Math 140
Quiz 4
Due Monday February 18 in class.

1. A child with flu like symptoms has developed a fever and his temperature is rising such that \( T(t) = 98.6 + \frac{4t}{t^2 + 1} \) where \( T \) is the child’s temperature in °F and \( t \) is time in hours since the fever began.

a. Find the rate at which the fever is changing over the first three hours.

\[
\frac{\Delta T}{\Delta t} = \frac{T(3) - T(0)}{3 - 0} = \frac{99.8 - 98.6}{3} = \frac{1.2}{3} = 0.4 \text{ °F/hr}
\]

b. Find the rate at which the child’s temperature is changing three hours after the fever began.

\[
T'(t) = 0 + \frac{(t^2 + 1)(4) - 4t(2t)}{(t^2 + 1)^2} = \frac{4t^2 + 4 - 8t^2}{(t^2 + 1)^2}
\]

\[
T'(3) = -\frac{4(3^2 - 1)}{(3^2 + 1)^2} = -\frac{32}{121} \text{ °F/hr}
\]

c. Using methods of calculus, find the child’s maximum temperature during the illness.

When \( T'(t) = 0 \)

\[-4(t^2 - 1) = 0 \]

\[t^2 - 1 = 0\]

\[t = \pm 1\]

\[t = 1 \text{ hour after fever began}\]

Max: \( T(1) = 98.6 + \frac{4}{1 + 1} = 100.6 \text{ °F} \)
2. The temperature in degrees Fahrenheit in Erie on a warm late summer day can be modeled by
\[ T(t) = -0.05t^2 + 1.6t + 60 \] where \( t \) is the time in hours \( t = 0 \) corresponds to midnight.

\[
\frac{\Delta T}{\Delta t} \quad \text{for} \quad 0 \leq t \leq 18 \quad \text{and explain its meaning in the context of the situation.}
\]

\[
\frac{T(18) - T(0)}{18 - 0} = \frac{72.6 - 60}{18} = \frac{12.6}{18} = 0.7^\circ F/hr
\]

From midnight \((t = 0)\) to 6 PM \((t = 18)\), the temperature increased at an average rate of \(0.7^\circ F/hr\).

b. Find \( T'(t) \) at \( t = 18 \) and explain its meaning in the context of the situation.

\[
T'(t) = -0.1t + 1.6
\]

\[
T'(18) = -0.1(18) + 1.6 = -0.2^\circ F/hr
\]

At 6 PM \((t = 18)\), the temperature was decreasing at a \( \text{(instantaneous)} \) rate of \(0.2^\circ F/hr\).