

# Economic Growth Among Pennsylvania Counties in A Neoclassical Framework

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## **Abstract**

The ability of the neoclassical model to explain income growth rates and levels among Pennsylvania counties is assessed. These relatively homogenous counties provide an excellent test of the model's prediction of convergence in living standards and specification of the factors accounting for differences in income levels across regions. However, the basic model is unable to explain the counties' growth experiences. The addition of human capital somewhat rescues the model: it explains 62 percent of the variation in income levels but its quantitative predictions do not fit the data, and convergence is detected at a statistically insignificant 0.9 percent a year.

Can the neoclassical growth model be usefully applied to the analysis of local area economic growth? Mankiw (1995, p. 308) asserts that “the neoclassical model is still the most useful theory of growth that we have.” Indeed, Mankiw, Romer, and Weil (1992) demonstrate that the two variables upon which the model focuses, the rates of savings and population growth, explain much of the cross-country variation in income per capita in a sample of 98 non-oil nations. Also, the neoclassical growth model predicts that, under certain conditions, poor regions will grow faster than rich regions so that living standards across all regions will eventually be the same, even though some regions may start out way behind, and Dowrick and Nguyen (1989) and Mankiw, Romer, and Weil (1992) both find that there has been a significant tendency towards convergence of per capita income among OECD countries while Barro and Sala-i-Martin (1992; 1995) find evidence of convergence across U.S. states and across Japanese prefectures.

On the other hand, McCallum (1996, p. 50) claims that the neoclassical growth model “fails to explain even the most basic facts of actual growth behavior.” The model suggests that national or regional growth rates are independent of national or regional characteristics and policies. Yet, Sala-i-Martin (1998?) finds that various regional, political, and religious variables are strongly correlated economic growth. Moreover, the elasticities implied by the model do not fit the data (Romer 1987; 1994), and convergence fails across a broad sample of countries (Mankiw, Romer, and Weil 1992; Romer 1994). This purpose of this paper is to assess the ability of the neoclassical model to explain income growth rates and levels across a set of fairly homogeneous local political units, the 67 counties of the state of Pennsylvania.

## I. The Neoclassical Growth Model

Assume a Cobb-Douglas production function in which the rates of saving, population growth, and technological progress are assumed to be exogenous so that output at time  $t$ ,  $Y_t$ , is given by

$$(1) \quad Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$

with  $0 < \alpha < 1$ .  $A$  is the level of technology,  $K$  is capital, and  $L$  is labor.  $L$  and  $A$  are assumed to grow exogenously at rates  $n$  and  $g$ :

$$(2) \quad L_t = L_0 e^{nt}$$

$$(3) \quad A_t = A_0 e^{gt}$$

Let  $k_t = K_t/L_t$  and  $y_t = Y_t/L_t$ . The production function can then be written as

$$(4) \quad y_t = A_0 e^{gt} k_t^\alpha.$$

The neoclassical model assumes that a constant fraction of income,  $s$ , is invested in capital goods so that the growth of  $k$  is given by

$$(5) \quad \Delta k_t = s y_t - (n + g + \delta) k_t = s A_0 e^{gt} k_t^\alpha - (n + g + \delta) k_t,$$

where  $\delta$  is the rate of depreciation.  $k$  converges to the steady-state value  $k^*$  defined by  $\Delta k_t = 0$  or

$$(6) \quad s A_0 e^{gt} k^{*\alpha} = (n + g + \delta) k^*.$$

Equation (6) can be solved for the steady-state capital-labor ratio

$$(7) \quad k^* = [s A_0 e^{gt} / (n + g + \delta)]^{1/(1-\alpha)}.$$

Substituting equation (7) into the production function and taking logs yields the steady-state income per capita:

$$(8) \quad \log y^* = 1/(1-\alpha)(\log A_0 + gt) + \alpha/(1-\alpha) \log s - \alpha/(1-\alpha) \log (n + g + \delta).$$

$g$ , the rate of technological improvement, reflects the advancement of knowledge and, along with the rate of depreciation and the term  $A_0$  is assumed to be constant across counties. So, let  $1/(1-\alpha)(\log A_0 + gt)$  equal  $a$ , where  $a$  is constant across counties. Then, for each county

$$(9) \quad \log y^* = a + \alpha/(1-\alpha) \log s - \alpha/(1-\alpha) \log (n + g + \delta).$$

## II. Income Growth Rates

The growth of the capital stock per worker is given by equation (5). Take the first-order Taylor expansion around the steady-state capital-labor ratio:

$$(10) \quad dk/dt = [s dy/dk^* - (n + g + \delta)](k - k^*).$$

In the steady-state,  $s$  is equal to  $(n + g + \delta)(k^*/y^*)$ , where  $y^*$  is the steady-state income per capita given by equation (9). Substitute this into equation (10):

$$(11) \quad dk/dt = [(k^*/y^*)(dy/dk^*) - 1](n + g + \delta)(k - k^*).$$

If capital earns its marginal product,  $(k^*/y^*)(dy/dk^*)$  equals  $\alpha$ , capital's share of income. Then,

$$(12) \quad dk/dt = (\alpha - 1)(n + g + \delta)(k - k^*)$$

Income converges to its steady-state level at the same rate as capital. So, if the actual level of income per capita at time  $t$  is  $y_t$ , the speed of convergence is found by

$$(13) \quad d \log (y_t)/dt = (n + g + \delta)(1 - \alpha)(\log y^* - \log y_t).$$

Then,

$$(14) \quad \log y_t = (1 - e^{-\lambda t}) \log y^* + e^{-\lambda t} \log y_0,$$

where  $\lambda = (n + g + \delta)(1 - \alpha)$  and  $y_0$  is per capita income at some starting date. Subtracting  $\log y_0$  from both sides and substituting for  $y^*$  yields

$$(15) \quad \log y_t - \log y_0 = (1 - e^{-\lambda t})(\alpha + \alpha/(1 - \alpha) \log s - \alpha/(1 - \alpha) \log (n + g + \delta)) \\ - (1 - e^{-\lambda t}) \log y_0.$$

Equation (15) says that the growth of income per worker is a function of the initial level of income and the determinants of the steady-state level of income: income growth is positively correlated with the savings rate and negatively related to the rate of population growth and the initial level of income. A negative coefficient on the log of initial income per capita would suggest there has been convergence of income per capita across Pennsylvania counties: poor counties grow faster than rich counties.

#### A. Data and Sources

I begin assessing the usefulness of the neoclassical model for understanding local area growth by estimating equation (15) and testing for evidence of convergence of income across the 67 Pennsylvania counties over the period 1970 to 1996. The appropriate income measure is not available: a county-level version of gross domestic product. Personal income data is available on a county basis. But use of

personal income is problematic if people work in one county and live in another or if people tend to own capital in other counties because the personal income accounts reported by the Bureau of Economic Analysis assign income to the county in which the owner of the inputs resides not to the county in which the income was earned. For instance, Lackawanna County received 7033 commuters from Luzerne County in 1990 while sending there 5175 commuters, and nearly 60,000 Delaware County residents worked in Philadelphia with an even larger number of Philadelphia commuters coming from out of state (Pennsylvania State Data Center web site). Also, the personal income measure includes transfer payments. So, personal income is not a good measure of economic activity because it includes both unearned income and income earned outside the county.

I use “total earnings by place of work” as the measure of county income because it attributes income to the county in which it was earned. Total earnings includes wages and salaries, other labor income, contributions for social insurance, and proprietors’ income. It excludes dividends, interest, rent, and transfer payments. Since the neoclassical model is expressed in per worker terms, I divide total earnings by total employment for each county. Data for total earnings and employment are taken from the Regional Economic Information System web page.<sup>1</sup>

If the level of income influences  $s$ ,  $n$ , and the human capital variable introduced below, then estimates of equation (15) using OLS are potentially inconsistent. To reduce the simultaneity problem, I take the values of the investment and human capital variables from the beginning of the sample period, 1970. The fraction of income invested in capital goods,  $s$ , is estimated for each county by dividing manufacturing capital expenditures in 1970 by county total earnings. Capital expenditures for each county are found in Table 56 of the 1971 *Pennsylvania Statistical Abstract*. The population growth rate is the average annual growth rate of county employment for the period 1970 to 1996. Finally, I assume that  $g + \delta$  is equal to 0.05 to match the available data.<sup>2</sup>

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<sup>1</sup> <<http://fisher.lib.virginia.edu/reis>>

<sup>2</sup> Mankiw, Romer, and Weil (1992, pp. 413-414) state that  $\delta$  is 0.03 for the U.S. and that  $g$  is about 0.02.

## B. Results

The first column of Table 1 reports on the estimation of

$$(16) \quad \log y_{1996} - \log y_{1970} = \text{constant} + \log y_{1970}.$$

The coefficient on the log of initial per capita income is negative, but both the coefficient and the adjusted  $R^2$  are essentially 0. There is no tendency for poor Pennsylvania counties to unconditionally grow faster on average than rich counties.<sup>3</sup> But, as equation (15) demonstrates, the neoclassical model does not imply unconditional convergence; the model predicts conditional convergence, convergence after controlling for the determinants of the steady-state level of income. Counties that are further from their own steady state income per capita grow faster than those that are closer to their individual steady states. The second column of Table 1 adds the logs of  $s$  and  $(n + g + \delta)$  to the right-hand side of equation (16). The additional coefficients are not significantly different from zero. The initial level of income is not strongly correlated with county economic growth.

## C. Adding Human Capital to the Model

Mankiw, Romer, and Weil (1992) find that a neoclassical growth model that includes human capital provides an excellent explanation cross-country economic growth. The production function can be modified to include the stock of human capital,  $H_t$ :

$$(17) \quad Y_t = A_t K_t^\alpha H_t^\beta L_t^{1-\alpha-\beta}.$$

Let  $h$  be the fraction of income invested in human capital. Then, per capita income growth between year 0 and year  $t$  is given by

$$(18) \quad \log y_t - \log y_0 = (1 - e^{-\lambda t})(a + \alpha/(1 - \alpha - \beta) \log s + \beta/(1 - \alpha - \beta) \log h - (\alpha + \beta)/(1 - \alpha - \beta) \log (n + g + \delta)) - (1 - e^{-\lambda t}) \log y_0.$$

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<sup>3</sup> In 1970, the standard deviation of the log of total earnings per worker across Pennsylvania counties was 0.13; in 1996, it was 0.16. Barro and Sala-i-Martin (1995, p. 383) call this  $\sigma$ -divergence.

The third column of Table 1 adds the log of a proxy for the rate of accumulation of human capital to the right-hand side regressors of equation (16). The variable COLLEGE is the proportion of the county population of persons 25 years and over who have a bachelor's degree or higher for the year 1970. In this regression, the coefficient of rate of human capital accumulation is positive but, like the other independent variables, not significantly different from zero. The implied speed of convergence, derived from the coefficient on  $\log y_{1970}$ , is less than 1 percent a year.

Table 2 presents an estimate of equation (18) using  $\log(\text{COLLEGE})$  with the restriction that the coefficients on  $\log s$ ,  $\log h$ , and  $\log(n + g + \delta)$  sum to zero. The restriction is rejected at conventional levels of significance. The implied speed of convergence of 0.009 indicates that the economy of the typical Pennsylvania county moves halfway to its steady-state in about 80 years. The implied value of  $\alpha$ , the share of income going to physical capital, is far, far below the expected value of 0.33. The estimate of  $\beta$ , 0.31, is more reasonable.  $\beta$  is the share of income going to human capital, and Mankiw, Romer, and Weil (1992, p. 417) suggest that  $\beta$  ought to be between 0.33 and 0.5. Nevertheless, the neoclassical growth model clearly provides an unsatisfactory explanation of local economic growth in Pennsylvania. The variables the model claims to be important have no explanatory power over cross-county growth rates.

### III. County Differences in Income Levels

The neoclassical growth model makes qualitative and quantitative predictions about the factors explaining differences in income levels across regions. From equation (9),

$$(9) \quad \log y_t = a + \alpha(1 - \alpha) \log s - \alpha(1 - \alpha) \log(n + g + \delta),$$

income per capita is positively related to the investment rate and negatively related to the population growth rate. Furthermore,  $\alpha$  should equal capital's share of income, approximately 0.33 in the United States.

The first column of Table 3 presents an estimate of equation (9) using the log of total earnings per worker in 1996 as the dependent variable. The population growth rate is the average annual growth rate of county employment for the period 1970 to 1996. The fraction of income invested in new capital goods is calculated for each county by taking the 1992 value of new capital expenditures from the *1992 Census of Manufactures* and dividing it by total earnings for 1992. The sample size drops to 60 because new capital expenditures data are not reported for seven small counties. Neither of the explanatory variables is significant.

The second column of Table 3 estimates a neoclassical model growth model augmented by the addition of human capital to the production function. In this case, cross-county differences in income per capita are given by

$$(19) \quad \log y_t = a + \alpha/(1 - \alpha - \beta) \log s - (\alpha + \beta)/(1 - \alpha - \beta) \log (n + g + \delta) + \beta/(1 - \alpha - \beta) \log h,$$

where  $h$  is the fraction of income invested in human capital. The fraction of the working age population in 1990 with a college degree is used to proxy  $h$ . The rate of investment in physical capital, although insignificant, and the rate of investment in human capital are positively related to earnings, and the employment growth rate is negatively related to county income differences. These three variables explain over 60 percent of the cross county variation in the level of earnings per worker.

Table 4 estimates the neoclassical model imposing the restriction that the sum of the coefficients on the independent variables is zero. The basic neoclassical model fails to explain the differences in per capita income levels among Pennsylvania counties. The two variables deemed important by the model account for none of the cross-county variation in income per capita. Adding the proxy for human capital accumulation in the form of a college education to the model dramatically improves its explanatory power, but the restriction is rejected and the implied value of physical capital's share of income is 0.05, much smaller than the expected value of 0.33. The implied value of human capital's share of income, 0.26, is also too small.

#### IV. Conclusions

This paper assesses the ability of the neoclassical model to explain income growth rates and income levels among the 67 Pennsylvania counties. The relatively homogenous counties of Pennsylvania provide an excellent test of the neoclassical growth model's prediction of convergence in living standards and specification of the factor's accounting for differences in income levels across local areas. The counties possess identical access to technology and have similar cultural and political institutions; the preferences of a representative Luzerne county household are going to be closer to the preferences of a Lehigh county resident than to those of, say, a representative English household. Given that, the failure of the neoclassical growth model to adequately explain the growth experiences of Pennsylvania counties is surprising. The values of the model's parameters are neither statistically nor economically significant.

Table 1  
Explaining Income Growth Rates

Dependent variable: log difference total earnings per worker 1970-1996

	Unconditional	Conditional on $s$ and $(n + g + \delta)$	Conditional on $s$ , $(n + g + \delta)$ , and COLLEGE
Constant	0.3213 (0.368)	0.4692 (0.513)	1.801 (1.535)
Log $y_{1970}$	-0.0362 (0.402)	-0.0059 (0.063)	-0.155 (1.236)
Log $s$		0.0061 (0.377)	0.0015 (0.095)
Log $(n + g + \delta)$		0.1508 (1.902)	0.0413 (0.414)
Log COLLEGE			0.0760 (1.770)
Adjusted $R^2$	-0.01	0.01	0.04
Standard error	0.10	0.10	0.09
Observations	67	67	67
Implied $\lambda$	0.0014	0.0002	0.0065

Note: Absolute values of t-statistics are in parentheses.  $Y_{1970}$  is total earnings per capita in 1970. The investment rate is for 1970 and the population growth rate is the average annual rate of employment growth for the period 1970-1996.  $(g + \delta)$  is assumed to equal 0.05. COLLEGE is the proportion of the county population of persons 25 years and over who had a bachelor's degree or higher in 1970.

Table 2  
Explaining Income Growth Rates, Restricted Regression

Dependent variable: log difference total earnings per worker 1970-1996

	Conditional on $s$ , $(n + g + \delta)$ , and COLLEGE
Constant	1.937 (1.643)
Log $y_{1970}$	-0.2034 (1.676)
Log $s - \text{Log}(n + g + \delta)$	-0.0056 (0.359)
Log COLLEGE - Log $(n + g + \delta)$	0.0916 (2.192)
Adjusted $R^2$	0.03
Standard error	0.09
Observations	67
p-value of test of restriction	0.003
Implied $\lambda$	0.0087
Implied $\alpha$	-0.02
Implied $\beta$	0.31

Note: Absolute values of t-statistics are in parentheses.  $Y_{1970}$  is total earnings per capita in 1970. The investment rate is for 1970 and the population growth rate is the average annual rate of employment growth for the period 1970-1996.  $(g + \delta)$  is assumed to equal 0.05. COLLEGE is the proportion of the county population of persons 25 years and over who had a bachelor's degree or higher in 1970.

Table 3  
Explaining Per Capita Income Level Differences

Dependent variable: log 1996 total earnings per worker

	basic neoclassical model	model augmented by the COLLEGE human capital variable
Constant	9.6723 (24.651)	9.3024 (38.854)
Log $s$	0.8540 (1.042)	0.7382 (1.494)
Log $(n + g + \delta)$	0.0034 (0.024)	-0.4055 (4.301)
Log COLLEGE		0.3768 (10.040)
Adjusted R <sup>2</sup>	-0.02	0.63
Standard error	0.16	0.10
Observations	60	60

Note: Absolute values of t-statistics are in parentheses.  $Y_{1970}$  is total earnings per capita in 1970. The investment rate is for 1992 and is equal to 1992 new capital expenditures divided by 1992 total earnings by place of work and the population growth rate is the average annual rate of employment growth for the period 1970-1996.  $(g + \delta)$  is assumed to equal 0.05. COLLEGE is the proportion of the county population of persons 25 years and over who had a bachelor's degree or higher in 1990.

Table 4  
Explaining Per Capita Income Level Differences, Restricted Regressions

Dependent variable: log 1996 total earnings per worker

	basic neoclassical model	model augmented by COLLEGE human capital variable
Constant	9.5758 (25.164)	9.221 (39.418)
$\text{Log } s - \text{Log } (n + g + \delta)$	0.0401 (0.299)	0.0649 (0.795)
$\text{Log COLLEGE} - \text{Log } (n + g + \delta)$		0.3783 (10.004)
Adjusted R <sup>2</sup>	0.00	0.62
Standard error	0.16	0.10
observations	60	60
p-value of test of restriction	0.48	0.008
Implied $\alpha$	0.04	0.05
Implied $\beta$		0.26

Note: Absolute values of t-statistics are in parentheses.  $Y_{1970}$  is total earnings per capita in 1970. The investment rate is for 1992 and is equal to 1992 new capital expenditures divided by 1992 total earnings by place of work and the population growth rate is the average annual rate of employment growth for the period 1970-1996.  $(g + \delta)$  is assumed to equal 0.05. COLLEGE is the proportion of the county population of persons 25 years and over who had a bachelor's degree or higher in 1990.

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