Problem 1. Consider the following game tree:

1.a) Come up with a “real life” story for it. That is, describe with words a strategic environment that could be modelled by it. Your description should specify payoffs. Feel free to rename the players and actions.

1.b) Write down a payoff matrix for a strategic form game representing the environment you described in (1.a). How many strategies does each player have?

Problem 2. Identify which of the following are valid trees with valid information structures. For those that are not valid you must specify why. For those that are valid please specify how many strategies each player has.

2.a)

2.b)
Problem 3. Consider the following strategic environment involving the owner of a firm, a manager and a worker. The owner first decides whether to hire the worker, to refuse hiring the worker or to delegate the hiring decision to the manager. If the owner lets the manager decide, then the manager must choose whether to hire the worker or not. If the worker is hired then he/she chooses whether to work diligently or to shirk. Suppose that the worker does not know whether the hiring decision was made by the manager or by the owner. If the worker is not hired then all three players get a payoff of 0. If the worker is hired and shirks then both the manager and the owner get a payoff of $-1$ and the worker gets a payoff of 1. If the worker is hired by the owner and works diligently, then the owner gets a payoff of 3 and both the worker and the manager get a payoff of 0. If the worker is hired by the manager and works diligently, then the owner gets 0, the manager gets 1 and the worker gets 2.

3.a) Write down an extensive form game that represents this situation

3.b) Write down a strategic form game that represents this situation

3.c) Which strategies are rationalizable?

3.d) [Bonus] Find all the Nash equilibria in pure strategies. Hint: There are three Nash equilibria in pure strategies and all of them yield the same payoffs

Problem 4. Anna and Bob are working as partners. During a given year they will both provide effort and the firm will generate revenues that depend on the levels of effort provided. They can provide any level of effort in $[0, 5]$. Let $A$ denote the level of effort provided by Anna and $B$ the level of effort provided by Bob. Providing effort is costly, the cost for Anna is $-\frac{1}{2}A^2$ and the cost for Bob is $-\frac{1}{2}B^2$. The total revenue of the firm is $2(A + B + \frac{1}{2}AB)$. Bob and Anna split the revenue halfway so that the total revenue that each one for them receives is $(A + B + \frac{1}{2}AB)$. Notice that we have a simultaneous move game with players \{Anna, Bob\}, strategy sets $S_{Anna} = S_{Bob} = [0, 5]$ and payoff functions $u_{Anna}(A, B) = A + B + \frac{1}{2}AB - \frac{1}{2}A^2$ and $u_{Bob}(A, B) = A + B + \frac{1}{2}AB - \frac{1}{2}B^2$.

4.a) Find an analytic solution for the best response functions and graph them in a clearly labelled figure. Hint: Notice that the problem is very similar to the Cournot competition example covered in class.

4.b) Can Anna rationalize choosing $A = 4$? How about $A = 2.5$ or $A = 1.5$? Justify your answer in detail.
Problem 5. Find the set of rationalizable strategies for the following game:

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