Curve Sketching

1. Sketch the graph of a function continuous for all real x, and having all of the following characteristics. Label and identify all relative extrema, and identify all points of inflection.

\[ f(0) = f(2) = f(4) = 0 \]
\[ f'(1) = f'(3) = 0 \]
\[ f'(x) > 0 \quad \text{for} \quad x < 1 \]
\[ f'(x) < 0 \quad \text{for} \quad 1 < x < 3 \]
\[ f'(x) > 0 \quad \text{for} \quad x > 3 \]
\[ f''(2) = 0 \]
\[ f''(x) < 0 \quad \text{for} \quad x < 2 \]
\[ f''(x) > 0 \quad \text{for} \quad x > 2 \]
2. Sketch the graph of a function continuous for all real \( x \), and having all of the following characteristics. Label and identify all relative extrema, and identify all points of inflection.

\[
\begin{align*}
  f(0) &= f(2) = 0 \\
  f(1) &= -2 \\
  f'(x) &< 0 \quad \text{for } x < 1 \\
  f'(1) &\text{ is undefined.} \\
  f'(x) &> 0 \quad \text{for } x > 1 \\
  f''(x) &< 0 \quad \text{for } x \neq 1 \\
  f''(1) &\text{ is undefined.}
\end{align*}
\]