Name: ______________________________ ID: ______________

1. Consider the surface defined by the equation

$$x^2 + y^2 - 2x - 6y - z + 10 = 0.$$

(a) Find the center or vertex of the surface by completing the square to identify it as a translated version of a surface with center/vertex at the origin.

**Solution:** Completing the square yields

$$\begin{aligned}(x-1)^2 - 1 + (y-3)^2 - 9 - z + 10 = 0,\\ (x-1)^2 + (y-3)^2 = z.\end{aligned}$$

The surface is centered at $(1, 3, 0)$.

(b) Identify the horizontal traces of the surface in the planes $z = k$. If the answer depends on the value of $k$, be sure to specify which values of $k$ give which answer.

**Solution:** Plugging in $z = k$, we find that

$$(x-1)^2 + (y-3)^2 = k.$$

This is the equation of a circle if $k > 0$. If $k = 0$, then it is the equation of the single point $(1, 3)$, so the trace contains a single point. If $k < 0$ then there are no solutions to the equation, so the trace is empty.

(c) Identify the vertical traces of the surface in the planes $x = k$. If the answer depends on the value of $k$, be sure to specify which values of $k$ give which answer.

**Solution:** Plugging in $x = k$, we find that

$$(k-1)^2 + (y-3)^2 = z.$$

This is the equation of a parabola for all values of $k$.

(d) Identify the vertical traces of the surface in the planes $y = k$. If the answer depends on the value of $k$, be sure to specify which values of $k$ give which answer.

**Solution:** Plugging in $y = k$, we find that

$$(x-1)^2 + (k-3)^2 = z.$$

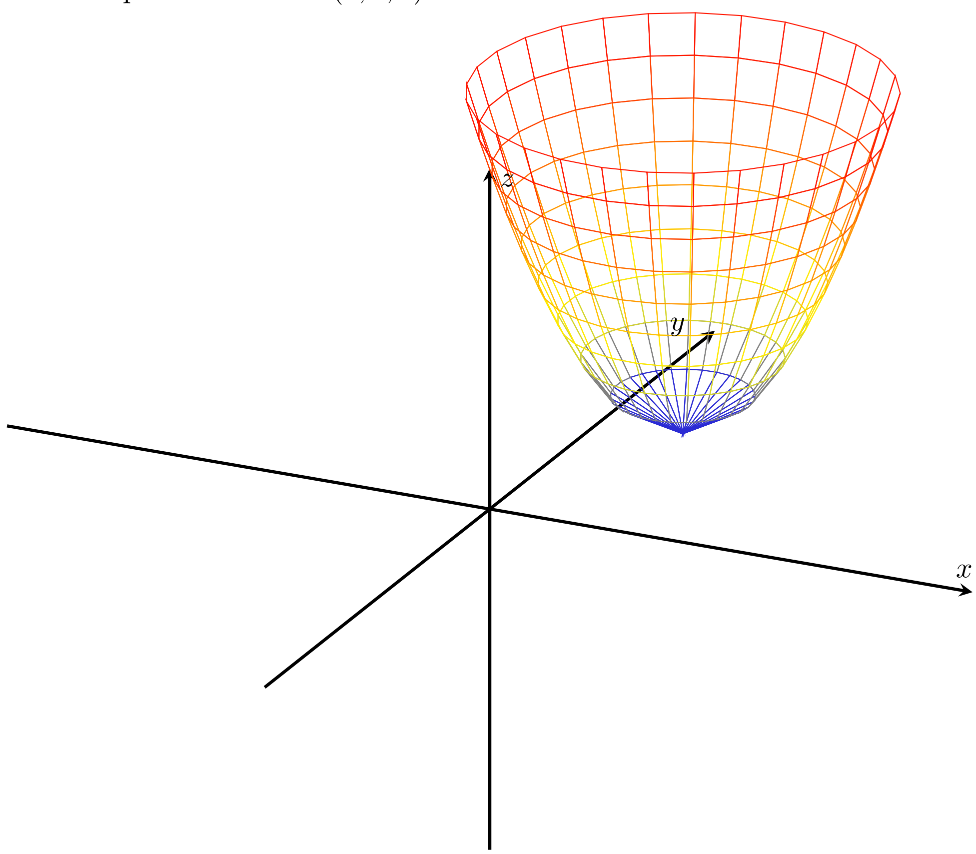
This is the equation of a parabola for all values of $k$.

(e) Identify the surface.

**Solution:** A surface with parabolic and elliptic traces is an elliptic paraboloid.

(f) Sketch the surface. Your sketch does not need to be quantitatively correct, but it should show the correct type of surface in the correct location with the correct orientation. If you feel like you need to, feel free to write a sentence to clarify the location and orientation.

**Solution:** Based on the traces with $z = k$, we see that the paraboloid opens in the positive $z$ direction. Indeed, when $k < 0$, our traces are empty, when $k = 0$, our trace is a point, and when $k > 0$, our traces are circles. Based on our answer above, the vertex of the paraboloid is at $(1, 3, 0)$.



2. Consider the curve defined by the equation

$$\mathbf{r}(t) = \left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle.$$

Find the unit tangent vector $\mathbf{T}(t)$. Be sure to simplify.

**Solution:** We compute

$$\begin{aligned}
\mathbf{r}'(t) &= \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle, \\
\left|\mathbf{r}'(t)\right| &= \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}, \\
\mathbf{T}(t) &= \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{e^t + e^{-t}} \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle.
\end{aligned}$$