Lecture 1

Axial Fans: Aerodynamic Design

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1.1 Axial Cascade Blade Element (BE)

Rotor diameter: \( D_r = 2r_s \)
Hub diameter: \( D_i = 2r_i \)
Radial extension of BE: \( \delta \)
Number of blades: \( z \)

Chord length: \( l \)
Circumferential spacing: \( t \)
Blade angle at leading edge (LE): \( \beta_1 \)
Blade angle at leading edge (LE): \( \beta_2 \)

1.2 Actual vs. Ideal Flow

Actual:
- 3D
- unsteady

Ideal:
- 1D
- steady
1.3 Frames of References

- $\mathbf{\dot{u}}$ = velocity of rotating system (relative system)
- $\mathbf{\dot{w}}$ = velocity of a particle in relative system

$\Rightarrow$ Observer in the absolute frame of reference sees absolute particle velocity

$$\mathbf{\dot{c}} = \mathbf{\dot{u}} + \mathbf{\dot{w}} \quad (1-1)$$

Velocity triangle

1.4 Co-ordinate System and Components of Velocity

Velocity $\mathbf{\dot{c}}$ and relevant components:

- in $z$-direction $\Rightarrow$ axial component $\mathbf{\dot{c}}_z$
- in $\varphi$-direction $\Rightarrow$ circumferential (= tangential) component $\mathbf{\dot{c}}_\varphi$
- in $r$-direction $\Rightarrow$ radial component $\mathbf{\dot{c}}_r$

$$\mathbf{\dot{c}} = \mathbf{\dot{c}}_z + \mathbf{\dot{c}}_\varphi + \mathbf{\dot{c}}_r \quad (1-2)$$

In addition useful: Meridional component: $\mathbf{\dot{c}}_m = \mathbf{\dot{c}}_z + \mathbf{\dot{c}}_r \quad (1-3)$
1.5 Velocity triangles on BE (turbine)

\[ C_{u} = C_{inlet} \]

1.6 EULER's Equation of Turbomachinery (Angular Momentum Analysis)

control volume

\[ M_{shaft} \]

\[ \delta F_u \]

\[ \delta r \]
Apply to angular momentum conservation to CV „blade element“ (BE)

\[
\int \left( \vec{r} \cdot \vec{v} \right) \rho \, dt + \int \rho \left( \vec{r} \times \vec{c} \right) \cdot \vec{n} \, dA = \sum M_{\text{shaft}} + \oint \sum \delta \tau \cdot \mathbf{n} \cdot d\mathbf{S}
\]

(1-4)

With incremental mass flow through BE

\[
\delta m = \rho c_{\omega} \delta A = \rho c_{\omega} 2\pi \delta r \quad (1-5)
\]

one gets the incremental torque at shaft

\[
\delta m (r, c_{\omega}) = \delta M_{\text{shaft}}
\]

change of angular momentum from entrance to exit of blade channel

Note:

Incremental tangential force is

\[
\delta F_{\tau} = -c_{\omega} \delta m \quad (1-6)
\]

With the incremental power

\[
\delta P_{\text{BE}} = \delta M_{\text{shaft}} \Omega \quad (1-7)
\]

and

\[
u = r \Omega \quad (1-8)
\]

one eventually obtains the specific work

\[
Y = \frac{\delta P_{\text{BE}}}{\delta m} = u_{c_{2}} \nu \quad (1-9a)
\]

or, if \(c_{\omega} = 0\)

\[
Y = u_{c_{2}} - u_{c_{1}} \quad (1-9b)
\]

EULER equation of turbomachinery (1754)

LEONHARD EULER
1707 - 1783

SEGNER’s waterwheel analyzed by L. EULER
1.7 Blade Element Design

1st way to get circumferential force on BE

Recall circumferential force on BE from angular momentum analysis

\[ \delta F_c = -c_w \delta m \]  

(1-6)

Now, CV enclosing only one blade and thus

\[ \delta m = \rho c_m \delta A = \rho c_m t \delta r \]

yields

\[ \delta F_c = -c_w \rho c_m t \delta r \]  

(1-10)

2nd way to get circumferential force on BE

Flow induced forces on one blade

- lift \[ \delta L = c_l \rho \frac{w^2}{2} l \delta r \]
- drag \[ \delta D = c_d \rho \frac{w^2}{2} l \delta r \]

For design conditions usually \( D \ll L \), i.e. the drag-lift ratio is

\[ \frac{\delta D}{\delta L} = \sin \varepsilon = \varepsilon \]  

(1-11)

and

\[ \delta F = \delta L \]

\[ \delta F_c = \sin(\beta_c - \varepsilon) \delta F \]

\[ \Rightarrow \delta F_c = \sin(\beta_c - \varepsilon) c_w \rho \frac{w^2}{2} l \delta r \]  

(1-12)
\[ w_\infty, \beta_\infty \]

For cascades with small flow deflection, i.e., for blades with small camber, replace up- and downstream velocities by their vector-mean:

\[ \hat{w}_\infty = \frac{1}{2}(\hat{w}_1 + \hat{w}_2) \quad (1-13) \]

Hence, from velocity triangles we get

\[ w_\infty \]

and the vector-mean flow angle \[ \beta_\infty \]

Combining Eqs. (1-10) and (1-12, -13) yields

\[ c_t = \frac{2\sigma r}{\epsilon} \]

Inserting in this equation

- the blade spacing

\[ t = \frac{2\sigma r}{\epsilon} \quad (1-14) \]

- the solidity

\[ \sigma = \frac{12 \epsilon r}{Z \sigma r} \quad (1-15) \]

\[ \sigma = f(\text{velocity triangles}, c_{1s}, c_{2s}) \quad (1-16) \]
1.8 Performance Analysis via Computational Fluid Dynamics (CFD)

- Computational domain and numerical grid

![Streamlines and Pressure Distribution](image)

- Typical CFD results (I)
• Typical CFD results (II)

Spanwise blade loading (pressure distribution)

• Typical CFD results (III)

Performance Curves
1.9 Prototyping and Scaling Up

CNC-Milling Machine

Prototyping

Experimental model test

Full scale
Short presentation of the University of Siegen Fan Design Code ....