Math 141
Exam 3
April 5, 2013

1. Use the series \( \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+1} \right) \), to find the following.

The type of series: ______________________

\[ S_4 = \]

\[ S_n = \]

\[ S, \] if convergent.

For problems, #2 - #3,

- **Name** the test utilized.
- **Test** the series for convergence or divergence.
- **Support** your conclusion.
- **If** convergent, find the sum, whenever possible.

<table>
<thead>
<tr>
<th>2. [ \sum_{n=1}^{\infty} \frac{n^2}{n^2 + 1} ]</th>
<th>3. [ \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{5^{n-1}} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Test Name:</strong></td>
<td><strong>Test Name:</strong></td>
</tr>
<tr>
<td><strong>Apply Test:</strong></td>
<td><strong>Apply Test:</strong></td>
</tr>
<tr>
<td><strong>Support/Reason:</strong></td>
<td><strong>Support/Reason:</strong></td>
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<td><strong>Converge/Diverge:</strong></td>
<td><strong>Converge/Diverge:</strong></td>
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<tr>
<td><strong>Sum, if applicable:</strong></td>
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</tr>
</tbody>
</table>
4. Find the center, radius, and the (open) interval of convergence for the power series. **DO NOT** test endpoints.

\[ f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{3^n \cdot n} \]

Center: \( C = \) \[ \]

Radius: \( R = \) \[ \]

(open) Interval of Convergence Radius: \( ( \quad , \quad ) \)

5. Find the center, radius, and the (open) interval of convergence for the power series. **DO NOT** test endpoints.

\[ f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{(n+1)!} \]

Center: \( C = \) \[ \]

Radius: \( R = \) \[ \]

(open) Interval of Convergence Radius: \( ( \quad , \quad ) \)
6. Use the given power series and the fact that the radius of convergence is \( R = 5 \), to find the interval of convergence, including testing endpoints, as necessary. That is, find the “final” interval of convergence.

\[
f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x - 2)^n}{5^n \cdot n^2}
\]

7. For the power series,

\[
f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x - 1)^n}{n!}
\]

find and SIMPLIFY THE SERIES NOTATION, if necessary, for each of the following: (Note: you do not need find the interval(s) of convergence.)

\[
f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x - 1)^n}{n!} = \frac{1}{0!} - \frac{(x - 1)}{1!} + \frac{(x - 1)^2}{2!} - \frac{(x - 1)^3}{3!} + ...
\]

a. \( f'(x) = \)

b. \( \int f(x) \, dx = \)
8. Suppose the interval of convergence for a particular power series $f(x)$ is $[-2, 1]$.

Answer the following, TRUE OR FALSE.

a. It is possible that the interval of convergence of $f'(x)$ is $(-2, 1)$. ____________.

b. It is possible that the interval of convergence of $f'(x)$ is $[-2, 1]$. ____________.

c. It is possible that the interval of convergence of $\int f(x) \, dx$ is $(-2, 1)$. ____________.

d. It is possible that the interval of convergence of $\int f(x) \, dx$ is $[-2, 1]$. ____________.

9. (4 points) If you hit “shuffle” on your iPod, what song do wish plays first? ____________________

Bonus: One of your so-called friends has just posted an unflattering photo of you in a compromising position on FB. Your famous photo has received 1000 “Likes” in the first hour it was posted. Each hour thereafter, it receives 75% of the number of likes it received in the previous hour. Write the series that gives the total number of likes your photo receives. Then determine the total number of likes it receives if the photo remains posted forever, if finite. If not finite, explain.

\[
\sum_{n=1}^{\infty} \text{Total number of likes, if finite: } ______________
\]