SARIMA Models
1 SARIMA
Seasonal ARMA($P, Q$) is used when seasonal (hence nonstationary) behavior is present in the time series. We use the model

$$\Phi_P(B^s)Z_t = \Theta_Q(B^s)a_t$$

where $s = 12$ if data is in months and $s = 4$ if data is in quarters, etc. Seasonal differencing may be in order if the seasonal component follows a random walk, as in

$$Z_t = Z_{t-12} + a_t$$

The seasonal difference of order $D$ is defined as

$$\nabla_s^D Z_t = (1 - B^s)^D Z_t$$
The seasonal autoregressive integrated moving average model of Box and Jenkins (1970) is given by

$$\Phi_P(B^s)\phi(B)\nabla^D_s\nabla^d Z_t = \alpha + \Theta_Q(B^s)\theta(B)a_t$$

and is denoted as an ARIMA$(p, d, q) \times (P, D, Q)_s$. 
Federal Reserve Board Production Index

```r
cpyod = ts(scan("mydata/prod.dat"), start=1948, frequency=12)
ts.plot(prod)

par(mfrow=c(2,1))
acf(prod, 48)
pacf(prod, 48)
```
Federal Reserve Board Production Index

par(mfrow=c(2,1)) \# (P)ACF of d1 data
acf(diff(prod), 48)
pacf(diff(prod), 48)

par(mfrow=c(2,1)) \# (P)ACF of d1-d12 data
acf(diff(diff(prod),12), 48)
pacf(diff(diff(prod),12), 48)
> prod.fit3 = arima(prod, order=c(1,1,1),
+ seasonal=list(order=c(2,1,1), period=12))
> prod.fit3 # to view the results

Call:
arima(x = prod, order = c(1, 1, 1), seasonal = list(order = c(2, 1, 1), period = 12))

Coefficients:
             ar1     ma1     sar1     sar2     sma1
       0.5753  -0.2709  -0.2153  -0.2800  -0.4968
sigma^2 estimated as 1.351:  log likelihood = -568.22,  aic = 1148.43

> tsdiag(prod.fit3, gof.lag=48) # diagnostics
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```r
> prod.pr = predict(prod.fit3, n.ahead=12)
> U = prod.pr$pred + 2*prod.pr$se
> L = prod.pr$pred - 2*prod.pr$se
> ts.plot(prod,prod.pr$pred, col=1:2, type="o", ylim=c(105,175), xlim=c(1975,1980))
> lines(U, col="blue", lty="dashed")
> lines(L, col="blue", lty="dashed")
```