Solving problem sets is one of the most efficient ways of learning the subject. You are encouraged to collaborate with fellow students and/or to consult senior students, local postdocs and me. But, please write the solution by yourself.

Homework is due at the end of the class on 12 April 2017. **No homework will be accepted after I post the solution on the course webpage** (usually right after the class following the due date).

1. (25 pts) [**WIMP mass and cross-section**] In this problem, you will summarize the properties of the cold relic WIMP particles in the mass($M_{\text{cdm}}$) - cross section($\langle \sigma v \rangle$) plane. In the log-scale plane of $x$ (the mass of cold dark matter from 1 GeV to $10^5$ GeV) and $y$ (thermal average of $\langle \sigma v \rangle$ from $10^{-38}$ cm$^2$ to $10^{-34}$ cm$^2$), plot regions satisfying following condition ($h = 0.7$):

   (a) $\Omega_{\text{cdm}} = 0.24$, s-wave annihilation, $\Omega_{\text{cdm}} = 1$, s-wave annihilation,
   (b) $\Omega_{\text{cdm}} = 0.24$, p-wave annihilation, $\Omega_{\text{cdm}} = 1$, p-wave annihilation,
   (c) Weak interaction via $W^\pm/Z^0$ bosons: $\langle \sigma v \rangle = G_F^2 m^2$
   (d) Weak interaction via heavier bosons with mass $M_{\text{new}} = 1$TeV: $\langle \sigma v \rangle = G_F^2 m^2(M_W/M_{\text{new}})^4$
   (e) Unitarity bound: $\langle \sigma v \rangle = m^{-2}$. What is the critical mass of the dark matter above which the dark matter energy density would over-close the universe (that is, $\Omega_m > 1$)?

2. (25 pts) [**Alpher, Herman, Gomow’s estimation of $T_{\text{cmb}} \approx 5$ K**] Around the year 1948 (that is, some 20 years **before** the CMB was discovered), Alpher, Herman and Gomow have written a serious of papers predicting the thermal CMB, and estimated the temperature of CMB $\approx 5 \sim 10$ K! Let us follow their line of thought to do the same estimation. **For this problre, do not use $T_{\text{cmb}} = 2.726$K!** They somehow figured out that the nucleosynthesis must happen at $T \approx 10^9$K at which the interaction $n + p \rightarrow D + \gamma$ has freezed out. They also knew that the cross section for the interaction is (at $T \sim 10^9$K) $\langle \sigma v \rangle \approx 10^{-20}$cm$^2$/s. By the way, they did not know about neutrinos, and only included photon as relativistic species.

   (a) From the freeze-out condition, and that the freeze-out temperature $T \approx 10^9$K, calculate the baryon number density at the time of freeze-out. Here, assume that all baryons are participating the interaction.
   (b) They then use the current matter (of course, baryon, because, back then, dark matter has not been discovered yet) density $\rho_{b0} \approx 10^{-30}$g/cm$^3$ to estimate the redshift corresponding to $T \approx 10^9$K.
   (c) What is the estimated temperature of photon at present time?