Homework #2: Due Sept. 12, 2008

1. Find an upper bound for

\[ \left| \sin(n) + \frac{1}{3} \sin(2n) + \frac{1}{3^2} \sin(3n) + \cdots + \frac{1}{3^{n-1}} \sin(n^2) \right|. \]

Justify your answer.

2. Let \( a_n = \ln(3n + 5) - \ln(n) \) for \( n \geq 1 \). Show that \( \{a_n\} \) converges. You may assume that \( \ln(x) \) is an increasing function.

3. Let \( a_n = \int_1^n e^{-x^2} \, dx \) for \( n \geq 1 \). Show that \( \{a_n\} \) converges. Hint: \( e^{-x^2} \leq xe^{-x^2} \) for \( x \geq 1 \).

4. (a) Find \( N(k) \) such that

\[ \left| \frac{n^2 - 1}{3n^2 + 7} - \frac{1}{3} \right| < k \]

for every \( n > N(k) \). Your answer, \( N(k) \), should be a function of \( k \). For what values of \( k \) is your function defined?

(b) Find a specific value of \( k > 0 \) such that

\[ \left| \frac{n^2 - 1}{3n^2 + 7} - \frac{1}{2} \right| < k \]

is not true for any value of \( n \geq 1 \). Justify your answer.