MATH 597 : Dynamical Systems on Nilmanifolds

Instructor: Zhiren Wang
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Lectures: 256 HHD Building, MWF 9:05-9:55am
Textbook: Lecture notes will be provided by the instructor
Topics:

Nilmanifolds are compact quotients of simply connected nilpotent Lie groups. Maps on nilmanifolds provide a rich source of examples in modern dynamical systems. Properties of dynamical systems differ a lot depending on whether the systems has zero entropy or positive entropy. Surprisingly, in both these categories, some of the most important constructions are built on nilmanifolds. In the zero entropy category, translations on nilmanifolds are basic examples of distal systems and Ratner's property. On the other hand, in the category of hyperbolic dynamics, which have positive entropy, it was conjectured that all Anosov maps take place on nilmanifolds and their finite-to-one covers. Studies of dynamics on nilmanifolds also have profound applications in several areas including hyperbolic dynamical systems, additive combinatorics and ergodic theory.

In this course, we shall discuss the following topics:

- Basic properties of nilpotent Lie groups and nilpotent Lie algebras
  - Baker-Campbell-Hausdorff formula
- Lattices of nilpotent Lie groups and nilmanifolds
  - Mal'cev basis, rationality of lattices
- Translations on nilmanifolds (nilsystems)
  - Criterion for ergodicity and unique ergodicity
  - Measure classification for non-ergodic maps
- Automorphism of nilmanifolds
  - Criterion for ergodicity and mixing
  - Calculation of entropy

If time allows, we shall choose some of the following topics to discuss:

- Relation to hyperbolic dynamics
  - Anosov diffeomorphism, Franks-Manning theorem
  - Some partial results on Franks' conjecture (Brin-Manning, Hammerlindl)
- Relation to additive combinatorics and ergodic theory
  - Szemeredi's theorem and Kronecker factors
  - Multiple ergodic averages and nilfactors (Host-Kra)
- Relation to number theory
  - Mobius disjointness for nilsequences (Green-Tao)

Homeworks and exams: There will be 5 homework assignments, including 10 problems in total. Each problem will count for 10% of the final grade. There will be no exams.

You may discuss with other students while working on the homeworks, but must write up the solution by yourself.