1. Use the series, \( \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right) \), to find the following.

The type of series: telescoping

\[
S_4 = \left( 1 - \frac{1}{3} \right) + \left( \frac{1}{2} - \frac{1}{4} \right) + \left( \frac{1}{3} - \frac{1}{5} \right) + \left( \frac{1}{4} - \frac{1}{6} \right) = 1 + \frac{1}{2} - \frac{1}{5} - \frac{1}{6}
\]

\[
S_n = \frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}
\]

\[
S_n \text{ converges.}
\]

For problems #2 - #3,

- Name the test utilized.
- Test the series for convergence or divergence.
- Support your conclusion.
- If convergent, find the sum, whenever possible.

2. \[ \sum_{n=1}^{\infty} \frac{n}{2n+1} \]

Test Name: \( n \text{-th term test} \)

Apply Test:

\[
\lim_{n \to \infty} \frac{n}{2n+1} = \frac{1}{2} \neq 0
\]

Support/Reason:

Converge/Diverge: diverges

Sum, if applicable:

3. \[ \sum_{n=0}^{\infty} \frac{(-1)^n 3^n}{5^{n+1}} \]

Test Name: geometric

Apply Test:

\[
\gamma = -\frac{3}{5}
\]

\[ \left| -\frac{3}{5} \right| < 1 \text{ converges} \]

Support/Reason:

Converge/Diverge: converges

Sum, if applicable:

\[
S = \frac{\frac{1}{5}}{1 - (-\frac{3}{5})} = \frac{1}{5} \times \frac{8}{5} = \frac{8}{25}
\]
4. Find the **center**, radius, and the (open) **interval of convergence** for the power series. You need **NOT** test endpoints.

\[ f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{5^n \cdot n^2} \]

**Center:** \(C = 1\)

\[ u_n = \frac{(x-1)^n}{5^{n+1}(n+1)^2} \]

\[ u_{n+1} = \frac{(x-1)^{n+1}}{5^{n+2}(n+2)^2} \]

\[ \lim_{n \to \infty} \left| \frac{(x-1)^{n+1}}{5^{n+1}(n+1)^2} \cdot \frac{5^n n^2}{(x-1)^n} \right| = \left| \frac{x-1}{5} \right| \lim_{n \to \infty} \left| \frac{n^2}{(n+1)^2} \right| = \left| \frac{x-1}{5} \right| < 1 \]

For conv. \( \left| \frac{x-1}{5} \right| < 1 \)

So \( |x-1| < 5 \)

Radius: \( R = 5 \)

(Open) Interval of Convergence Radius: \((-4, 6)\)

5. Find the **center**, radius, and the (open) **interval of convergence** for the power series. You need **NOT** test endpoints.

\[ f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^n}{(n-1)!} \]

**Center:** \(C = 0\)

\[ u_n = \frac{x^n}{(n-1)!} \]

\[ u_{n+1} = \frac{x^{n+1}}{n!} \]

\[ \lim_{n \to \infty} \left| \frac{x^{n+1}}{n!} \cdot \frac{(n-1)!}{x^n} \right| = |x| \lim_{n \to \infty} \left| \frac{1}{n} \right| = |x| < 1 \]

For all \( x \)

Radius: \( R = \infty \)

(Open) Interval of Convergence Radius: \((-\infty, \infty)\)
6. Use the given power series and the fact that the radius of convergence is \( R = 2 \), to find the interval of convergence, including testing endpoints, as necessary. That is find the “final” interval of convergence.

\[
 f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n(x-1)^n}{2^n \cdot n}
\]

For the power series, find \((-1, 3)\) → “find” interval \([-1, 3]\)

\[ x = -1 \]
\[
 f(-1) = \sum_{n=1}^{\infty} \frac{(-1)^n(-2)^n}{2^n \cdot n} = \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{div} \ p-series \ p = 1 \leq 1
\]

\[ x = 3 \]
\[
 f(3) = \sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^n}{2^n \cdot n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \quad \text{conv. AST} \quad \lim_{n \to \infty} \frac{1}{n} = 0
\]

7. For the power series, \( f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n(x-2)^{n+1}}{(n+1)!} \), find and SIMPLIFY THE SERIES NOTATION, if necessary, for each of the following: (Note: you do not need find the interval(s) of convergence.)

\[
 f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n(x-2)^{n+1}}{(n+1)!} = \frac{(x-2)}{1!} - \frac{(x-2)^2}{2!} + \frac{(x-2)^3}{3!} - \ldots
\]

\[ f'(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{n+1} \]

\[ f''(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+2}}{(n+2) \cdot n!} \]

---

b. \( \int f(x) \, dx = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+2}}{(n+2)!} \)
8. Suppose the interval of convergence for a particular power series \( f(x) \) is \((-1,3]\).

Answer the following. TRUE OR FALSE.

a. It is possible that the interval of convergence of \( f'(x) \) is \((-1,3)\). TRUE

b. It is possible that the interval of convergence of \( f'(x) \) is \([-1,3]\). TRUE

c. It is possible that the interval of convergence of \( \int f(x) \, dx \) is \((-1,3)\). FALSE

d. It is possible that the interval of convergence of \( \int f(x) \, dx \) is \([-1,3]\). TRUE

9. (4 points) If you hit "shuffle" on your iPod, what song do wish plays first? Answer varies.

BONUS: SpongeBob and Patrick are jellyfishing. At noon, they have netted 36 jellyfish in Jellyfish Fields. Each hour after noon they net 50% of number of jellyfish as during the previous hour. As time passes on and on (infinitely), do they net a finite number of jellyfish or an infinite number of jellyfish? Support your answer by writing a series, and determining if the series converges or diverges, and the results of your testing. If the number is finite, find the total number of jellyfish they net.

\[
\sum_{n=0}^{\infty} 36 \left( \frac{1}{2} \right)^n
\]

Common geometric series

\[
r = \frac{1}{2} < \frac{1}{2} < 1
\]

\[
S = \frac{36}{1 - \frac{1}{2}} = 72 \text{ jellyfish}
\]