“Never confuse motion with action.”
- Benjamin Franklin

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PHYS 211
Generalize to 3D

Position, displacement, velocity and acceleration can be generalized to 3D using vectors.

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We can also generalize two of our constant acceleration equations.

\[ v(t) = v_0 + at \quad \rightarrow \quad \vec{v}(t) = \vec{v}_0 + \vec{a}t \]
\[ x(t) = x_0 + v_0t + \frac{1}{2}at^2 \quad \rightarrow \quad \vec{r}(t) = \vec{r}_0 + \vec{v}_0t + \frac{1}{2}\vec{a}t^2 \]
We can also generalize two of our constant acceleration equations.

\( \vec{v}(t) = \vec{v}_0 + \vec{a}t \)

\( \vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}t^2 \)

\( \vec{v}_x = v_{0,x} + 2a_x \Delta x \)

\( \vec{v}_y = v_{0,y} + 2a_y \Delta y \)

\( \vec{v}_z = v_{0,z} + 2a_z \Delta z \)
Lecture Question 4.1

When an object is thrown (ignoring air drag), after it has left the thrower’s hand,

(a) $v_x$ and $v_y$ are constant.

(b) $v_x$ and $v_y$ change with time.

(c) $v_x$ changes with time but $v_y$ is constant.

(d) $v_x$ is constant but $v_y$ changes with time.
Projectile motion is a very common example of 2D motion where objects move under the influence of gravity.
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This ball is also rotating — we’ll get to that later (Ch 10).
In projectile motion, the acceleration in the horizontal direction is 0 m/s².
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If we pick $+x$ as right, $a_x = 0$ m/s$^2$. 
In projectile motion, the acceleration in the vertical direction is \( g = 9.81 \text{ m/s}^2 \).
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If we pick \(+y\) as up, \( a_y = -9.8 \text{ m/s}^2 \).
Projectile Motion

In projectile motion, the horizontal and vertical motion are independent of each other.
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We use our standard equations:

\[
\begin{align*}
x(t) &= x_0 + v_{0,x} t + \frac{1}{2} a_x t^2 \\
y(t) &= y_0 + v_{0,y} t + \frac{1}{2} a_y t^2
\end{align*}
\]
Lecture Question 4.2
A bullet is aimed at a target on the wall a distance $L$ away from the firing position and the bullet strikes the wall a distance $\Delta y$ below the mark. If the distance $L$ was half as large, and the bullet had the same initial velocity, how would $\Delta y$ change?

(a) $\Delta y \rightarrow 2\Delta y$
(b) $\Delta y \rightarrow 4\Delta y$
(c) $\Delta y \rightarrow \frac{\Delta y}{2}$
(d) $\Delta y \rightarrow \frac{\Delta y}{4}$
(e) Need more information.
An object is in **uniform circular motion** when its speed is constant and it travels in a circle.
Uniform Circular Motion

An object moving in a circle experiences acceleration (even if it’s moving at constant speed!).
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The acceleration vector always points toward the center.

The velocity vector is always tangent to the path.
Uniform Circular Motion

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If the object moves faster, should the acceleration be larger or smaller?
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Uniform Circular Motion

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$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$
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\vec{v} = v_x \hat{i} + v_y \hat{j} = [-v \sin(\theta)] \hat{i} + [v \cos(\theta)] \hat{j}
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\vec{v} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j} = [-v \sin(\theta)] \hat{i} + [v \cos(\theta)] \hat{j} = \left(-\frac{vy}{r}\right) \hat{i} + \left(\frac{vx}{r}\right) \hat{j}
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\[
\vec{a} = \frac{d\vec{v}}{dt}
\]
Uniform Circular Motion

\[ \vec{v} = \frac{v}{r} ( -yi + xj ) \]
Uniform Circular Motion

\[ \vec{v} = \frac{v}{r} (-y \hat{i} + x \hat{j}) \]

\[ \vec{a} = \frac{d\vec{v}}{dt} = \frac{v}{r} (-v_y \hat{i} + v_x \hat{j}) \]
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\[ \vec{v} = \frac{v}{r} (-yi + xj) \]

\[ \vec{a} = \frac{d\vec{v}}{dt} = -\frac{v}{r} (v_y\hat{i} + v_x\hat{j}) \]

\[ a = \sqrt{a_x^2 + a_y^2} \]
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\[ a = \sqrt{a_x^2 + a_y^2} = \frac{v}{r} \sqrt{v_y^2 + v_x^2} \]
Uniform Circular Motion

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\[ a = \frac{v^2}{r} \text{ (uniform circular motion)} \]
Lecture Question 4.3

A steel ball is whirled on the end of a chain in a horizontal circle of radius $R$ with a constant period $T$. If the radius of the circle is then reduced to $0.75R$, while the period remains $T$, what happens to the centripetal acceleration of the ball?

(a) Centripetal acceleration increases.
(b) Centripetal acceleration decrease.
(c) Centripetal acceleration stays the same.
(d) Not enough information.
Relative Motion

The velocity of an object depends on the reference frame from which it is measured.
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- frame A (Alice) is stationary
- frame B (Bob) moves with some constant velocity
- object P (Parakeet) is measured
Relative Motion

- $x_{BA}$: position of Bob relative to Alice
- $x_{PB}$: position of Parakeet relative to Bob
- $x_{PA}$: position of Parakeet relative to Alice
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\[ x_{PA} = x_{PB} + x_{BA} \]
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v_{PA} = v_{PB} + v_{BA}
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x_{PA} &= x_{PB} + x_{BA} \\
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- \( \vec{r}_{BA} \): position of Bob relative to Alice
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