1. (10 points) Let \( \Omega \in L^1(S^{d-1}) \) have mean zero. Prove that, if the operator \( T_\Omega \) defined by:
\[
T_\Omega f(x) := \text{p.v.} \int_{\mathbb{R}^d} \frac{\Omega(y/|y|)}{|y|^d} f(x - y) \, dy, \quad f \in \mathcal{S}(\mathbb{R}^d),
\]
maps \( L^p \) to \( L^q \) boundedly, then necessarily \( p = q \).

2. (10 points) Assume that \( T \) is a linear operator acting on measurable function in \( \mathbb{R}^d \) with the property that, if \( f \) is supported on a cube \( Q \), then \( Tf \) is supported on a fixed multiple of the cube, say \( NQ \). Prove that, if \( T \) is strong type \((p, p)\) for some \( p \), then \( T \) is also weak type \((1, 1)\).