Math 231, Fall 2008
Review sheet for the final Exam

Our final exam will cover Chapters 13, 14, and 15. This review sheet deals only with Chapter 15 since the midterm review sheet covered the main topics from Chapters 13 and 15.

Some important skills

15.1. Functions of several variables.

• Finding the domain and the range of a function of two or more variables.
• Sketch the domain of a function of two variables.
• Finding the level curves of a function of two variables.

15.2. Limits and continuity.

• Finding the limit of a function at a point, provided that it exists or show that the function has no limit at the given point.
• Finding the set of points where a given function is continuous.

15.3. Partial derivatives.

• Finding the first partial derivatives of a function of two or more variables.
• Finding all second partial derivatives of a function of two or three variables.
• Apply Clairaut’s Theorem, if it is applicable.
• Finding the indicated (higher) partial derivatives.

15.4. Tangent planes and linear approximations.

• Finding an equation of the tangent plane to a surface at a given point.
• Finding an equation of the normal line to a surface at a given point.
• Finding the linearization $L$ of a function $f$ at a given point $P_0 = (x_0, y_0, z_0)$.
• Use the linearization function at $P_0$ to estimate the value of the function at some point $P$.
• Finding the total differential of a function.
15.5. The Chain Rule.

- Chain Rule for function of two variables Case 1
- Chain Rule for function of two variables Case 2
- Applying the Chain Rule for a function of two or three variables.

15.6. Directional Derivatives and the Gradient Vector.

- How to find the directional derivative at a point if the direction is given by a non-unit vector?
- What is the maximum rate of change of a function of two or more variables at a given point?
- Finding the directions in which the directional derivative has a given value.
- Finding the tangent plane and the normal line to a given level surface \( F(x, y, z) = k \) at a given point.

15.7. Maximum and Minimum Values.

- How to find the critical points of a function of two or three variables?
- How to find the extreme values (local minima and maxima) and the saddle points of a function of two or three variables?
- Apply the first derivative Test for finding critical points
- Classify a critical point using the Second derivative test.
- How to find the absolute minimum and maximum values of \( f(x, y) \) on a given closed and bounded set?

15.8. Lagrange Multipliers.

- Applying the Method of Lagrange Multipliers for functions of two or three variables subject to one constraint.
- Finding the absolute minimum or maximum values of a function \( f \) of two or three variables on the boundary of a given region \( D \) in 3-space using the Method of Lagrange Multipliers.
Other important facts:

1. The directional derivative $D_u f$ of a function $f$ of three variables at a point $P = (x, y, z)$ in the direction of a given unit vector $u$ can be written as:

   $$D_u f = \nabla f \cdot u \quad \text{or} \quad D_u f = |\nabla f| \cos \theta,$$

   where $\theta$ is the angle between $\nabla f$ and $u$.

2. The gradient vector $\nabla f$ at $P$ gives the direction of most rapidly increase of $f$ at that point.

3. The gradient vector $\nabla f$ at $P$ is normal to the level surface of $f$ passing through $P$. 