Name: ______________________________ ID: ______________

1. Consider the triangle determined by the points $P(2,-1,0)$, $Q(4,1,1)$, and $R(4,-5,4)$. Compute the lengths of the sides of this triangle. Is it a right triangle? Is it an isosceles triangle?

Solution: Using the distance formula, we compute that

$$\begin{aligned} |PQ| &= \sqrt{2^2+2^2+1^2} = 3, \\ |QR| &= \sqrt{0^2+6^2+3^2} = 3\sqrt{5}, \\ |RP| &= \sqrt{2^2+4^2+4^2} = 6. \end{aligned}$$

These side lengths are all different, so the triangle is not isosceles. We can compute that $3^2+6^2 = (3\sqrt{5})^2$, so the triangle is a right triangle by the Pythagorean theorem.

2. Consider the line passing through the points $A(2,4,-3)$ and $B(3,-1,1)$.

   (a) Find parametric equations and symmetric equations of the line.

   Solution: We can compute a vector parallel to the line to be

   $$\mathbf{v} = \overrightarrow{AB} = \langle 1, -5, 4 \rangle.$$

   We can use either A or B as our start point. Using A, we obtain the parametric equations

   $$x = 2 + t, \qquad y = 4 - 5t, \qquad z = -3 + 4t.$$

   Isolating t, we obtain the symmetric equations

   $$x - 2 = -\frac{y-4}{5} = \frac{z+3}{4}.$$

   (b) At what point does this line intersect the xy-plane?

   Solution: The xy-plane contains the points with $z = 0$. Thus, $0 = -3 + 4t$, from which we find that $t = \frac{3}{4}$. Plugging that in to our equations for x and y, we find that $x = \frac{11}{4}$ and $y = \frac{1}{4}$. Thus, our intersection point is $\left(\frac{11}{4}, \frac{1}{4}, 0\right)$.