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Secular Cycles and Millennial Trends

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Secular Cycles and Millennial Trends

Abstract

We consider what effects could be produced by the long-term interaction of millennial macro-trends of the World System development and shorter-term cyclical dynamics. Among other things this will make it possible for us to demonstrate how even rather simple mathematical models of pre-industrial political-demographic cycles could help us to account for a paradox that has been encountered recently by political anthropologists.

Our research confirms that, notwithstanding recent arguments to the contrary, population density was a major determinant of warfare frequency in pre-industrial societies. However, the relationship between the two variables is dynamic, and could only be adequately described by nonlinear dynamic models. Hence, we confront a rather paradoxical situation. On the one hand, we observe a millennial trend leading to the growth of both population density and warfare frequency. As a result, from a long-term perspective, we observe a very strong positive correlation between these two variables. But on the other hand, we also observe secular demographic-warfare cycles, which produce negative correlations both for individual cultures and for subsamples of cultures with similar levels of technological and/or political development. So finally, if we make a straightforward cross-cultural test of the linear relationship between the two variables using a world-wide sample including cultures with all levels of technological and political development, we do not find any significant correlation at all. However it appears that hiding behind this “non-correlation” is the presence of an extremely strong and significant dynamic non-linear relationship.
Chapter 4

Secular Cycles and Millennial Trends

Initially, we want to consider what effects could be produced by the long-term interaction of millennial macrotrends of the World System development and shorter-term cyclical dynamics. Among other things this will make it possible for us to demonstrate how even rather simple mathematical models of pre-industrial political-demographic cycles could help us to account for a paradox that has been encountered recently by political anthropologists.

At least since 1798 when Thomas Malthus published his *Essay on the Principle of Population* (1798) it has been commonly assumed that in pre-industrial societies the growth of population density tends to lead to increase in warfare frequency. For example, in Anthropology this assumption forms one of the foundations of the "warfare theory" of state formation. Indeed, according to this theory population growth leads to an increase in warfare which can lead, under certain circumstances, to political centralization (Sanders and Price 1968: 230–2; Harner 1970: 68; Carneiro 1970, 1972, 1978, 1981, 1987, 1988, 2000a: 182–6; Harris 1972, 1978; 1997: 286–90; 2001: 92; Larson 1972; Webster 1975; Ferguson 1984; 1990: 31–3; Johnson and Earle 1987: 16–8).

However, some anthropologists have raised doubts regarding this relationship (Vayda 1974: 183; Cowgill 1979: 59–60; Redmond 1994; Kang 2000). On the other hand, Wright and Johnson (1975) have shown that in South-West Iran population density in the state formation period did not grow, but rather declined, while Kang's analysis of data for the state formation period Korea reveals only a very weak correlation between warfare frequency and population density (Kang 2000). Finally, cross-cultural testing performed by Keeley (1996: 117–21, 202) did not confirm the existence of any significant positive correlation between the two variables, which seems to have convinced Johnson and Earle to drop, from the second edition of their book, the mention of population pressure as a major cause of warfare in pre-industrial cultures (Johnson and Earle 2000: 15–6).

1 The first half of this chapter has been prepared on the basis of an article written by the first author of this monograph in collaboration with Peter Turchin (Turchin and Korotayev 2006).

2 Indeed, Malthus (1798/1978) saw war as one of the standard consequences of overpopulation along with disease and famine.
We tested this hypothesis ourselves using the Standard Cross-Cultural Sample database (STDS 2002). The first test seems to have confirmed the total absence of any significant correlation between population density and warfare frequency (see Diagram 4.1):

**Diagram 4.1.** Population Density × Warfare Frequency. Scatterplot with fitted Lowess line. For the Standard Cross-Cultural Sample (Murdock and White 1969)

**Note:** $\rho = 0.04, p = 0.59$ (2-tailed)

1. Use of Spearman's *Rho* (and associated measures of significance) reflects the fact that we were dealing with variables measured on an ordinal scale.
The absence of a significant correlation between the two variables seems to be entirely evident. However, let us ask ourselves the following question: Does the proposition that growing population density tends to lead to a rise in warfare's frequency necessarily imply the presence of a positive correlation between the two variables under consideration? Some scholars certainly seem to think exactly this way. Note, e.g., that Keeley (1996: 117–21, 202) thinks that he has refuted the hypothesis under consideration precisely by demonstrating the absence of such a correlation. However, we believe that to operationalize the hypothesis under consideration this way is naïve.

The point is that, as we shall see below, the growth of population density in preindustrial cultures does lead to a rise in warfare's frequency. But how would the rise of warfare frequency affect population density? Certainly, it would lead to the decline of population density. This is not only because frequent warfare would decrease the number of people in any given zone. The other important point is that frequent warfare also would reduce the carrying capacity of the given zone. Within such zones affected by constant warfare, people would tend to inhabit places that can be made defensible (see, e.g., Wickham 1981; Earle 1997: 105–42; Turchin 2003b: 120). Fearful of attack, people cultivate (or exploit in other ways) only a fraction of the productive area, specifically, the fraction in proximity to fortified settlements.

In fact, it is quite clear that we have a dynamic relationship here which can be described by non-linear dynamic models: the rise of population density leads to the rise of warfare frequency, but the growth of warfare frequency leads in its turn to a decline of population density. What kind of linear correlation would such a relationship produce?

The simplest model describing this type of relationship is the "predator – prey" one (Lotka 1925; Volterra 1926). There are good grounds to expect this model to be relevant in our case, because the relationship between population density and warfare frequency appears to have the same basic logic as the one between prey and predator populations.

This simple model looks as follows:

\[
\frac{dx}{dt} = Ax - Bxy, \\
\frac{dy}{dt} = Cxy - Dy,
\]

where \(x\) is population density ["prey"], \(y\) is warfare frequency ["predator"], and \(A, B, C, D\) are coefficients.

Note that though this model implies a perfect 1.0 level nonlinear correlation between the variables, it predicts that tests of linear relationship for the
same data will detect a weak negative correlation between warfare frequency and population density (especially if the linear ranked correlation is measured).

For example, with $A = 0.02$ (which is, by the way, the normal unlimited annual demographic growth rate for preindustrial cultures [Turchin 2003b]), $B = 0.02$, $C = 0.025$, and $D = 0.1$, the temporal dynamics appear as follows (see Diagram 4.2):

**Diagram 4.2.** Temporal Dynamics of Population Density $X$ (thick curve) and Warfare Frequency $Y$ (thin curve) with $A = 0.02$, $B = 0.02$, $C = 0.025$, and $D = 0.1$

The scatterplot of relationship between the two variables under consideration in this case will look as follows (see Diagram 4.3):

**Diagram 4.3.** Scatterplot of Relationship between Population Density and Warfare Frequency predicted by the model with $A = 0.02$, $B = 0.02$, $C = 0.025$, and $D = 0.1$
Quite predictably with such a shape of distribution the test of linear relationship for 500 cases detects beyond any doubt a weak but highly significant negative ranked correlation ($Rho = -0.19, p = 0.00001$, 2 tailed).

With $A = 0.02$, $B = 0.04$, $C = 0.04$, and $D = 0.1$, the temporal plot of the variables under consideration looks as follows (see Diagram 4.4):

**Diagram 4.4.** Temporal Dynamics of Population Density $X$ (thick curve) and warfare frequency $Y$ (thin curve) with $A = 0.02$, $B = 0.04$, $C = 0.04$, and $D = 0.1$

The scatterplot of relationship between the two variables under consideration in this case will look as follows (see Diagram 4.5):

**Diagram 4.5.** Scatterplot of Relationship between Population Density and Warfare Frequency predicted by the model with $A = 0.02$, $B = 0.04$, $C = 0.04$, and $D = 0.1$
With this shape of distribution the test of linear relationship for 500 cases detects a much stronger and even more highly significant negative ranked correlation between the variables under consideration ($\text{Rho} = -0.45$, $p = 0.000002$, 2-tailed).\(^4\)

This negative correlation is easy to explain. What is more, the model seems to correspond quite well to known facts. After all, population growth will be to some extent inhibited by frequent warfare; hence, periods of significant population growth should almost by definition coincide with periods of relatively low warfare frequency. On the other hand, as was mentioned above, the growth of warfare frequency above a certain level will lead to immediate and rapid decline of population density, whereas the drop of warfare frequency would lead to immediate increase in the population density.\(^5\)

The other important point is that warfare has a certain amount of inertia, and so does not decline immediately after the drop of population. For most preindustrial cultures this inertia has a very straightforward explanation. As has been shown by Keeley (1996), the most frequent cause of warfare is simply revenge. Now, suppose that as a result of a sharp decline of population density caused by extensive warfare the population-pressure-induced reasons for war have disappeared; yet, the very high level of warfare in the previous period of time would almost by definition imply a high level of warfare in the given period, since the earlier period would have left a large number of killings and other hostile acts still to be avenged.\(^6\)

Thus we appear to have two time lags which would tend to produce negative correlations. As a result, we would have relatively many cases of high population density accompanied by low warfare frequency, as well as low population density accompanied by high warfare frequency, whereas the number of cases with high population density accompanied by high warfare frequency would be zero, or very close to zero; hence, such lag effects would result in negative linear correlations.

Due to reasons which we will discuss at the end of this chapter, this negative correlation is observed especially among cultures with relatively similar technological bases resulting in fairly similar values of the carrying capacity of land. In the Standard Cross-Cultural Sample this correlation can be detected for a subsample of chiefdoms (i.e., cultures with 1 or 2 levels of political integra-

\(^4\) Negative correlations for this kind of model are much more likely to be detected if one measures Spearman’s Rho, rather than Pearson’s $r$. It might be argued, then, that we have “stacked the deck” by using the former rather than the latter. Note, however, that both the population density and the warfare frequency variables available in cross-cultural databases are ordinal-level, so that rank correlation (Spearman’s Rho) turns out to be simply more appropriate methodologically.

\(^5\) This is almost inevitable as the drop of the population density in the preceding prolonged period of frequent warfare would mean that the remaining population would have abundant resources immediately after the end of this period.

\(^6\) Hence, those versions of the model above with the coefficient combinations implying such a time lag seem to correspond to historical reality rather closely.
tion above that of the individual community, having mostly fairly similar technological bases). This test detected the actual presence of a significant negative correlation between the population density and warfare frequency for chiefdoms ($Rho = -0.26, p = 0.02$). Note that the correlation between population density and external warfare frequency for this subsample turned out to be stronger ($Rho = -0.40$). Predictably it turned out to be particularly strong when the sample was further split into simple and complex chiefdoms (see Diagrams 4.6 and 4.7):


NOTE: $Rho = -0.44, p = 0.002$ (1-tailed). For sources and codes see Diagram 4.1.


As we had clear hypotheses predicting the correlation directions, and as all the correlations tested here and below turned out to be in the predicted direction, we used 1-tailed significance tests.
NOTE: $\rho = -0.41, p = 0.04$ (1-tailed). For sources and codes see Diagram 4.1.

A similar correlation is also observed, for example, for cultures based on extensive agriculture (see Diagram 4.8):

**Diagram 4.8.** Population Density × Warfare Frequency. Scatterplot with linear regression line. For the Standard Cross-Cultural Sample, subsample of cultures based on extensive agriculture\(^\text{10}\), scatterplot with a fitted regression line

NOTES: $\rho = -0.3, p = 0.025$ (1-tailed). For sources and codes see notes to Diagram 4.1.

The fact that population density and warfare frequency are characterized by an extremely close non-linear dynamic relationship, which on the surface appears as a relatively weak negative linear correlation, is especially clear when we

\(^{10}\) Cultures based on extensive agriculture were selected using the variable INTENSITY OF CULTIVATION (Murdock 1967, 1985; Murdock et al. 1986, 1990, 1999–2000; STDS 2002: file STDS10.SAV [v232]). We selected cultures with value "3" of this variable ("Extensive or shifting cultivation, as where new fields are cleared annually, cultivated for a year or two, and then allowed to revert to forest or bush for a long fallow period" [Murdock 1967: 159, etc.]).
have more or less exact long-term data for population density and warfare frequency for concrete pre-industrial cultures (Turchin 2003b, Turchin and Korotayev 2006, see, e.g., Diagram 4.9):

**Diagram 4.9.** Dynamics of Population (solid line) and Internal Warfare (broken line) in China from 200 BCE to 300 CE, temporal plot (Turchin and Korotayev 2006)
NOTES: (a) trajectories of population and internal warfare index; (b) variable dynamics in the phase space (X axis – logarithm of population; Y axis – logarithm of internal warfare index): $r = -0.37, p < 0.01$.

It might be revealing to compare these figures with the ones illustrating a classical prey-predator relationship. The data document population oscillations of prey, a caterpillar that eats the needles of larch trees in the Swiss Alps, and its predators, parasitic wasps. The caterpillar population goes through very regular population oscillations with the period of 8–9 years. Predators (here measured by the mortality rate that they inflict on the caterpillars) also go through oscillations of the same period, but shifted in phase by 2 years with respect to the prey (Diagram 4.10a). Almost 95% of variation in caterpillar numbers is explained by wasp predation (Turchin 2003a), but when we plot the two variables against each other we see only a weak, and negative correlation (Diagram 4.10b). If we plot predators against the lagged prey numbers, then we clearly see the positive correlation (Diagram 4.10c):

Diagram 4.10. Population dynamics of the caterpillar (larch budmoth) and its predators (parasitic wasps). (a) Population oscillations of the caterpillar (solid curve) and predators (broken curve). (b) A scatter plot of the predator against the prey. The solid line is the regression. Broken lines connect consecutive data points, revealing the presence of cycles. (c) A scatter plot of the predator against prey lagged by two years (Turchin 2003a)
However, why did our first cross-cultural test, which employed a sample that included societies with all possible degrees of cultural complexity stretching from the !Kung Bushmen to the Modern Chinese, fail to reveal any significant correlation between the variables under consideration?

To answer this question, let us try first to model how the military-demographic dynamics could change with the growth of political centralization. First of all, note that the growth of political centralization (that is, the transition from independent communities to simple, and then complex, chiefdoms and later to states and empires, which is accompanied by a hyperbolic growth of polity sizes) leads to the decrease of the relative "lethality" of warfare (see, e.g.: Nazaretyan 1995, 1999a, 1999b, 2001).

Indeed, on the one hand, for a small (< 50) independent local group the maximum value of the warfare frequency index ("17 = occurs almost constantly and at any time of the year") implies an almost inevitable and very serious depopulation. On the other hand, any accurate coder of cross-cultural data could hardly fail to assign to, say, Russia between 1820 and 1860 exactly the maximum value of this index (at least because of the Caucasus War that continued between 1817 and 1864, let alone numerous other wars, like the Crimean one). However, in this case this maximum value of the warfare frequency index was not accompanied by anything even remotely similar to the depopula-
tion of Russia; what is more, throughout this exact period, Russia experienced a very rapid demographic growth (see, e.g., Nefedov 2005). The explanation for this apparent paradox is, of course, very simple. The point is that within the developed states (even if still pre-Industrial), wars (especially external ones) were usually waged by relatively small professional well armed and trained armies. As a result, a country could be nominally in the state of war for many decades without experiencing any depopulation at all.

Note that such external wars would not normally lead to any significant decrease of the carrying capacity with respect to the territory in which the majority of the respective state's population lives.11 In general, the larger the polity, the lower the negative influence of this polity's external wars on the carrying capacity of the territory controlled by this polity. Indeed, along the borders between those polities that wage constant wars against each other we usually observe considerable belts of economically unexploited (or underexploited) territories (see, e.g., Blanton et al. 1999).

It appears possible to demonstrate the effect of polity size growth in reducing external warfare's negative influence on carrying capacity with the help of the following model (see Diagram 4.11):

**Diagram 4.11. Influence of Frequent External Warfare on Carrying Capacity in a Zone Consisting of Simple Chiefdoms (a) and Complex Chiefdoms (b), a Model**

(a) Simple Chiefdoms

(b) Complex Chiefdoms

---

11 This is valid more for external than for internal warfare. Thus, within supercomplex agrarian societies it is the internal warfare frequency that turns out to be connected with population density along the "predator – prey" model's lines. On the other hand, the external warfare frequency dynamics also turn out to be connected with the population density dynamics, but, as we shall see below, according to a totally different model.
NOTE: Borders between polities are marked with solid lines. This model assumes that strips along the borders (marked with grey filling) are not exploited economically. As a result, frequent warfare decreases carrying capacity of a zone consisting of simple chiefdoms by almost 89%, whereas in an analogous zone consisting of complex chiefdoms it decreases the carrying capacity by less than 56%.

During the course of human history, the polity sizes have increased by 5 to 6 orders of magnitude (see, e.g.: Taagapera 1968, 1978a, 1978b, 1979, 1997; Carneiro 1978; Graber 1995 etc.); therefore the influence of this factor should have been very sizeable. The long-term trend towards the growth of political centralization tended to reduce the negative influence of frequent warfare on demographic dynamics in two ways: (1) through the professionalization of warfare and (2) through the growth of polity sizes, which entailed reduction of the negative influence of frequent warfare on carrying capacity. Is it possible to use the Lotka – Volterra equations to model how warfare's frequency would be affected by the long-term decrease of warfare's "lethality" that accompanied growing political centralization? This could be done very easily through the reduction of the value of coefficient $B$, which is none other than a coefficient of the lethal influence of the predator/warfare presence on the prey population/population density. If this is done, what will happen to the predator population dynamics? *Ceteris paribus* (that is with the same values of initial prey and predator populations, as well as with the same values of coefficients $A$, $C$, and $D$), the decrease of the value of coefficient $B$ will lead to an upward drift of the system singular point (center)\(^2\) in the phase space (see Diagram 4.12):

**Diagram 4.12.** Drift of the "Prey – Predator" System Center with Decrease of Lotka – Volterra Coefficient $B$ Value from 1.5 to 0.5

\(^2\) With a considerable degree of oversimplification it can be said that in this case the system singular point (center) can be considered as that point around which the system rotation takes place in the phase space during the cycles.
NOTE: The values of coefficients $A$, $C$, and $D$ were taken as 1.0 for all three simulations. The initial values of prey and predator populations were also identical for all three simulations.

The upward singular point (center) drift within such a phase space implies that the predator population increases for all the cycle phases; thus, the average predator population increases for any given year of the cycle.\(^\text{13}\)

Hence, for traditional cultures\(^\text{14}\) we have sufficient reason to expect the presence of a positive correlation between the level of political complexity and warfare frequency. Our cross-cultural test employing the same database has shown that such a correlation is actually observed (see Diagram 4.13):

**Diagram 4.13.** Correlation between Political Complexity Level and Warfare Frequency, for the Standard Cross-Cultural Sample, scatterplot with a fitted regression line

\(^{13}\) Within real ecological systems this may be observed, for example, if the predators’ size decreases due to some evolutionary pressures. Indeed, if in order to support its survival a predator needs just one prey animal a day (instead of, say, five analogous prey animals), then the same prey population may support reproduction of a much larger predator population.

\(^{14}\) As we shall see below, for modern cultures the Lotka – Volterra model turns out to be inapplicable (at least in the context that is of interest for us here).
NOTES: $Rho = +0.262$, $p = 0.002$. Sources and codes for the warfare frequency index are described in notes to Diagram 4.1. The number of supracommunal political integration levels has been determined on the basis of the variable JURISDICTIONAL HIERARCHY BEYOND LOCAL COMMUNITY (Murdock 1967, 1985; Murdock et al. 1999–2000; STDS 2002: file stds10.sav [v237]).

Political centralization, however, is only one of several relevant millennial trends. No less relevant, for us here, is a trend already considered by us in detail earlier (Korotayev, Malkov, and Khaltourina 2006): the long-term increase in carrying capacity that has been driven by the millennial trend of technological growth.

Is it possible to use the Lotka – Volterra equations to model how this long-term trend would be expected to affect the warfare's frequency? Let us recollect the basic model of population dynamics developed by Verhulst (1838):

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right), \quad (4.2)$$

where $N$ is population, $K$ is carrying capacity, and $r$ is the intrinsic population growth rate in the absence of any resource limitations.

According to this model, an increase in carrying capacity, in the absence of predators, will lead (ceteris paribus) to an increase in the relative growth rate of the prey population. Hence, the influence of carrying capacity growth on military-demographic dynamics can be easily modeled by increasing the value of coefficient $A$ in the Lotka – Volterra model. As this coefficient is increased, what will happen with the predator population dynamics?

Ceteris paribus (that is, with the same values for initial prey and predator populations, as well as for coefficients $B$, $C$, and $D$), the increase in the value of coefficient $A$ will also lead to an upward drift of the system singular point (cen-
ter) in the phase space, as we have already observed for a decrease in coefficient $B$ (see Diagram 4.14):

**Diagram 4.14.** Drift of the "Predator – Prey" System Center with the Increase in Lotka – Volterra Coefficient $A$ Value from 0.5 to 2.0

NOTE: The value of coefficients $B$, $C$, and $D$ was taken as 1.0 for all three simulations. The initial values of prey and predator populations were also identical for all three simulations.

In this case too, the upward singular point (center) drift within the phase space implies that the predator population increases for all the cycle phases; thus, the average predator population increases for any given year of the cycle.$^{15}$

Thus, for traditional societies$^{16}$ we have definite grounds to expect a positive correlation between the technologically determined carrying capacity and warfare frequency. To test this hypothesis cross-culturally we can compare warfare frequencies in pre-agricultural societies with those in cultures practicing casual, extensive, and intensive agriculture. The transition to agriculture and its intensification was accompanied by a radical increase in the carrying capacity. Therefore our hypothesis may be operationalized in the following way: the highest warfare frequency should be observed among the intensive agriculturalists, the frequency should be significantly lower among the extensive agriculturalists, and so on. Our cross-cultural test employing the same database indicates that such a correlation is actually observed (see Diagram 4.15):

$^{15}$ It is easy to understand why such a dynamics will be observed in actual ecological systems. Indeed, if the carrying capacity increased $n$ times with respect to the prey population, it would mean that the respective zone could support $n$ times more prey animals, and, hence, it would tend to be able to support $n$ times more predators.

$^{16}$ In modern (and even "protomodern") societies, as we shall see this below, the technologically determined increase in carrying capacity tends to decrease rather than to increase warfare frequency.
Diagram 4.15. Correlation between Intensity of Subsistence Economy and Warfare Frequency, for the Standard Cross-Cultural Sample, scatterplot with a fitted regression line

NOTES: \( Rho = +0.136 \), \( p = 0.044 \) (1-tailed). Sources and codes for warfare frequency index are described in notes to Diagram 4.1. The intensity of subsistence economy has been determined on the basis of the variable INTENSITY OF CULTIVATION (Murdock 1967, 1985; Murdock et al. 1999–2000; STDS 2002: file stds10.sav [v232]).

As we see, our cross-cultural test has detected the presence of a statistically significant correlation in the predicted direction. It may be no coincidence, moreover, that in the present test the correlation is only about half as strong as in the previous test (see Diagram 4.13). The point is that the growth of political complexity tends to be accompanied by the growth of carrying capacity. In fact, these variables are dynamically related with each other. The intensification of subsistence economy creates powerful stimuli towards the growth of political complexity (of course, the presence of such stimuli does not always lead to the actual growth of political complexity, but it is by no means infrequently that it does so). On the other hand, the growth of political complexity creates powerful stimuli towards the intensification of the subsistence economy (see, e.g., Korotayev 1991). As a result, we observe a rather pronounced correlation between the intensity of subsistence economy and political complexity (see Table 4.1):

Table 4.1. Correlation between Intensity of Subsistence Economy and Political Complexity

<table>
<thead>
<tr>
<th>Subsistence Economy Index</th>
<th>Political Complexity Index = Number of Political Integration Levels over Community</th>
<th>Total</th>
</tr>
</thead>
</table>

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<table>
<thead>
<tr>
<th>Intensity Index</th>
<th>0 (independent communities)</th>
<th>1 (simple chiefdoms)</th>
<th>2 (complex chiefdoms)</th>
<th>3 (simple states)</th>
<th>≥ 4 (complex states/empires)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (agriculture is absent)</td>
<td>34</td>
<td>8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2 (incipient agriculture)</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3 (extensive agriculture)</td>
<td>20</td>
<td>21</td>
<td>11</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4 (intensive agriculture)</td>
<td>8</td>
<td>5</td>
<td>5</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>5 (intensive irrigated agriculture)</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Total</td>
<td>73</td>
<td>42</td>
<td>20</td>
<td>19</td>
<td>12</td>
</tr>
</tbody>
</table>

NOTE: \( \text{Rho} = +0.540, \quad p = 0.0000000000000006 \)
\( \text{Gamma} = +0.612, \quad p < 0.0000000000000001 \)

As we see, all the above mentioned advances in subsistence technology correlate rather strongly with the growth of political complexity.

Consistent with this is the fact that preagricultural societies appear almost never to have been organized as states; intensive agriculturalists, conversely, had, as their most typical form of political organization, precisely the state.

As we see from Table 4.1, a certain degree of subsistence economy intensification is an absolutely necessary condition for the formation of a complex political organization; however, even a very high level of subsistence economy intensification is not a sufficient condition for the development of very complex forms of political organization, such as the state and its alternatives.

On the other hand, as we have seen in earlier chapters, the growth of political complexity can be regarded in itself as one of the possible ways to increase the carrying capacity.

In this regard it is remarkable that more politically complex societies tend to have higher population density than societies with the same subsistence economy intensity, but with a simple political organization (see, e.g., Table 4.2):

<table>
<thead>
<tr>
<th>Table 4.2. Correlation between Political Complexity and Population</th>
</tr>
</thead>
</table>

\( ^{17} \) Or alternative forms of political organization of an equivalent complexity level.

\( ^{18} \) Or alternative forms of political organization of an equivalent complexity level.

\( ^{19} \) Or alternative forms of political organization of an equivalent complexity level (on the forms of political organization that are alternative to chiefdoms and states see, e.g., Bondarenko 2001; Kradin et al. 2000; Kradin 2001; Bondarenko and Korotayev 2000, 2002; Bondarenko, Grinin, and Korotayev 2002; Grinin 2003; Grinin et al. 2004 etc.).
Density (for societies with subsistence economy based on intensive irrigated agriculture)

<table>
<thead>
<tr>
<th>Population Density</th>
<th>Political Complexity Index = Number of Political Integration Levels over Community</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1 person per 5 sq. mile</td>
<td>0 (independent communities) 1 1 1 0 0 3</td>
<td></td>
</tr>
<tr>
<td>1–5 persons per sq. mile</td>
<td>1 3 0 0 0 4</td>
<td></td>
</tr>
<tr>
<td>6–25 persons per sq. mile</td>
<td>0 0 0 0 2 2</td>
<td></td>
</tr>
<tr>
<td>26–100 persons per sq. mile</td>
<td>2 0 0 1 0 3</td>
<td></td>
</tr>
<tr>
<td>101–500 persons per sq. mile</td>
<td>1 1 1 1 1 5</td>
<td></td>
</tr>
<tr>
<td>&gt; 500 persons per sq. mile</td>
<td>0 0 2 4 5 11</td>
<td></td>
</tr>
<tr>
<td>Итого</td>
<td>5 5 4 6 8 28</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: \( Rho = +0.53, p = 0.00002 \)

Though a higher level of subsistence economy intensity does not necessarily imply the presence of a radically more complex political organization, a radical growth of political complexity is almost inevitably accompanied (at least in a long-term perspective) by a radical increase in carrying capacity. This appears to explain, to a considerable degree, why warfare frequency correlates with political complexity more strongly than it does with subsistence economy intensity. Indeed, within the Lotka – Volterra model the highest predator population will be observed if we increase the value of coefficient \( A \), while simultaneously decreasing coefficient \( B \) (which in social reality would correspond to the simultaneous growth of carrying capacity and decrease of warfare "lethality"), see Diagram 4.16:

**Diagram 4.16.** Drift of the "Prey – Predator" System Center with the Simultaneous Increase in Lotka – Volterra Coefficient \( A \) Value from 0.5 to 3.0 and Decrease in Coefficient \( B \) Value from 3.0 to 0.5

\(^{20}\) It seems necessary to stress again that these two variables are connected with each other by a mutual positive dynamic relationship; thus it appears to be incorrect to define either of them as independent, while considering the other as unequivocally dependent.
NOTES: The value of coefficients \( C \) and \( D \) was taken as 1.0 for all three simulations. The initial values of prey and predator populations were also identical for all three simulations.

Subsistence economy intensification will be inevitably accompanied only by carrying capacity growth (i.e., by "increase in the value of coefficient \( A \)"), whereas a radical increase in political complexity is almost inevitably accompanied simultaneously both by carrying capacity growth (i.e., by "increase in the value of coefficient \( A \)"") and by decrease of warfare "lethality" (i.e., by the "decrease of the value of coefficient \( B \)""); this appears to account for political complexity's correlating more strongly with warfare frequency than does subsistence economy intensity.

Throughout human history we observe a long-term trend towards (1) the growth of carrying capacity, closely associated with trends towards (2) increase in population density, and (3) growth of political complexity. As we have seen above, at least trends (1) and (3) should correlate with the growth of warfare frequency. Thus, should we not expect that in the long-term perspective we should also observe a rather strong correlation between population density and warfare frequency? Of course we should.

However, in order to detect this correlation we should alter the unit of comparison and compare the mean values of population density and warfare frequency that are typical for societies with different levels of political complexity (see Diagram 4.17):

**Diagram 4.17.** Correlation between Population Density and Warfare Frequency, for cultures with different levels of political complexity.
Mean value of warfare frequency index

Mean value of population density index

NOTE: $r = + 0.984$, $\alpha = 0.002$. Sources and codes for the warfare frequency index are described in notes to Diagram 4.1. The diagram's five points can be interpreted as roughly corresponding to the positions of the most typical singular points (centers) of military-demographic cycles for respective types of cultures.

Thus, our hypothesis has been supported. Indeed, the tightly interconnected millennial trends towards the development of subsistence technologies, growth of political complexity, carrying capacity, and population density were accompanied by a pronounced increase in frequency of warfare (which was, incidentally, the principal means through which the growth of political complexity was taking place [see, e.g., Carneiro 1970, 1981, 1987, 1991, 2000a, 2000b; Graber 1995 etc.]). This trend could finally result in such situations as that described, for example, for Ancient Rome, where "the doors of the Janus Temple (that according to the Roman traditions should have been kept open when the Roman polity was in the state of war) remained open for more than 200 years" (Knabe 1983: 80).

It appears necessary to stress that warfare frequency correlates positively with population density only when technological growth fails, for a considerable period, to raise carrying capacity as rapidly as population is growing. After the carrying capacity starts growing substantially faster than population, the demographic growth is no longer accompanied by increase in population pres-

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21 For an interesting mathematical model describing the relationship between population density and political complexity see Graber 1995.
sure, and warfare, as described in a mathematical model presented at the end of this chapter, begins to decline.

Thus, our research has confirmed that, notwithstanding recent arguments to the contrary, population density was a major determinant of warfare frequency in pre-industrial societies. However, the relationship between the two variables is dynamic, and could only be adequately described by nonlinear dynamic models. Hence, we confront a rather paradoxical situation. On the one hand, we observe a millennial trend leading to the growth of both population density and warfare frequency. As a result, from a long-term perspective, we observe a very strong positive correlation between these two variables. But on the other hand, we also observe secular demographic-warfare cycles, which produce negative correlations both for individual cultures and for subsamples of cultures with similar levels of technological and/or political development. So finally, if we make a straightforward cross-cultural test of the linear relationship between the two variables using a world-wide sample including cultures with all levels of technological and political development, we do not find any significant correlation at all. However it appears that hiding behind this "non-correlation" is the presence of an extremely strong and significant dynamic non-linear relationship.

Models of the type analyzed in the previous chapters could be used as a basis for development of a new generation of models accounting both for "secular cycles" and "millennial trends". In order to do this, we suggest altering some basic assumptions of the earlier generations of demographic cycle models, specifically, their assumptions that both the carrying capacity of land and the polity size are constant. Carrying capacity, cultural complexity, and polity size, for example, far from being constant, are nothing less than the variables with pronounced trend dynamics that the new generation of models needs to account for. Demographic cycle models account at present only for cyclical dynamics; the new modeling should be able to interconnect trend with phase dynamics.

For example, there are both theoretical and empirical grounds to maintain that the carrying capacity not only experienced a long-term upward hyperbolic trend (as has been analyzed in detail in Korotayev, Malkov, and Khaltourina 2006), but also that the innovations contributing to this trend occurred in particular phases of demographic-political cycles. Incentives created by the relative abundance of resources in the initial upward growth phases of agrarian state demographic-political cycles tend to be insufficient to create the innovations leading to the rise of the carrying capacity (these phases, however, were also very important, since during them there were strong incentives for the innovations that eventuated in the rise of the productivity of labor). Rather, innovations raising the carrying capacity tended to occur during intermediate growth phases before the demographic collapse phase. While such innovations usually acted only to delay demographic collapses, they secured the existence
of a very important upward trend, which could be accounted for, to some extent, as a by-product of sociodemographic cycle dynamics.

Another trend is the one towards larger polity sizes (see Diagrams 4.18 and 4.19 for West and East Asia).

**Diagram 4.18.** Growth Trend of Largest State/Empire
Territory Size (millions of square km) in West Asian/Mesopotamia Centered System,
3000 BCE – 750 CE

SOURCES: Taagepera 1968; 1978a; 1978b; 1979; 1997:
This trend seems to be accounted by the fact that infrastructures created by empires do not usually disappear entirely with the collapse of the systems that created them. Hence, new empires frequently do not have to build imperial infrastructures entirely anew and can rely to a considerable extent on preexisting infrastructures, making it more likely that later empires will outgrow earlier ones.

It is revealing to compare the diagram above with the one depicting population dynamics of China during the same period (see Diagram 4.20):

Diagram 4.20. Chinese Population Dynamics (in millions), 400 BCE – 1850 CE

NOTE: The diagram reproduces estimates surveyed above in Chapter 2
While comparing Diagrams 4.19 and 4.20 it is difficult not to notice a rather close fit between demographic cycles and cycles of territorial expansion/contraction. We do not think this is a coincidence.

What theoretical expectations might we have for the relationship between phases of these cycles? It has turned out that a considerable number of relevant theoretical predictions can be generated by Turchin's demographic-fiscal model (Turchin 2003: 121–7), based on Goldstone (1991). Let us recollect the main logic of this model, which can be outlined as follows:

During the initial phase of a demographic cycle we observe relatively high levels of per capita production and consumption, leading not only to relatively high population growth rates, but also to relatively high rates of surplus production. As a result, during this phase the population can afford paying taxes without great problems, the taxes are quite easily collectable, and the population growth is accompanied by growth of state revenues. During the intermediate phase, the increasing overpopulation leads to decrease of per capita production and consumption levels, it becomes more and more difficult to collect taxes, and state revenues stop growing, whereas state expenditures grow due to the growth of the population controlled by the state. As a result, during this phase the state starts experiencing considerable fiscal difficulty. During the final pre-collapse phases the overpopulation leads to further decrease of per capita production, the surplus production further decreases, and state revenues shrink, whereas the state requires more and more resources to control the population (which is still growing, though at lower and lower rates). Eventually this leads to state breakdown and demographic collapse, after which a new demographic cycle begins.

What kind of territorial expansion/contraction pattern would be generated by such demographic-fiscal dynamics? During the initial phase state revenues are high and continue to grow, making it possible for a state to support large armies and to undertake active territorial expansion. Note that this is only valid for unipolar regional systems, i.e., systems with a single strong state. In multipolar regional systems comprising a few equally strong states we can only expect that the composite states will try to undertake attempts at territorial expansion. However, there are naturally no guarantees that the attempts of any particular state will be successful. What is more, within a fairly balanced multipolar system such attempts undertaken by a few states could result in a stalemate, as a result of which none of the participant states would enjoy considerable territorial gains.

During the intermediate phase the state starts experiencing fiscal problems, its ability to support large and effective armies decreases. Thus, we have grounds to expect that during this phase imperial territorial expansion will slow down.

During the final pre-collapse phase state revenues considerably decrease, which leads to a considerable decrease of the size and effectiveness of military
forces supported by the state. Hence, we have grounds to expect that during this phase imperial territorial expansion will stop. What is more, during this phase the state territory is likely to start contracting.

To test these predictions we have used Taagepera’s database on historical dynamics of empire sizes (Taagepera 1968; 1978a; 1978b; 1979; 1997), as well as Nefedov’s (1999b, 1999d, 2000a, 2001b, 2003) and Turchin’s (2003b) data on population and consumption dynamics.\footnote{Note that in both cases we are not really dealing with a sample, but rather with the general population of all cases, for which empirical data are available.}

The first (and the least counterintuitive) prediction tested by us is that the phases of relatively rapid population growth should correlate with phases of relatively rapid territorial expansion.

The test has supported this hypothesis: the correlation has turned out to be in the predicted direction, very strong, and significant beyond any doubt (see Diagram 4.21 and Table 4.3):

**Diagram 4.21.** Population Growth Rate × Territorial Expansion/Aggressive External Warfare (scatterplot with fitted Lowess line)
<table>
<thead>
<tr>
<th>Population Growth Rate (direct and indirect evidence)</th>
<th>Territorial Expansion/Aggressive External Warfare</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (stagnation)</td>
<td>-1 (mostly defensive warfare)</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>0 (almost absent)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 (relatively low)</td>
<td>3</td>
</tr>
<tr>
<td>1 (relatively low)</td>
<td>-1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2 (intermediate)</td>
<td></td>
</tr>
<tr>
<td>2 (intermediate)</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3 (relatively high)</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>23</td>
</tr>
</tbody>
</table>

NOTES: For all cases: $\tau = +0.81$, $p < 0.0000000000000000001$; $\rho = +0.83$, $p = 0.00000002$. For cases with direct evidence on population growth rate (all Chinese): $\tau = +0.79$, $p < 0.0000000000000000001$; $\rho = +0.87$, $p = .000003$. For non-Chinese cases (Roman, direct evidence; Babylonian, indirect evidence): no correlation coefficient can be computed due to the small number of cases, but the contrastive periods for the Roman and Babylonian cases are consistent with the hypothesis.

Note, however, that for the direct test of this hypothesis we had to rely almost exclusively on the Chinese data, as East Asia is the only region (and the only unipolar region) for which we have direct data on historical population dynamics. On the other hand, it has proven possible to collect sufficient extra-Chinese data to test our next hypothesis, namely, the one linking relative per capita con-

21 Roman Empire 120 – 200 CE, Western Han 40 BCE – 10 CE, Eastern Han 105 – 157 CE (Roman type is used for cases for which direct evidence is available for both variables, and italics where evidence is indirect for one or both variables).
22 Babylonia 556 – 539 BCE, T’ang 733 – 754, Ming 1450 – 1620.
23 Qing 1800 – 1830.
24 Moghol Empire 1620 – 1670, Western Han 110 – 40 BCE.
25 Ming 1410 – 1450.
26 Moghol Empire 1670 – 1690, Roman Empire 50 – 120 CE.
27 Sung 1000 – 1066.
28 Ming 1410 – 1450.
29 Qing 1720 – 1750.
30 Qing 1750 – 1800.
sumption levels and territorial expansion/contraction. What are our theoretical predictions in this case?

To start with, the demographic cycle models predict that the relatively fast population growth should correlate with relatively high consumption levels. Our empirical test of this assumption has confirmed its validity (see Diagram 4.22 and Table 4.4):

**Diagram 4.22.** Relative Consumption Rate × Territorial Expansion/Aggressive External Warfare (scatterplot with fitted Lowess line)
### Table 4.4. Population Growth Rate (direct and indirect evidence) × Relative Consumption Rate (direct and indirect evidence)

<table>
<thead>
<tr>
<th>Population Growth Rate (direct and indirect evidence)</th>
<th>Relative Consumption Rate (direct and indirect evidence)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (stagnation)</td>
<td>1 (relatively low)</td>
<td></td>
</tr>
<tr>
<td>2 (intermediate)</td>
<td>3 (relatively high)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

For all cases: \( \text{Tau-b} = +0.88, \quad p < 0.0000000000000001 \)

\( \text{Rho} = +0.92, \quad p = 0.0000000009 \)

For cases with direct evidence on population growth rate:

\( \text{Tau-b} = +0.85, \quad p = 0.000004 \)

\( \text{Rho} = +0.92, \quad p = 0.000000009 \)

Against the background of our first test, this of course suggests that relatively high consumption levels should correlate positively with relatively rapid territorial expansion. Note that as the consumption rates are usually measured through the real wages of unskilled workers (the amount of staple food an unskilled worker could buy from daily wages), the operationalization of this hypothesis may sound especially counterintuitive: we claim that if we know the relative real wages of unskilled workers, at least in the center of a unipolar system, we can predict with a very high degree of confidence whether the respective empire experienced a relatively rapid expansion, expanded slowly, or contracted. In fact, there is nothing mysterious in this relationship. Relatively high consumption levels should correlate positively with relatively rapid territorial expansion.

53 Roman Empire 120 – 200 CE, Western Han 40 BCE – 10 CE.
54 Eastern Han 105 – 157 CE.
55 Babylon 556 – 539 BCE, Roman Empire 50 – 120 CE, Moghol Empire 1670 – 1690, T'ang 733 – 754, Ming 1450 – 1620, Qing 1800 – 1830.
56 Western Han 110 – 40 BCE.
57 Moghol Empire 1620 – 1670, Sung 1000 – 1066, Ming 1410 – 1450.
58 Qing 1720 – 1750.
59 Qing 1750 – 1800.
real wages imply that labor is a scarce resource, hence, there is no overpopulation; the population, being well provided with resources, rapidly grows, producing large surpluses; and the state has high and growing revenues with which to support a large, effective army and to undertake rapid territorial expansion. On the other hand, relatively low real wages imply that labor is overabundant, hence, there is overpopulation; the population, being insufficiently provided with resources, grows slowly, producing almost no surplus, and the state having low and shrinking revenues is unable to support a large, effective army and to undertake a rapid territorial expansion; indeed, it may fail even to organize adequate defense of territories under its control.

The test has supported this hypothesis: the correlation has turned out to be in the predicted direction, very strong, and significant beyond any doubt (see Diagram 4.23 and Table 4.5):

**Diagram 4.23. Relative Consumption Rate × Territorial Expansion/Aggressive External Warfare**
### Territorial Expansion/Agressive External Warfare

<table>
<thead>
<tr>
<th>Relative Consumption Rate</th>
<th>Territorial Expansion/Agressive External Warfare</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1 (mostly defensive warfare)</td>
<td>0 (almost absent)</td>
</tr>
<tr>
<td>0 (very low)</td>
<td>241</td>
<td>0</td>
</tr>
<tr>
<td>1 (relatively low)</td>
<td>42</td>
<td>143</td>
</tr>
<tr>
<td>2 (intermediate)</td>
<td>146</td>
<td>147</td>
</tr>
<tr>
<td>3 (relatively high)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

For all cases: $\tau_b = +0.83, p < 0.0000000000000001; \rho = +0.90, p = 0.00000001$  
For cases with direct evidence on population growth rate:  
$\tau_b = +0.85, p < 0.0000000000000001; \rho = +0.91, p = 0.00000002$  
For non-Chinese cases: $\tau_b = +0.93, p < 0.0000000000000001; \rho = +0.96, p = 0.0001$

The population of the core area is smallest during the initial phase of the demographic cycle and is highest during the final pre-collapse phase. This re-

---

41 Roman Empire 120 – 200 CE, Western Han 40 BCE – 10 CE.  
43 Qing 1800 – 1830.  
44 Roman Empire 50 – 120, Moghol Empire 1670 – 1690.  
45 Western Han 110 – 40 BCE.  
46 Sung 1000 – 1066.  
47 Qing 1720 – 1750.  
48 Moghol Empire 1620 – 1670, Qing 1750 – 1800.  
sults in another counterintuitive hypothesis – the relatively higher the population of the empire core area, the lower its expansion rate. Our empirical test has supported this hypothesis too (see Diagram 4.24 and Table 4.6):

Diagram 4.24. Core Area Population × Territorial Expansion/Aggressive External Warfare

Core Area Population (direct and indirect evidence)
1 – low in comparison with other phases of respective cycle
2 – intermediate in comparison with other phases of respective cycle
3 – high in comparison with other phases of respective cycle

50 The core area is defined here as the area of the central polity of a unipolar region before the start of its expansion at the beginning of a political-demographic cycle.
Table 4.6. Correlation between Core Area Population (direct and indirect evidence) and Territorial Expansion/Aggressive External Warfare

<table>
<thead>
<tr>
<th>Core Area Population (direct and indirect evidence)</th>
<th>Territorial Expansion/Aggressive External Warfare</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1 (mostly defensive warfare)</td>
<td>9^51</td>
</tr>
<tr>
<td></td>
<td>0 (almost absent)</td>
<td>9</td>
</tr>
<tr>
<td>1 (relatively low)</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2 (intermediate)</td>
<td>2^32</td>
<td>5</td>
</tr>
<tr>
<td>3 (relatively high)</td>
<td>7^54</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

NOTE: Tau-b = −0.94, \( p < 0.0000000000000001 \); Rho = + 0.97, \( p = 0.000000000000001 \).

Hence, these findings support a point made earlier, that the structure of millennial trends cannot be adequately understood without secular cycles being taken into account. At a certain level of analysis, millennial trends appear to be virtual byproducts of the demographic cycle mechanisms, which turn out to incorporate certain trend-creating mechanisms. Demographic-political cycle models can serve as a basis for the development and testing of models accounting not only for secular cycles but also for millennial trends. In order to do this, we suggest altering the basic assumptions of earlier generations of demographic cycle models (such as that both the carrying capacity and the polity size are

52 Roman Empire 50 – 120 CE, Ming 1410 – 1450.
53 Moghol Empire 1620 – 1670, Western Han 110 – 40 BCE, Qing 1750 – 1800.
55 Qing 1800 – 1830.
56 Moghol Empire 1670 – 1690.
constant). These are variables with long-term trend dynamics in the rise of carrying capacity, cultural complexity, and empire sizes that the new generation of models needs to account for.

An interesting mathematical model that describes both secular political-demographic cycles and millennial growth trends has been proposed by Komlos and Nefedov (Komlos and Nefedov 2002). However, note that irrespective of all its merits it does not describe the hyperbolic trend analyzed in the first part of our Introduction to Social Macrodynamics (Korotayev, Malkov, and Khaltourina 2006).

At the end of this chapter we would like to propose our own preliminary model designed to describe both secular cycles and millennial trends. We have developed it on the basis of our "secular cycle" model presented in the previous chapter, basically by adding to it Kremer's equation of technological growth:

$$\frac{dT}{dt} = aNT,$$  \hspace{1cm} (0.8)

where $T$ is the level of technology, $N$ is population, and $a$ is average technologically innovating productivity per person.

Let us remind you that actually Kremer uses here the following key assumption of the Endogenous Technological Growth theory, which we have already used above for the development of the first compact macromodel (Kuznets 1960; Grossman and Helpman 1991; Aghion and Howitt 1992, 1998; Simon 1977, 1981, 2000; Komlos and Nefedov 2002; Jones 1995, 2003, 2005 etc.):

"High population spurs technological change because it increases the number of potential inventors... All else equal, each person's chance of inventing something is independent of population. Thus, in a larger population there will be proportionally more people lucky or smart enough to come up with new ideas" (Kremer 1993: 685); thus, "the growth rate of technology is proportional to total population" (Kremer 1993: 682).

We also model the "Boserupian" effect (Boserup 1965). As was shown by Boserup relative overpopulation creates additional stimuli to generate and apply carrying-capacity-of-land-raising innovations. Indeed, if land shortage is absent, such stimuli are relatively weak, whereas in conditions of relative overpopulation the introduction of such innovations becomes literally a "question of life and death" for a major part of the population, and the intensity of the generation and diffusion of the carrying capacity enhancing innovations significantly increases. In our model this effect is modeled in the following way:

$^{57}$ "This implication flows naturally from the nonrivalry of technology... The cost of inventing a new technology is independent of the number of people who use it. Thus, holding constant the share of resources devoted to research, an increase in population leads to an increase in technological change" (Kremer 1993: 681).
The influence of technological growth on production is taken into account through the introduction of multiplier $T$ into (3.1), producing the following equation:

$$\text{Harvest}_i = H_0 \times \text{random number}_i \times T_i.$$  \hspace{1cm} (4.5)

Thus, in our "trend-cyclical" model, the per unit yield in year $i$ depends not only on the climatic conditions in year $i$ (simulated with random number$_i$), but also on the level of subsistence technology achieved at this year.

The extension of model (3.2)-(3.4) with equations (4.3)-(4.5) alters the dynamics generated by the model in a very significant way (see Diagrams 4.25–27):

**Diagram 4.25.** Dynamics Generated by the Compact Trend-Cyclical Model: population (black curve, persons) and food reserves (grey curve, in MAFRs)

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**NOTE:** the diagram reproduces the results of a simulation with the following values of parameters and initial conditions: $N_0 = 50,000,000$ peasants; $A_{total} = 100,000,000$ units, it is assumed that one unit produces, under average climatic conditions and initial technological level ($T_0 = 1$), one mini-

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$58$ Minimum annual food ration, an amount of food that is barely sufficient to support one person for one year.
mum annual food ration (MAFR), \(i.e.,\) an amount of food that is barely sufficient to support one person for one year, that is, \(H_0 = 1\) MAFR per unit per year; \textit{random number} range is between 0.85 and 1.15, thus \(\text{Harvest, } (\text{per unit yield in year } i)\) randomly assumes values in the range 0.85 to 1.15 MAFR per unit per year; \(\text{Food}_0 = 1\) MAFR; \(R_0 = 1000\) bandits; \(S_0 = 0\) MAFR; \(\alpha = 0.04\) MAFR\(^2\); \(\text{tax} = 0.1\); \(\text{out} = 0.1\); \(\beta = 0.03\); \(\text{rob} = 0.000000001\); \(\alpha\) (innovation productivity coefficient) = 0.000000000005.

This model describes not only cyclical, but also the hyperbolic trend dynamics. Note that it also describes the lengthening of growth phases detected in Chapter 2 for historical population dynamics in China, which was not described by our simple cyclical model. The mechanism that produces this lengthening in the model (and apparently in reality) is as follows: the later cycles are characterized by a higher technology, and, thus, higher carrying capacity and population, which, according to Kremer's technological development equation embedded into our model, produces higher rates of technological (and, thus, carrying capacity) growth. Thus, with every new cycle it takes the population more and more time to approach the carrying capacity ceiling to a critical extent; finally it "fails" to do so, the technological growth rates begin to exceed systematically the population growth rates, and population escapes from the "Malthusian trap" (see Diagram 4.26):

\textbf{Diagram 4.26. Dynamics Generated by the Compact Trend-Cyclical Macromodel: per capita production, in MAFR}

A numerical investigation of the effect of parameter values on the dynamics of our model indicates that that the main parameters that affect the period of the
cycle are still the proportion of resources accumulated in counter-famine reserves ($\text{tax}$), peasant-bandit transformation rate ($\alpha_{\text{out}}$), and the magnitude of climatic fluctuations ($M_c$). Longer cycles still obtain for higher values of $\text{tax}$ and lower values of $\alpha_{\text{out}}$ and $M_c$, whereas the cycles' length becomes shorter with lower $\text{tax}$, and higher $\alpha_{\text{out}}$ and $M_c$. Thus we find again that the length of the cycles would increase with the growth of the strength of counter-famine and law-enforcement subsystems, and could be decreased by the increase in the magnitude of climatic fluctuations. Of special importance is that our numerical investigation indicates that with shorter average period of cycles a system experiences a slower technological growth, and it takes a system longer to escape from the "Malthusian trap" than with a longer average cycle period. The implications of this finding will be studied in more detail in the subsequent volume of our *Introduction to Social Macrodynamics* (Korotayev and Khaltourina 2006).

Of special interest for us here are the internal warfare dynamics described by the compact trend-cyclical model (see Diagram 4.27):

**Diagram 4.27.** Dynamics Generated by the Compact Trend-Cyclical Macromodel: internal warfare intensity, number of "peasants" killed by "bandits" per year

As we see, the intensity of internal warfare observed during demographic collapse phases tends to grow significantly. The mechanism that generates this ef-
fect in the model is produced by the "prey – predator" model logic whose elements are incorporated into the trend-cyclical model: a higher population of "peasants" can support a higher "bandit population" that in its turn would kill more "peasants". On the other hand, note that the population's "escape from the Malthusian trap" leads to internal warfare's "extinction": the recurrent out-breaks of internal warfare completely disappear.

Naturally, the trend-cyclical model describes historical dynamics in a more accurate way when it is extended along the lines suggested in model (0.20)-(0.14)-(0.21) (see Introduction) so that it can also describe the system's withdrawal from the blow-up regime via the demographic transition. This is done by adding to the model an equation describing literacy dynamics:

\[ l_{i+1} = l_i + b \times dF_i \times l_i \times (1 - l_i), \]  

(4.6)

where \( l_i \) is the proportion of population that is literate in year \( i \), \( dF_i \) is per capita surplus, and \( b \) is a constant. Of course, equation (4.6) is simply a modified version of equation (0.21), substantiated in the Introduction, as well as in Kortayev, Malkov, and Khaltourina 2006. The influence of literacy\(^\text{59}^\) on the demographic transition is expressed through the addition to (3.3) of the multiplier \( (1 - l) \), which results in equation (4.7):

\[ N_{i+1} = N_i \times (1 + \alpha \times dF' \times (1 - l) - dR_i - rob \times N_i \times R_i). \]  

(4.7)

Typical dynamics generated by the resulting model is presented in Diagram 4.28:

\(^{59}\text{And other dimensions of the human capital level that strongly correlate with literacy.}\)
Diagram 4.28. Dynamics Generated by the Extended Trend-Cyclical Model: population (black curve, persons) and food reserves (grey curve, MAFRs)

NOTE: The diagram reproduces the results of a simulation with the following values of parameters and initial conditions: \( N_0 = 100,000,000 \) peasants; \( A_{\text{out}} = 150,000,000 \) units, it is assumed that one unit produces under average climatic conditions and initial technological level \( (T_0 = 1) \) one minimum annual food ration (MAFR), i.e., an amount of food that is barely sufficient to support one person for one year, that is, \( H_0 = 1 \) MAFR per unit per year; random number range is between 0.65 and 1.35, thus \( \text{Harvest} \) (per unit yield in year \( i \)) randomly assumes values in the range 0.65\( T \) to 1.35\( T \) MAFR per unit per year; \( \text{Food}_{\text{in}} = 1 \) MAFR; \( B_0 = 1000 \) bandits; \( S_0 = 0 \) MAFR; \( \alpha = 0.04 \) MAFR\(^{-1} \); tax = 0.1; \( \alpha_{\text{out}} = 0.2 \); \( \beta = 0.03 \); rob = 0.000000001; \( a \) (innovation productivity coefficient) = 0.0000000000065; \( b \) (literacy growth coefficient) = 0.01.

Of course, these models can be only regarded as first steps towards the development of effective models describing both secular cycles and millennial upward trend dynamics.