both steady flow simulations using the multigrid method and acoustic radiation simulations from a turbofan engine inlet are presented.

4.1 Steady State Solutions

In multigrid work the RMS residual or the error is usually plotted versus the number of work units so that the convergence rates can be compared in the same norms with those given by using single meshes. This is because one has to perform more operations in a multigrid cycle than in a single mesh cycle (equivalent to one time step or iteration for single mesh). The increase in the total number of operations is usually proportional to \( P = (p + \sum_{q=0}^{2} 1/2^q) \), where \( p \) is the overhead cost and the sum is the increase due to performing calculations on multiple grids. The overhead is usually associated with the restriction and prolongation operations as well as the application of the solid wall, far-field, and the fan-face boundary conditions in the case of an engine inlet after each of the transfer operations. The total cost increase factor \( P \) for the current code is about 1.8. Therefore, the ratio of the multigrid convergence rate to single mesh convergence rate per fine mesh iteration must be over at least 1.8 so that one can talk about convergence improvement using the multigrid method.

In other words, the convergence improvement per time step using multigrid method must not be offset by the introduced overhead and the increase in the number of operations. One good measure would be the convergence rate given per CPU time. However, the CPU time is usually machine dependent. Therefore, we present the results showing the convergence versus the number of cycles curve.

4.1.1 RAE 2822 Airfoil

First, we present results for the steady state flow around the RAE 2822 airfoil at a free stream Mach number of \( M_\infty = 0.725 \) and an angle of attack of \( \alpha = 2.92^\circ \). The solution was performed on 3 different size domains using the Euler equations together with 2 different far-field boundary conditions sets, namely the Riemann type boundary conditions and the Bayliss-Turkel type non-reflecting boundary conditions. The fine meshes of the multigrid runs all included 192 x 96 x 1 cells (including the ghost cells). Due to their C topology, these meshes had high aspect ratio cells in the far-field, especially in relatively large domains.

Figure 4 shows the convergence histories of the single grid and multigrid runs. The convergence improvements by the multigrid application are clear. For these cases the Riemann type boundary conditions were used. It is evident that the domain size (indicated by 5c, 10c and 25c in the figure; \( c = \) chord length) does not have much effect on the single-grid convergence of the solution. However, the multigrid convergence is influenced by the domain size. This is mainly due to the strong stretching of the grid points in the far-field region on the relatively large domains.

Figure 4: Convergence histories of the Euler runs on varying-size domains. RAE2822 airfoil, \( M_\infty = 0.725, \alpha = 2.92^\circ \).

Figure 5 illustrates the Mach contours about the airfoil. Although the mesh was relatively coarse in the chord-wise direction over the airfoil, the shock was captured very well, as evident from the figure. Figure 6 indicates that the numerical pressure distribution over the airfoil is highly affected by the computational domain size. The differences between the Euler solutions and the experiment observed in this figure are mainly due the viscous effects plus more importantly the wind tunnel effects that the experimental measurements involve. Therefore, pressure coefficient comparisons are usually performed at modified angles of attack matching the experimental lift coefficient (see, for example, Coakley). However, this figure is significant to validate the multigrid solutions. They agree perfectly with the single grid solutions. Figure 7 indicates a very interesting point. It is clear that, as the domain size is increased, the use of the Riemann type far-field boundary conditions results in increasingly over-predicted pressure distributions, while the use of the Bayliss-Turkel type boundary conditions results in decreasingly over-predicted solutions as compared to the experiment. However, both type boundary conditions yield converging solutions as the domain size is increased further and further.

Figure 5: Mach contours about the RAE2822 airfoil, \( M_\infty = 0.725, \alpha = 2.92^\circ \).
4.1.2 JT15D inlet

The low Mach number cases presented in this section pertain to the JT15D inlet geometry\(^\text{12}\) (without a centerbody) at zero angle of attack. All mesh levels were generated using conformal mapping.\(^\text{3}\) The far-field boundaries were placed as close as only 1.5-2.0 inlet diameters from the inlet lip. At the fan, 1-D characteristic based nonreflecting boundary conditions were used.\(^\text{3,23}\)

Figure 8 compares the convergence histories (density residual) of the single mesh and multigrid runs for a free stream Mach number of 0.204 and a mass flow rate (MFR) of 14 kg/s. The fine mesh for this case had 192 \(\times 32 \times 1\) cells with aspect ratios varying from nearly 1 to 5. Clearly there is a significant improvement in convergence per time step using the multigrid method. It is extremely important to drive the numerical errors to very low levels so that they do not contaminate the acoustic solutions. Therefore, in noise prediction calculations, the steady state flow residual is usually driven 10-13 orders of magnitude down from its initial value. It has been experienced that the single grid runs do not usually drive the residual down to these levels at a constant rate. Sometimes the solution does not converge at all when these levels are aimed.

Through multigrid iterations the residual is expelled more quickly out of the domain as mentioned earlier. This is evident from the mass flow ratio histories of the single grid and multigrid runs, as shown in Figure 9. The mass flow ratio with the multigrid method came very near the specified mass flow ratio and stabilized within only 250 time steps while this took ap-
approximately 1700 steps for the single grid. This behavior of multigrid methods is crucial in quick aerodynamic design analyses. The 1-D characteristic fan-face boundary condition implementation described in Ozyörük yielded an extremely accurate mass flow ratio at convergence. The computed mass flow ratio is 0.594 as compared to the specified 0.596.

In Figure 10 the convergence histories for \( M_\infty = 0.192 \) and \( MFR = 22 \text{ kg/s} \) are presented. The fine mesh for this case had \( 384 \times 96 \times 1 \) cells with aspect ratios ranging from nearly 1 to 5. This mesh was designed to resolve waves at up to \( 4.8 \text{ KHz} \) in the near-field. It is extremely difficult to attain a steady state flow field in reasonable number of time steps on this kind of fine meshes using the fourth-order accurate algorithm, as indicated by the single grid convergence curve. However, using the FAS multigrid technique a significant improvement was observed. It is evident by comparing Figures 8 and 10 that the improvement tends to increase as the mesh is further refined. This can be attributed to the increase in the transferred error bandwidth during the restriction and prolongation operations.

Figure 11 illustrates the histories of the mass flow ratios. Again the mass flow ratio for the multigrid case settled very quickly compared to the single grid case. The final mass flow ratio at convergence is 0.935, while 0.938 was specified as the operating condition. The error is well below 1%.

The flow field Mach number contours for this case at convergence are shown in Figure 12. The flow accelerates in the throat region due to the high mass flow rate. The Mach number in this region reaches 0.27. This means that the wavelength of the acoustic waves in the upstream direction is shortened by a factor of 0.73 \((= 1.0 - M)\). This is a disadvantage in the ducted fan noise computations. Notice that the contour lines in this figure are extremely smooth across the centerline showing no signs of a singularity problem. This can be further observed when contours of the ratio of the local total enthalpy to the free stream total enthalpy are examined in Figure 13. The ratio is extremely close to unity everywhere except near the solid wall where the effect of artificial dissipation is observed. This effect, however, is only slight. These results are extremely important in terms of showing that the method can accurately predict steady inlet flow fields.

4.2 Inlet Radiation Cases

The ultimate test of the current code is carried out in this section. Far-field noise of a spinning mode of the JT15D engine is predicted at actual flight conditions.
The JT15D engine configuration has an array of 41 rods placed in front of the 28-blade rotor, which produces mode (13,0) interaction tones at the BPF. The flight Mach number \(M_\infty\) is 0.204 and the inlet duct carries a mass flow of 17.297 kg/s at an engine speed of 8120 revolutions per minute (RPM). There is no angle of attack. The (13,0) mode at these conditions has a BPF of 3789.3 Hz and cut-off ratio of \(\xi_{13,0} = 1.27\). This frequency corresponds to the dimensionless frequency parameter \((2\pi BPF) \times (r_f/c_f) = 18.65\), where \(r_f\) and \(c_f\) are the duct radius and the speed of sound at the fan stage, respectively.

The steady state part of the problem was solved on an axisymmetric, 384 x 96 x 1-cell mesh using the multigrid technique. This mesh is the same as the one used in the previous section and is shown in Figure 14. The steady state solution was then spread out onto one of the 13 periodic 3-D grids to start the time-accurate part of the solution, using periodic boundary conditions in the circumferential direction. One indeed does not need to solve this problem on a fully 360° mesh, since a single spinning mode generates periodic pressure patterns in the circumferential direction. The (13,0) mode generates periodic patterns at every 360°/13 degrees. Therefore, the problem was solved on a 384 x 96 x 16 mesh, having 16 cells per circumferential lobe. The time-accurate R-K iterations were carried out using a time step size \(\Delta t = 0.25/(384 \times BPF)\), and the Kirchhoff integrations were performed at every 16th step of the R-K iterations.

Figure 15 shows the location of the Kirchhoff surface and the steady state pressure contours for this case. A snapshot of the acoustic pressure contours in the vertical plane of the inlet is shown in Figure 16, where also shown are the RMS acoustic pressure contours. The directivity trend of the radiating (13,0) mode is evident from this figure. The far-field sound pressure levels as calculated for 24 observer points at a distance of 30.48 m at every 3° are shown in Figure 17 along with the experimental data and the finite element-wave envelope (FE-WE) solution of Eversman et al.

Figure 18 shows the far-field sound pressure level (SPL) for the conditions, \(M_\infty = 0.200\), \(MFR = 19.275\) kg/s and \(BPF = 4360\) Hz. The frequency parameter for this case is \(k_f r_f = 21.48\). For this case a time step size of \(\Delta t = 0.4/(384 \times BPF)\) was used and the Kirchhoff integrations were performed at every 12th R-K iteration.

The comparison of the current simulations with the experimental data reveals very good agreement in general, although the predictions have the SPL peaks at somewhat higher angles. Similar differences are observed between the FE-WE solutions and experiment, as well. However, the current simulations have about a 2-degree better prediction of the peak SPL angles than the FE-WE method. The reason for the differences between the experiment and the simulations is not clear. At this point we can only speculate about the differences, such as that the source model used is only approximate and does not exactly represent the real acoustic source, namely the interactions of the rods with the rotor. Nonetheless the source has been modeled reasonably well.

## 5 Conclusions

Convergence acceleration is essential in the current approach of the ducted fan noise prediction method solv-
Every 3rd grid line shown

Figure 14: The mesh system and two typical Kirchhoff surfaces around the JT15D inlet.

ing the full, time-dependent Euler equations together with nonreflecting boundary conditions. For this purpose, a multigrid convergence acceleration technique that retains the high-order accuracy of a typical, explicit, high-resolution CAA algorithm has been developed and implemented. Particularly, Jameson’s full approximation storage method has been utilized. The method has been applied successfully both to the time-dependent Euler and nonreflecting boundary conditions equations. Although only 3 mesh levels with V-cycles are used, the improved convergence characteristics of the hybrid code with the multigrid method make it possible to attain steady state flows at very low Mach numbers on very fine grids. Example calculations showing the use of the method have been carried out for both external and internal flow problems. Far-field noise simulations have been performed for an actual turbofan engine inlet and comparisons with flight data and other numerical results have indicated very good agreement.

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JT15D at $M_\infty = 0.204$, $MFR = 17.297 \text{ kg/s}$
(13,0) mode, $BPF = 3789.3 \text{ Hz}$

Figure 16: Instantaneous and RMS acoustic pressure contours in the vertical plane. (13,0) mode, JT15D inlet, $M_\infty = 0.204$, $MFR = 17.297 \text{ kg/s}$, $BPF = 3789.3 \text{ Hz}$.


Figure 17: Far-field sound pressure level of the (13,0) mode. JT15D inlet, $M_\infty = 0.204$, $MFR = 17.297 \text{ kg/s}$, $BPF = 3789.3 \text{ Hz}$.


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