Homework #12: Due Dec. 5, 2008

1. Assume that \( f(x) \) is differentiable on \( \mathbb{R} \). Show that if \( f(x) \) is even, then \( f'(x) \) is odd.
   
   Hint: Use the Chain Rule.

2. Assume that \( f(x) \) is differentiable and periodic on \( \mathbb{R} \). Show that \( f'(x) \) is periodic.

3. Suppose that \( f'(x) \) is bounded on \((a, b)\). Show that \( f(x) \) is also bounded on \((a, b)\).
   
   Hint: Note that the Extreme Value Theorem does not apply to \( f(x) \) on \((a, b)\). Use the Mean Value Theorem.

4. Suppose that \( f(x) \) is continuous on \([a, b]\) and \( f'(x) \) is continuous and positive on \((a, b)\). Show that \( f(x) \) is strictly increasing on \([a, b]\).
   
   Hint: We already know that \( f(x) \) is strictly increasing on \((a, b)\). It remains to show that \( f(a) < f(z) < f(b) \) for all \( z \in (a, b) \).

5. Show that \( \lim_{x \to \infty} \frac{x^t}{e^x} = 0 \) for all \( t \in \mathbb{R} \).
   
   Hint: Use L'Hospital's rule, when appropriate.