1. Use the series, \( \sum_{n=1}^{\infty} \left( \frac{1}{n} - \frac{1}{n+2} \right) \), to find the following.

The type of series: __________________________

\( S_4 = \) _____

\( S_n = \) _____

\( S \), if convergent.

For problems, #2 - #3,

- Name the test utilized.
- Test the series for convergence or divergence.
- Support your conclusion.
- If convergent, find the sum, whenever possible.

2. \( \sum_{n=1}^{\infty} \frac{n}{2n+1} \)

Test Name: __________________________

Apply Test: __________________________

Support/Reason: __________________________

Converge/Diverge: __________________________

Sum, if applicable: __________________________

3. \( \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{5^{n+1}} \)

Test Name: __________________________

Apply Test: __________________________

Support/Reason: __________________________

Converge/Diverge: __________________________

Sum, if applicable: __________________________
4. Find the **center**, **radius**, and the (open) **interval of convergence** for the power series. You need **NOT** test endpoints.

\[ f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{5^n \cdot n^2} \]

Center: \( C = \) 

Radius: \( R = \) 

(open) Interval of Convergence Radius: \( ( , ) \)

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5. Find the **center**, **radius**, and the (open) **interval of convergence** for the power series. You need **NOT** test endpoints.

\[ f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{(n-1)!} \]

Center: \( C = \) 

Radius: \( R = \) 

(open) Interval of Convergence Radius: \( ( , ) \)
6. Use the given power series and the fact that the radius of convergence is $R = 2$, to find the interval of convergence, including testing endpoints, as necessary. That is find the “final” interval of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n (x-1)^n}{2^n \cdot n}$$

7. For the power series, $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)!}$, find and SIMPLIFY THE SERIES NOTATION, if necessary, for each of the following: (Note: you do not need find the interval(s) of convergence.)

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{n+1}}{(n+1)!} = \frac{(x-2)}{1!} - \frac{(x-2)^2}{2!} + \frac{(x-2)^3}{3!} - ...$$

a. $f'(x) = \quad$ 

b. $\int f(x) \, dx = \quad$
8. Suppose the interval of convergence for a particular power series \( f(x) \) is \((-1,3]\).

Answer the following, **TRUE OR FALSE**.

a. It is possible that the interval of convergence of \( f'(x) \) is \((-1,3)\)._______________.

b. It is possible that the interval of convergence of \( f'(x) \) is \((-1,3]\)._______________.

c. It is possible that the interval of convergence of \( \int f(x) \, dx \) is \((-1,3)\)._______________.

d. It is possible that the interval of convergence of \( \int f(x) \, dx \) is \([-1,3]\)._______________.

9. (4 points) If you hit “shuffle” on your **Ipod**, what song do wish plays first? ________________

**BONUS:** **SpongeBob and Patrick** are jellyfishing. At noon, they have netted 36 jellyfish in **Jellyfish Fields**. Each hour after noon they net 50% of number of jellyfish as during the previous hour. As time passes on and on (infinitely), do they net a finite number of jellyfish or an infinite number of jellyfish? Support your answer by **writing a series**, and **determining if the series converges or diverges**, and **the results** of your testing. If the number is finite, **find the total number of jellyfish** they net.

[Image of SpongeBob and Patrick]