1. Recall that \( s(n) \) represents the sum of the prime factors of the integer \( n \). Show that for every integer \( k > 0 \), \( 1/k \) is a cluster point of \( \{s(n)/n\} \).

2. Show that for all sequences \( \{a_n\} \), the sequence \( \{b_n\} \) defined by
\[
b_n = \frac{1 + a_n}{1 + a_n^2}
\]
has a convergent subsequence. Hint: Use the methods of Calculus to show that \( f(x) = \frac{1+x}{1+x^2} \) is bounded (i.e., find the extreme values of \( f(x) \)). Explain what this fact about \( f(x) \) has to do with the existence of a convergent subsequence of \( \{b_n\} \).

3. Suppose that \( \{a_n\} \) for \( n \geq 0 \) has the following property: there exists constants \( N \) and \( K \) where \( 0 < K < 1 \) such that
\[
|a_n - a_{n+1}| < K|a_{n-1} - a_n|
\]
if \( n > N \). Prove that \( \{a_n\} \) is a Cauchy sequence. Hint: If \( m > n \), then
\[
a_n - a_m = a_n - a_{n+1} + a_{n+1} - a_{n+2} + a_{n+2} - \cdots + a_{m-1} - a_m.
\]

4. Refer to Exercise 6.5 #3. Use parts (a), (c), (e), (f) and (g) to prove the following:
   
   (a) If \( A \subseteq B \) then \( \inf(A) \geq \inf(B) \).
   (b) If \( c < 0 \) then \( \inf(cA) = c \sup(A) \).
   (c) \( \inf(A + B) \geq \inf(A) + \inf(B) \)

   Note: You do not need to prove parts (a), (c), (e), (f) and (g). The proofs of parts (a), (c), (e), (f) and (g) are posted online. Do not simply alter these proofs to prove the above statements.