THE CONCEPT OF "MILITARY ECONOMIES OF SCALE" *

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Abstract

Military economies of scale exist if an increase of x percent in all inputs increases an army's destructive capability by more than x percent. Economies of scale did not exist in ancient and medieval warfare. Perceived instances of military scale economies are actually the expected outcome of the application of a superior weapon system. With the development of firearms, combat came to involve more than the front ranks alone. When every soldier is able to fire on every other soldier, an army's capability increases by the square of its size. By 1700, scale economies were available to all modern armies.

THE CONCEPT OF "MILITARY ECONOMIES OF SCALE"

I. INTRODUCTION

The concept of "military economies of scale" occupies a prominent position in many theories of grand strategy,[1] yet past usage of the term has been plagued by several difficulties. The term is often defined loosely and imprecisely or in a manner that is inconsistent with its use in the theory of the firm. Furthermore, the sources of military scale economies have not been adequately discussed. What causes sheer numbers of soldiers to have a disproportionate influence on military capabilities? Failure to properly address this question has led some researchers to detect military economies of scale where none, in fact, actually existed. Most importantly, the definition of military scale economies differs from researcher to researcher with the result that the concept has become hopelessly vague and nearly meaningless, sometimes amounting to nothing more than a tautology. The purposes of this paper are to provide a rigorous definition of the concept of "military economies of scale" that is consistent with its meaning in microeconomic theory, to describe the sources of these economies, and to establish their existence over time.

II. PITFALLS EXHIBITED IN THE LITERATURE

Lane (1942; 1958) argues that the violence-controlling, violence-using industry has a natural monopoly over a given area.

This implies the existence of military scale economies at the socially efficient level of protection. But, Lane does not discuss how or why these economies arise. He notes that changes in the art of war and in transportation, among other things, affect the economies and diseconomies of scale for violence-using enterprises, but he fails to define the concept and does not discuss why or how these factors would be expected to influence the degree of military economies of scale.

Some researchers use the term "military economies of scale" to refer to the decreasing monetary costs of defending a given area. Olson (1986, p. 125) notes that the per capita cost of providing a military capability of $200 billion is $10,000 for a population of 20 million, while for a population of 200 million it would be $1,000 per capita and concludes that "the economies of national scale in military power are staggering". Bean (1973) argues that military scale economies arise from the geographic fact that area expands faster than borders needing defense. However, these are both a misapplication of the concept of military economies of scale.

While it is hazardous to generalize about the purposes to which military power has been put, the preponderance of military force in ancient and medieval times was applied in pursuit of nonmilitary goals such as raids for booty, slaves, or captives for ransom. Such a nonmilitary strategy was not aimed at inflicting casualties on the enemy army. Modern military strategy, on the other hand, typically strives to gain political, economic, or other nonmilitary objectives through warlike means directed at the enemy armed forces. The ultimate purpose of modern military power is to inflict casualties on an adversary in battle. Even if one believes that the final output of the military sector is national security or protection, this output is produced by either inflicting casualties on adversaries or possessing the ability to do so. The concept of military economies of scale ought to refer, then, to the application of violence on the battlefield.

Dudley (1990, p. 244) measures the destructive power of an army by the ratio of casualties suffered by the two sides when a smaller force comes up against a larger one. Anything which either reduces the casualties an army will itself suffer or increases the number of casualties it can inflict contributes to military scale economies in Dudley's scheme. Dudley cites the Parthian victory over the Romans at Carrhae in 53 B.C. as evidence of the existence of military scale economies and diseconomies in ancient warfare. Crassus led an army numbering about 36,000, including 4,000 cavalry and 4,000 light infantry. The Parthians waited to engage the Romans until they were on the level, sandy terrain of Mesopotamia. Crassus deployed his army in a square, but the Parthians did not charge with their heavy cavalry. Instead, the Parthian light cavalry circled the Roman heavy infantry square and began to shoot arrows at them.

> "The Parthians now placing themselves at distances began to shoot from all sides . . .

> The position of the Romans was a very bad one from the first; for if they kept their ranks, they were wounded, and if they tried to charge, they hurt the enemy none the more, and themselves suffered none the less. For the Parthians threw their darts as they fled . . ."[2]

Most of Crassus' army were either killed or captured. According to Dudley, innovations in cavalry warfare in the form of stronger horses and the saddle had rendered heavy cavalry more effective by making it easier for small contingents of cavalry to inflict casualties on larger numbers of infantry. Therefore, there were increasing returns to the use of heavy cavalry and decreasing returns in the application of heavy infantry on the battlefield. The rising costs of defending the existing borders of the Roman empire with heavy infantry led to a shrinkage of the empire's effective frontiers.

Dudley's use of the concept of military scale economies is subject to two immediate objections. First, although it measures an army's destructive capability, it refers to how a change in the mix of military inputs affects the ratio of casualties not to how an increase in all military inputs affects the casualty ratio. This usage is inconsistent with the concept's customary microeconomic definition. Second, Dudley confuses military economies of scale with a superior weapon system. The Parthian victory at Carrhae was not due to military scale economies in the use of cavalry. Rather, it was the expected outcome of an encounter between the light cavalry weapon system and heavy infantry.

Dudley stresses the role of the Parthian heavy cavalry in forcing the Roman infantry into tight formations where they made easier targets for the mounted archers. Yet, without their light cavalry the Parthians could not have prevailed since disciplined heavy infantry can successfully withstand a heavy cavalry charge. "Against skillful Roman soldiers, thoroughly armored, protected by large, strong shields, and formed six ranks deep, even the armored Parthian heavy cavalry with their long lances could not prevail."[3] "The heavy cavalry aided the Parthian victory; but with sufficient arrows, which their commander had carefully provided, their light cavalry could have won the battle unaided."[4]

Light cavalry constitutes a superior weapon system when pitted against heavy infantry because the cavalry is able to keep its distance and destroy the enemy with its missiles. The superior mobility of cavalry enables it to avoid a charge by heavy infantry so that heavy infantry has no means of inflicting damage upon light cavalry. The tactical power of light cavalry against heavy infantry had been demonstrated long before the Roman debacle at Carrhae.[5] In 328 B.C., Alexander the Great sent an expedition consisting of about 800 light cavalry and 1,500 heavy infantry to relieve the besieged town of Maracanda. The Sogdianian rebels retreated, pursued by the Macedonians. The rebels made a stand on a level plain near the desert. When the Macedonian infantry attacked, the rebel light cavalry "rode round and discharged arrows at the phalanx of infantry."[6] The Macedonian cavalry was too exhausted by the pursuit to play any role in the battle. The horse archers attacked the Macedonians "whether they halted or retreated. Many of them were wounded by the arrows, and some were killed."[7] The Macedonians formed a square and retreated into a river where soon "they were entirely surrounded by the [rebel] cavalry,[8] and all killed with arrows, except a few of them, whom they reduced to slavery."[9]

Light infantry, however, does possess the means to inflict damage on light cavalry. In fact, light infantry is a superior weapon system when matched against light cavalry because the earth provides a more stable platform for firing a bow or launching any type of missile than does a horse. The foot archer could shoot with greater speed and accuracy than the mounted archer.

The Roman light infantry at Carrhae contained few archers or slingers and, outnumbered by the Parthian horse archers with their more powerful bows, failed to affect the outcome at all. But, in 38 B.C., a Roman army in Syria containing a large number of slingers decisively defeated the Parthians.

> "The Parthian horsemen, accustomed to arrow practice without reprisal fire, became aware that the legion had developed a sting. Horsemen began to go down under the

> relentless shower of stones. Stunned, riderless horses screamed as they plunged on the field. The Parthian arrow flow slackened as it drew effective counterfire. Accustomed to deliberate bow practice at the expense of the legion, they began to grow unsteady."[10]

The outcome of this battle demonstrates that if the Roman army at Carrhae had contained a larger proportion of foot archers, the Romans would have been able to withstand the Parthian attack.[11]

Even more than fourteen hundred years later, light infantry was able to demonstrate its innate superiority over light cavalry. In 1385, the King of Castile invaded Portugal to put down a revolt against a union with Spain. The Portuguese and their English allies offered battle at Aljubarotta. The Portuguese crossbowmen and English longbowmen were stationed behind barriers on the flanks of dismounted heavy cavalry. This army thwarted both mounted and dismounted Castilian attacks. The Castilian javelin-armed light cavalry "suffered very severely from the flanking fire of arrows, bolts, and javelins. So many horses were shot down that 'in forty places the ravine was passable over their heaped-up carcasses.'"[12]

Assuming a featureless plain and soldiers of comparable quality, light cavalry in ancient and medieval warfare is tactically superior to heavy infantry while light infantry is superior to light cavalry.[13] It was noted above that heavy infantry constitutes a superior weapon system when matched against heavy cavalry. Although heavy infantry lacks the mobility to force combat on cavalry, disciplined, properly formed heavy infantry can resist a heavy cavalry charge. "Depth, the better fighting platform of the earth, and the ability to give undivided attention to the combat . . . meant that cavalry could not match the coordinated group action of foot soldiers."[14] In 552, a Byzantine army was sent to reconquer Italy from the Goths. The two armies met at Taginae. Narses, the Byzantine commander, sent fifty heavy infantry to occupy a hill on his left flank. They took up position on a path, "standing shoulder to shoulder and arrayed in the form of a phalanx . . ."[15] A contingent of Gothic heavy cavalry attempted to dislodge them, "but the Romans drew up together into a small space and, making a barrier with their shields and thrusting forward their spears, held their ground."[16] The Goths had difficulty managing "horses that did not in the least obey their urging"[17] and were repulsed several times by the heavy infantry "packed so closely together and not giving an inch of ground."[18]

The introduction of the stirrup did not alter the fundamental relationship between heavy infantry and heavy cavalry. White's (1962, p. 37) claim that the Norman heavy cavalry made William the Conqueror's victory at Hastings an inevitability is not supported by the course of the battle. The English heavy infantry withstood numerous cavalry charges. The Norman light infantry was the decisive arm. The Norman heavy cavalry became effective only after the archery attack had thrown the English ranks into disarray.

Light infantry is a superior weapon system relative to heavy infantry. "On a billiard table terrain of infinite extent, light infantry could defeat heavy infantry if it had enough missiles and patience and adhered to its light infantry tactics of avoiding shock combat in which the heavy infantry specialized."[19] The Athenian heavy infantry was decimated by light infantry during an invasion of Aetolia during the Peloponnesian War. "[The Aetolians] came running down from the hills on all sides, hurling their javelins, falling back whenever the Athenian army advanced, and coming on again as soon as it retired. . . . [T]he soldiers [became] tired out with having constantly to make the same wearisome manoeuvres; the Aetolians pressed hard upon them with their volleys of javelins, so that finally they turned and fled."[20] David's victory against Goliath is an even more vivid demonstration of the ability of the missile-equipped light infantryman to defeat heavy infantry that relied on shock combat. "The Philistine then moved to meet David at close quarters . . . David put his hand into the bag and took out a stone, hurled it with the sling, and struck the Philistine on the forehead. . . . Thus David overcame the Philistine with sling and stone; he struck the Philistine mortally, and did it without a sword."[21]

Light infantry in ancient and medieval warfare was decidedly inferior to heavy cavalry. Protected by armor, heavy cavalry can quickly move through the area in which the arrows fall and use the lance and sword to run down the lightly armored foot archers.

The English light infantry was deployed on the right flank against the Scots during a battle in 1314. The Scottish King Robert sent his reserve of 500 heavy cavalry against them. "They charged in diagonally upon the bowmen, and scattered them with great loss . . ."[22]

Light cavalry is just slightly more mobile than heavy cavalry, but the difference is sufficient to enable light cavalry to refuse combat with heavy cavalry while continuing to rain missiles upon them. At the Battle of Manzikert in 1071, the Byzantine heavy cavalry advanced against the Turkish horse archers. The Turks retreated while continuing to shower the Byzantine cavalry with arrows. Many Byzantine mounts were killed and wounded. The Turks were driven back to their camp and even beyond. Commenting on Byzantine strategy, Fuller (1954, vol. 1, p. 402) notes that this advance "was an unwise move . . . because the Turks had unlimited ground in which to fall back, and unless Romanus [the Byzantine emperor] could reach some spot where water was to be found for his men and horses, he would be compelled to retire and then most certainly would be counter-attacked." As twilight approached, the Byzantine emperor ordered a retreat, but the wings did not understand the signal and the ranks fell into disorder. The Turkish counterattack destroyed the Byzantine army.

To sum up the merits of the four basic weapon systems of ancient and medieval combat, each having its own special capabilities and relative superiorities: light cavalry is superior to both heavy infantry and heavy cavalry, heavy infantry is superior to heavy cavalry, heavy cavalry is superior to light infantry, and light infantry is superior to heavy infantry and light cavalry.[23]

The size of armies mattered little through medieval times; what was important was the combination of weapon systems employed. The casualty ratio obtained by a force of heavy infantry facing another group of heavy infantry may be, say, one to one. An army composed of light cavalry may be able to inflict fifty casualties against heavy infantry for every one it suffers. But, one cannot then conclude that there are economies of scale in the application of light cavalry because if the light cavalry had instead been opposed by light infantry, the casualty ratio may have been the reverse: light cavalry suffering fifty casualties for every one it inflicts. Thus, as defined by Dudley (1990), the existence of military economies of scale is entirely dependent on the combination of weapon systems engaged.

The Parthians attempted to follow up their victory at Carrhae by invading Syria. But, while "they are almost invincible in their own [flat, treeless] country and in any that has similar characteristics",[24] the dense Syrian forests rendered their cavalry useless. The Parthians even contemplated cutting down a forest so their cavalry could approach one Syrian town. If military scale economies is to be a useful concept they should not disappear when facing a superior weapon system or exist only on certain types of terrain.[25]

A definition of the concept "military economies of scale" must (1) describe the output response to a proportionate increase of all military inputs where (2) the output of the military sector is the number of casualties it is able to inflict in battle and (3) be independent of specific circumstances such as the combination of weapon systems employed or the terrain of the battlefield.

III. MILITARY SCALE ECONOMIES DEFINED

Economies of scale exist if an increase of x percent in all inputs leads to an increase of more than x percent in output. Thus, if a doubling of the size of a military force more than doubles the number of casualties it is able to inflict, then military economies of scale can be said to exist. More formally, let z represent a vector of inputs into the military's production function and assume that output is the capability to inflict casualties. A military production function, $f(z)$, exhibits economies of scale if

$$(1) \quad f(tz) > tf(z) \quad \text{for all } t>1.$$

Military economies of scale did not exist to an appreciable extent in ancient times when one heavy infantry phalanx battled another. The number of men actually wielding their weapons at any given instant was roughly the same on both sides as long as the lines remained unbroken. Large numbers did not provide a disproportionate advantage as skill counted more than numbers.[26] One man would find himself opposed to one man and the battle, in essence, consisted of a number of individual duels.[27] Adding more soldiers to the phalanx did not increase an army's destructive power. Assuming that both armies were equally skilled, armies could expect to suffer similar casualties. Sparta and Argos once agreed to settle a quarrel over some disputed territory by having three hundred picked men a side fight it out. "So closely was it contested that of the six hundred men only three were left alive - two Argives . . . and one Spartan . . ."[28]

Competent generals anchored their army's flanks on obstacles so that fronts remained equal. Lopsided casualty figures usually resulted only when an army's line was broken or when they were outflanked and surrounded. Thus, at the Battle of Granicus, the Macedonian heavy cavalry attacked the rear and flanks of the Persian heavy infantry inflicting perhaps 20,000 casualties while suffering only 115.[29] Hannibal attempted to encircle the 70,000 man Roman army at Cannae with only 50,000 men. The Carthaginian light infantry attacked the flanks of the Roman infantry while the light and heavy cavalry attacked the rear. 48,000 Romans were killed while Carthaginian casualties numbered at least 5,700.[30]

The ability of an army to inflict casualties using the tactics and weapons of ancient and medieval warfare was directly proportional to its size. The military production function took the form

$$(2)\quad f(z) = az$$

and exhibited constant returns to scale. A larger army possessed the capability to inflict more casualties than an equally skilled smaller army, but not disproportionately more. Economies of scale did not exist in ancient and medieval combat. The absence of scale economies on the battlefield did not dissuade states from fielding large armies because a large army would defeat an equally skilled but smaller army and sometimes even a more skilled but smaller army.

Skill mattered more than numbers up through the sixteenth century until the improvement of firearms.[31] Missile weapons enable two or more soldiers to simultaneously attack a single enemy soldier. Unlike the bow and arrow, the musket was capable of piercing any personal armor at a distance of 80 meters. Thus, unlike the archer, the musketeer possessed the ability to kill any enemy soldier within range on the battlefield.

The use of muskets and artillery gradually came to involve more than the front ranks alone. Turning and enveloping movements extended battle lines enabling an army to bring a larger proportion of its men into action at any given time. With long range missile weapons, the numerically inferior force finds itself under a far heavier fire than it is able to return. Under modern combat conditions, the concentration of superior numbers gives an immediate superiority in the number of active combatants.

This insight about the advantage conferred by numerical superiority when troops have missile weapons is embodied in F.W. Lanchester's (1916) N-square law. The number of casualties a force suffers per unit of time will be directly proportional to the numerical strength of the opposing force. Let a equal the numerical strength of army A and b represent the same for army B. Then,

(3) $da/dt = -bc$ and $db/dt = -ak$,

where c and k are constants representing the fighting values of the individual units of the force or the efficiency of the weapon system and t stands for time. The two armies will suffer losses of equal proportion when

(4) $(da/dt)/a = (db/dt)/b$.

This simplifies to

(5) $ka^2 = cb^2$,

which states that "the fighting strengths of the two forces are equal when the square of the numerical strength multiplied by the fighting value of the individual units are equal."[32]

This leads to the conclusion that the fighting strength of a military force is proportional to the square of its numerical strength multiplied by the fighting value of its individual units. This is Lanchester's N-square law. To see how this result comes about suppose that force A has 4,000 men and B has 1,000. Assuming that the forces are equal in skill, B will receive four times the bullets and suffer four times the casualties compared to A. In the first moment of combat A will lose one soldier while B will have lost four. The ratio between the two forces is no longer 4 to 1: A would have 3,999 and B 996. The ratio is now 4.015 to 1. The larger army's relative advantage becomes cumulative as the battle progresses. In the last moment of combat, force A will possess a relative advantage of around 70 to 1. A will eventually destroy all of B's force, but will suffer only about 130 casualties not the 1,000 expected under conventional shock combat.

The major assumption underlying this result is that every soldier could combat every other. That is, the law assumes that all of the members of the force are able to fire on the opposing army. This assumption obviously lacked reality when the ancient Greeks fought one another hand to hand in deep formations. Even eighteenth century musketeers sometimes failed to meet this assumption. But, with the use of long-range weapons using indirect fire, warfare began to conform to Lanchester's assumption. In addition, the law assumes that both forces are on the offensive on a level, plain battlefield. The law could be modified to take account of the augmented effectiveness of forces fighting on the defensive in favorable terrain.[33]

At the Battle of Albuera in 1811, the British had an average of 1,850 men firing against 800 French.[34] The British suffered 1,150 casualties attributable to musket fire. This amounts to a little over half of the 2,000 French losses attributable to small arms fire. Nelson was able to pit twenty-seven of his ships against twenty-three French vessels at Trafalgar.[35] The British advantage according to Lanchester's law was 729 to 529. The tactics successfully used by the British suggest that "Nelson, if not actually acquainted with the n-square law, must have had some equivalent basis on which to figure his tactical values."[36]

There are several other examples of the workings of Lanchester's N-square law. Lee faced McClellan's 70,000 effectives at Antietam with about 50,000 troops. Union losses were 12,400; the Confederates lost 13,700.[37] During the Franco-Prussian War, a Prussian army of 89,000 suffered casualties of 10,500 at Froeschwiller while the French had 11,000 killed and wounded and 9,000 taken prisoner out of a force of 42,000.[38] In a battle off Port Arthur during the Russo-Japanese War, the Russians had disabled five of the seventeen heavy guns in the Japanese fleet while losing only four of their own twenty-three big guns.[39] The Russian heavy gun superiority was becoming cumulative just as the N-square law predicts, but the Russian admiral was killed and his flagship damaged. This threw the Russian fleet into confusion. The German fleet was able to concentrate sixteen guns against only two British long range guns at the Battle of Coronel in 1914. The Germans suffered no losses while sinking two British vessels.[40] The losses suffered by the French and Germans during the tank battles of May 1940 are in line with the predictions of Lanchester's law.[41]

IV. IMPLICATIONS OF THE N-SQUARE LAW FOR SCALE ECONOMIES

Lanchester's N-square law implies that any commander able to concentrate his forces is able to attain an advantage that is more than proportional to his numbers. The N-square law can be used to specify the military production function. When the assumptions underlying the law are satisfied, the military production function takes the form

$$(6)\quad f(z) = z^2.$$

Suppose that the quantity of military inputs is multiplied by a scalar $t>1$. Then,

$$(7)\quad f(tz) = t^2z^2$$

and

$$(8)\quad tf(z) = tz^2.$$

Since $f(tz)$ is greater than $tf(z)$, the military production function will exhibit economies of scale when the N-square law is valid. This means that the doubling of the size of an army more than doubles its fighting capability. In fact, any increase in the size of an army causes a more than proportional increase in its ability to inflict casualties. Therefore, military economies of scale are available to all modern armies.

Modern warfare continues to approach the conditions necessary for the N-square law to be totally valid. Longer range weapons and dispersed armies have brought combat closer to the situation in which each soldier can shoot at every enemy soldier. The perfection of heat sensing weaponry and more powerful missile projectiles renders traditional methods of concealment useless and will continue the trend towards the case in which every weapon can fire on every other weapon, thereby generating additional scale economies on the battlefield.

V. DATING THE EMERGENCE OF MILITARY SCALE ECONOMIES

Economies of scale did not exist in ancient and medieval warfare. The number of soldiers on each side actually wielding their weapons was roughly equal at any given moment. Only the front ranks were able to combat the opposing force. Also, some weapon systems were ineffective against other systems. Since its basic assumption that every soldier can fire on every other soldier at any given instant is violated by the weapons and tactics of ancient and medieval combat, Lanchester's N-square law does not apply to ancient and medieval warfare. A larger army would defeat an equally skilled, but smaller army using the same weapon system. However, both armies would suffer similar numbers of casualties. Large numbers did not bring a disproportionate advantage in the ability to inflict casualties. There existed positive returns to quality and skill but not to scale in ancient and medieval combat.

Firearms were responsible for the generation of military scale economies on the battlefield. Contrary to Blum and Dudley (1989), artillery was not a significant source of military economies of scale. Losses due to cannon fire were often very slight. At the battle of Fornovo in 1495, an eyewitness estimated that fewer than ten men on both sides were killed by artillery fire, while total losses neared 4,000.[42] The basic assumptions of Lanchester's N-square law began to be satisfied in the early 1500's with the introduction of reliable and effective firearms. The linear formations devised in the 1590's by Maurice of Orange allowed a greater proportion of an army to fire on the opposition at a given moment. By the turn of the seventeenth century, musketeers had driven all but the pikeman from the battlefield and, by mid century, the ratio of muskets to pike in most units was three or four to one.[43] The pikeman was made obsolete by the development of the socket bayonet. The socket bayonet was in universal use by 1700.[44] It was at this point that all soldiers possessed the potential to simultaneously and successfully combat the enemy. Thus, the assumptions of Lanchester's N-square law were at last met. By 1700, significant military economies of scale were available on the battlefield.

Nations responded to the existence of these economies of scale by fielding larger armies in an attempt to capture them. Battlefield size grew fast in the seventeenth century. "[T]he rise in combatants is obvious when one compares battles like Pavia (1525) and Nieuwpoort (1600), with 10,000 combatants on either side, and a battle like Malplaquet (1709), with 200,000 men involved."[45]

VI. CONCLUSIONS

Military economies of scale exist if an increase of x percent in all inputs increases an army's destructive capability by more than x percent. Defined in this manner, economies of scale did not exist in ancient and medieval warfare. What are perceived as instances of military scale economies are actually the expected outcome of the application of a superior weapon system. The military production function exhibited no more than constant returns to scale. With the development of firearms, combat came to involve more than the front ranks alone. When two or more soldiers are able to simultaneously combat a single enemy soldier, there exist scale economies in the application of military force on the battlefield. Technological advances and tactical innovations over the last several hundred years have gradually enabled a larger proportion of an army to combat the enemy at any given moment. Sheer numbers of soldiers began to matter. When every soldier can fire on every other soldier, the larger army has a larger number of active combatants and is able to inflict a larger number of casualties. Lanchester's N-square law implies that an army's destructive capability increases by the square of its size. Tactical innovations such as the widespread adoption of linear formations and technological advances in the form of more powerful and reliable firearms and the introduction of the bayonet began to bring combat into line with the assumptions of the N-square law during the seventeenth century. By 1700, nations were fielding large armies in an attempt to capture the significant scale economies available on the battlefield. The assumptions of the law are coming closer and closer to being fully satisfied. Under conditions of modern combat all nations have access to military economies of scale on the battlefield. The important thing is being able to exploit these potential scale economies.

FOOTNOTES

1. A recent example is Wittman (1991). The term "grand strategy" refers to "the overall plan for defending the security and integrity of the state, including when necessary or desirable the expansion of territory over which the state rules. Diplomatic and economic means as well as military may be used in the implementation of a state's grand strategy" (Ferrill, 1985, p. 44).

2. Plutarch (no date, pp. 666-667).

3. Jones (1987, p. 39).

4. Jones (1987, p. 38).

5. And long before the saddle which Dudley (1990) partially credits with strengthening the relative position of cavalry was introduced into the West.

6. Arrian, IV, ch. 5 (1942, vol. 2, p. 505).

7. Arrian, IV, ch. 5 (1942, vol. 2, p. 505).

8. Note that heavy cavalry played no role whatsoever in the Sogdianian victory.

9. Arrian, IV, ch. 5 (1942, vol. 2, p. 505).

10. Hurley (1975, p. 107).

11. The Romans would, however, have only had a defensive capability because they would still have lacked the means to force combat on the Parthians.

12. Oman (1969, vol. 2, p. 194).

13. The best discussion of the relative merits of the four weapon systems and which I have heavily relied upon is in Jones (1987) especially pages 39-45, 144-147, 195-199, and 622-630.

14. Jones (1987, p. 39).

15. Procopius, VIII, ch. 29 (1954, vol. 5, p. 357).

16. Procopius, VIII, ch. 29 (1954, vol. 5, p. 357).

17. Procopius, VIII, ch. 29 (1954, vol. 5, p. 359).

18. Procopius, VIII, ch. 29 (1954, vol. 5, p. 359).

19. Jones (1987, p. 20).

20. Thucydides, III, 97-98 (1954, p. 252).

21. 1 Samuel 17, 48-50.

22. Oman (1969, vol. 2, p. 95).

23. The tactical capabilities of the four weapon systems discussed above are valid through medieval times. Heavy and light cavalry merged into a dual purpose cavalry called reiters in the sixteenth century. Reiters carried swords for shock combat and pistols for missile warfare. Reiters were tactically superior to both heavy and light infantry. The development of the socket bayonet made the pikeman obsolete by combining the attributes of light and heavy infantry in one soldier. By the turn of the nineteenth century, cavalry was inferior to infantry. Infantry's muskets dominated cavalry's pistols while a line of musketeers with bayonets could withstand a cavalry's charge with sabres. The development of more powerful and faster loading rifles drove cavalry off the battlefield by the end of the century. The traditional relationships among the four weapon systems returned in the 1930's with antitank guns being the modern equivalent of heavy infantry, tanks as heavy cavalry, aircraft as light cavalry, and antiaircraft guns as light infantry, but with the added complications of regular infantry and artillery and the fact that "inferior" weapons could successfully combat other systems.

24. Dio Cocceianus, XL, 15.4-5 (1954, vol. 3, p. 427).

25. Blum and Dudley's (1989) contention that military scale economies were responsible for Charles VII's dramatic successes against the English and French nobles is subject to a similar objection. Charles VII relied upon artillery and, during one sixteen month period, conducted sixty successful siege operations (Brodie and Brodie, 1973, p. 51). While they are undoubtedly correct in attributing this success to his artillery, they confuse economies of scale with a superior weapon system. The majority of strongholds which came under siege dated from the times before gunpowder. By 1530, however, fortifications were built that could withstand artillery (Duffy, 1979, pp. 15-22). Low, sloping dirt ramparts, unlike stone walls, were capable of absorbing cannonballs with little damage. The superiority of a weapon system ought to be and has been affected by such things as technological change, the terrain, or the weapon system of an adversary. Economies of scale, to which the fall of the Roman empire (Dudley, 1990) and the expansion of the power of the French monarchy (Blum and Dudley, 1989) are attributed, ought not to be so ephemeral as to disappear before mounds of dirt.

26. Skill refers not only to that of the individual soldier, but also to the tactical skills of the commander such as exploiting a superior weapon system or turning the enemy's flank.

27. "Meanwhile the plain was filled by human combatants and sparkled with the bronze of infantry and horse. The earth shook beneath their feet as the two forces rushed towards each other. And now . . . their two great champions, Aeneas son of Anchises and the godlike Achilles, came together bent on single combat . . ." (Homer, XX, 1950, p. 370).

28. Herodotus, I, 82 (1954, p. 73).

29. Delbruck (1975, vol. 1, p. 189).

30. Delbruck (1975, vol. 1, p. 320).

31. Firearms were initially less accurate than bows because the ball fitted only loosely in the barrel of the arquebus (Jones, 1987, p. 154). In addition to this inherent inaccuracy of the musket, "the dense clouds of white smoke that gunpowder produced when it exploded" (Hughes, 1974, p. 64) further inhibited the accuracy of the firearm. But, accuracy was not important on a battlefield full of massed infantry formations. Therefore, weapons designers stressed speed and reliability in loading so as to increase the rate of fire. Since aiming was not a factor in hand gun training an individual could become an effective musketeer after a fairly short period of training (O'Connell, 1989, p. 111).

32. Lanchester (1916, p. 48).

33. Men on the defense have the advantage of maintaining continuous fire without the interruption of the advance. The assumption that soldiers can fire on every enemy soldier is violated if the enemy is concealed behind ditches and walls, in shell holes, et cetera.

34. Jones (1987, p. 374).

35. Jones (1987, p. 379).

36. Lanchester (1916, p. 66).

37. Dupuy and Dupuy (1970, p. 879).

38. Jones (1987, p. 400).

39. Jones (1987, p. 430).

40. Dupuy and Dupuy (1970, p. 946).

41. Jones (1987, p. 532).

42. Contamine (1984, p. 200).

43. Parker (1988, p. 18).

44. Jones (1987, p. 268).

45. Parker (1976, p. 207).

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