Cosmology (ASTRO/PHYS 545)  
Spring 2017  
Problem Set 9  
Due 19 April 2017  

Solving problem sets is one of the most efficient ways of learning the subject. You are encouraged to collaborate with fellow students and/or to consult senior students, local postdocs and me. But, please write the solution by yourself.

Homework is due at the end of the class on 19 April 2017. No homework will be accepted after I post the solution on the course webpage (usually right after the class following the due date).

1. (20 pts) [BBN constraint of standard model] As discussed in class, BBN can be used as a probe of possible deviations from the standard cosmological model and the standard model of particle interactions. The purpose of this problem is for you to work through some of these constraints.
   Use the measured value of Helium abundance $Y_p = 0.2534 \pm 0.0083$, but, you don’t have to reproduce the central value from the estimation. Just use that $Y_p$ is measured in 3.28% accuracy. Assume that all neutrons at the time when $X_D^{(eq)} \approx 0.01$ end up in the Helium nucleus.
   (a) Explain how the primordial $^4$He abundance would change if the neutron lifetime were longer or shorter. Suppose some particle theorists speculate that Fermi’s constant $G_F$ might actually be a function of time. Based on $\tau_n \propto G_F^{-2}$, estimate the BBN constraint of the value of Fermi’s constant at the time of nucleosynthesis?
   (b) It is plausible that there is a neutrino-antineutrino asymmetry in the Universe, and therefore that the cosmological neutrino number density today is greater than it is in the canonical picture. What is the upper limit to the current neutrino density $\Omega_\nu$ provided by BBN (assume the neutrinos are massless)? How is that translated to the chemical potential of neutrinos ($\xi \equiv \mu/T$)?

2. (10 pts) [Helium recombination] The binding energy of $\text{He}^+$ and $\text{He}$ atoms are, respectively, 54.4 eV and 24.6 eV. Assuming the full equilibrium, find out the recombination redshift of $\text{He}^+$ and $\text{He}$ atoms using $Y_p = 0.24$ and $\Omega_b h^2 = 0.023$. 
