Homework #9: Due Oct. 31, 2008

1. Suppose that $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ where $a_0 > 0$. Show that $f(x)$ is locally positive at 0. In other words, show that there exists a $\delta > 0$ such that $f(x) > 0$ for all $x \in (-\delta, \delta)$.

   Hint: You may use the fact that $\lim_{x \to 0} cx^n = 0$ for any real number $c$ and positive integer $n$.

2. Show that $\lim_{x \to 0} \frac{1 - x}{x^2 + 1} = 1$ using only the definition of the limit of a function.

3. Show that $\frac{x}{x + 1}$ is continuous at $x = 1$ using only the definition of continuity of a function.