There are 20 multiple choice questions. Each problem is worth 5 points. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. **Mark only one choice; darken the circle completely** (you should not be able to see the letter after you have darkened the circle).

**THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.**

**PLEASE SHOW YOUR PSU ID CARD TO THE PROCTOR WHEN YOU FINISH.**

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 20 PROBLEMS ON 11 PAGES (INCLUDING THIS ONE).
1. Find the solution to the following linear system:

\[
\begin{align*}
  x_1 & - 2x_2 + 4x_3 = 1 \\
  3x_1 & - 8x_2 + 10x_3 = 7 \\
  2x_1 & - 3x_2 + 9x_3 = 0
\end{align*}
\]

a) \( x_1 = -3 \), \( x_2 \) is free, \( x_3 = -2 + x_2 \).

b) \( x_1 = -3 - 6x_3 \), \( x_2 = -2 - x_3 \), \( x_3 \) is free.

c) \( x_1 = -9 \), \( x_2 = -3 \), \( x_3 = -1 \).

d) There is no solution.

2. Which of the following statements is \textbf{always} true?

a) A consistent linear system has only one solution.

b) If a homogeneous linear system has at least one free variable, then the system has many solutions.

c) The augmented matrix of a linear system has a unique echelon form.

d) If an echelon form of the augmented matrix of a linear system has a zero row, then the system is consistent.
3. Let \[
\begin{bmatrix}
1 & -1 & 2 & 2 \\
1 & -3 & 1 & k \\
3 & 1 & -2h & 5
\end{bmatrix}
\] be the augmented matrix of a linear system. For which value(s) of \( h, k \) is the system inconsistent?

a) \( h = -4, k = \frac{5}{2} \).

b) \( h \neq -4, k = \frac{5}{2} \).

c) \( h = -4, k \neq \frac{5}{2} \).

d) \( h \neq -4, k \neq \frac{5}{2} \).

4. Which of the following matrices is in reduced echelon form?

a) \[
\begin{bmatrix}
1 & 2 & 0 & 1 \\
0 & 1 & 0 & 3 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]

b) \[
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

c) \[
\begin{bmatrix}
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

d) \[
\begin{bmatrix}
1 & 0 & 1 & 0 \\
0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]
5. Let \( \mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \), \( \mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \). Which of the following vectors is in the span of \( \{\mathbf{u}, \mathbf{v}\} \)?

a) \( \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix} \)

b) \( \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} \)

c) \( \begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix} \)

d) \( \begin{bmatrix} -5 \\ -1 \\ 1 \end{bmatrix} \)

6. Let \( \begin{bmatrix} 1 & -2 & 0 & b_1 \\ 0 & 4 & 0 & b_1 + b_2 \\ 0 & 0 & b_2 & b_1 + 4b_2 + 2 \end{bmatrix} \) be the augmented matrix of a linear system, then which of the following conditions implies that the linear system is consistent?

a) \( b_1 = 1, b_2 = 0. \)

b) \( b_1 \neq -2 - 4b_2, b_2 = 0. \)

c) \( b_1 = -2 - 4b_2, b_2 \neq 0. \)

d) The system is always consistent.
7. Let \( A = \begin{bmatrix} 2 & -4 & 4 \\ 0 & 2 & 1 \\ -1 & 8 & 1 \end{bmatrix} \), then which of the following is the solution of \( Ax = 0 \) in parametric vector form?

a) \( \begin{bmatrix} 6 \\ 1 \\ -2 \end{bmatrix} \)

b) \( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \)

c) \( x_3 \begin{bmatrix} -3 \\ -0.5 \\ 1 \end{bmatrix}, \ x_3 \in \mathbb{R} \)

d) \( x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \ x_2, x_3 \in \mathbb{R} \)

8. Let \( A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix} \). Suppose that for some \( \mathbf{b} \) in \( \mathbb{R}^2 \), \( \mathbf{p} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \) is one particular solution to the nonhomogeneous equation \( Ax = \mathbf{b} \). What is the general form of the solution to this equation (\( Ax = \mathbf{b} \))?

a) \( \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \)

b) \( \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \ x_3 \in \mathbb{R}. \)

c) \( x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \ x_3 \in \mathbb{R}. \)

d) \( \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \ x_3 \in \mathbb{R}. \)
9. Which of the following statements is always true?

a) If every row of $A$ has a pivot position, then $Ax = 0$ has only the trivial solution.

b) If $Ax = 0$ has nontrivial solutions, then $Ax = b$ is consistent for any $b$.

c) If the solution to $Ax = b$ is not unique, then $Ax = 0$ has nontrivial solutions.

d) If the solution to $Ax = b$ is unique, then $Ax = 0$ has no solution.

10. Which of the following sets of vectors is linearly independent?

a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\}$.

b) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$.

c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

d) $\left\{ \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.
11. Let \( A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -4 \\ -1 & 0 & 2 \end{bmatrix} \). Which of the following is a nontrivial solution to \( Ax = 0 \)?

a) \( \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \).

b) \( \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \).

c) \( \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \).

d) \( \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \).

12. Let \( A = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \). What is the geometric interpretation of the transformation \( T(x) = Ax \)? (Note that \( \cos(\pi/4) = \frac{\sqrt{2}}{2} \).)

a) A rotation by \( \frac{\pi}{4} \) about the origin in the counterclockwise direction.

b) A shearing in the \( x_1 \) direction by a factor of \( \frac{\sqrt{2}}{2} \).

c) A contraction by the factor of \( \frac{\sqrt{2}}{2} \).

d) A reflection in the \( x_1 \)-axis combined with a contraction by \( \frac{\sqrt{2}}{2} \).
13. Let $T$ be a linear transformation, and let $u, v$ be two vectors satisfying $T(u) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $T(v) = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$. Then $T(3u - v)$ is:

a) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$.

b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

c) $\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$.

d) There is not enough information to compute $T(3u - v)$.

14. Find the standard matrix of $T$ where $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_3 \\ 2x_1 + x_3 \\ x_1 - 5x_2 \end{bmatrix}$.

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 5 \end{bmatrix}$.

b) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & -5 \end{bmatrix}$.

c) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 1 & -5 & 0 \end{bmatrix}$.

d) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -5 \\ 1 & 1 & 0 \end{bmatrix}$. 
15. Consider \( T(x) = Ax \), where \( A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{bmatrix} \). Which of the following statements is true?

a) \( T \) is one-to-one and onto.

b) \( T \) is onto but not one-to-one.

c) \( T \) is one-to-one but not onto.

d) \( T \) is neither one-to-one nor onto.

16. Let \( T \) be the transformation defined by \( T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \), then which of the following vectors is in the range of \( T \)?

a) \( \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \).

b) \( \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \).

c) \( \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \).

d) \( \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \).
17. Which of the following statements regarding an $n \times n$ matrix $A$ is not equivalent to the other three?

(i) $A$ has linearly dependent columns.
(ii) The linear transformation $T(x) = Ax$ is not onto.
(iii) $A$ is not invertible.
(iv) $A$ is row equivalent to the identity matrix $I_n$.

a) (i)
b) (ii)
c) (iii)
d) (iv)

18. Let $A$, $B$, and $C$ be all $n \times n$ matrices. Which of the following is equal to $(AB)^T C^T$?

a) $C^T A^T B^T$
b) $C^T A B$
c) $A^T B^T C^T$
d) $C^T B A$
19. Find the inverse of the matrix \( A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \).

   a) \( \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} \)

   b) \( \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix} \)

   c) \( \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \)

   d) \( \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \)

20. Suppose \( B \) is a \( 3 \times 3 \) invertible matrix and that \( 2A = 5B \), then what is \( A^{-1} \)?

   a) \( \frac{2}{5} B^{-1} \)

   b) \( \frac{5}{2} B^{-1} \)

   c) \( 10B^{-1} \)

   d) None of above, since \( A \) is not necessarily invertible.
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