

Math 253A Midterm 1

February 13, 2020

Name: _____ ID: _____

- Each page has a space at the top for the last 4 digits of your student ID. Make sure that you fill that out on at least one side of every sheet of paper.
- Show enough work that your solution would convince a skeptical peer that your answer is correct.
- The questions are ordered by topic, not by difficulty.
- Each question is worth the same number of points.
- You may not use any tools or resources other than writing implements. In particular, no calculators, phones, notes, and so forth.

1. What does the equation $x = 4$ represent in \mathbb{R}^2 ? What does it represent in \mathbb{R}^3 ? Illustrate with sketches.

2. Let \mathbf{v} be the vector in the first quadrant that makes an angle of $\pi/3$ with the positive x -axis and has magnitude $|\mathbf{v}| = 4$. Write down \mathbf{v} in component form.

3. Let $\mathbf{a} = \langle -1, 4, 8 \rangle$ and let $\mathbf{b} = \langle 12, 1, 2 \rangle$.
- (a) Find the scalar projection of \mathbf{b} onto \mathbf{a} .

- (b) Find the vector projection of \mathbf{b} onto \mathbf{a} .

4. Consider the four points $P(-2, 1, 0)$, $Q(2, 3, 2)$, $R(1, 4, -1)$, and $S(3, 6, 1)$. Find the volume of the parallelepiped with adjacent edges PQ , PR , and PS .

5. Find the equation of the plane that contains the line

$$x = 1 + t,$$

$$y = 2 - t,$$

$$z = 4 - 3t$$

and is parallel to the plane

$$5x + 2y + z = 1.$$

6. Consider the surface defined by the equation

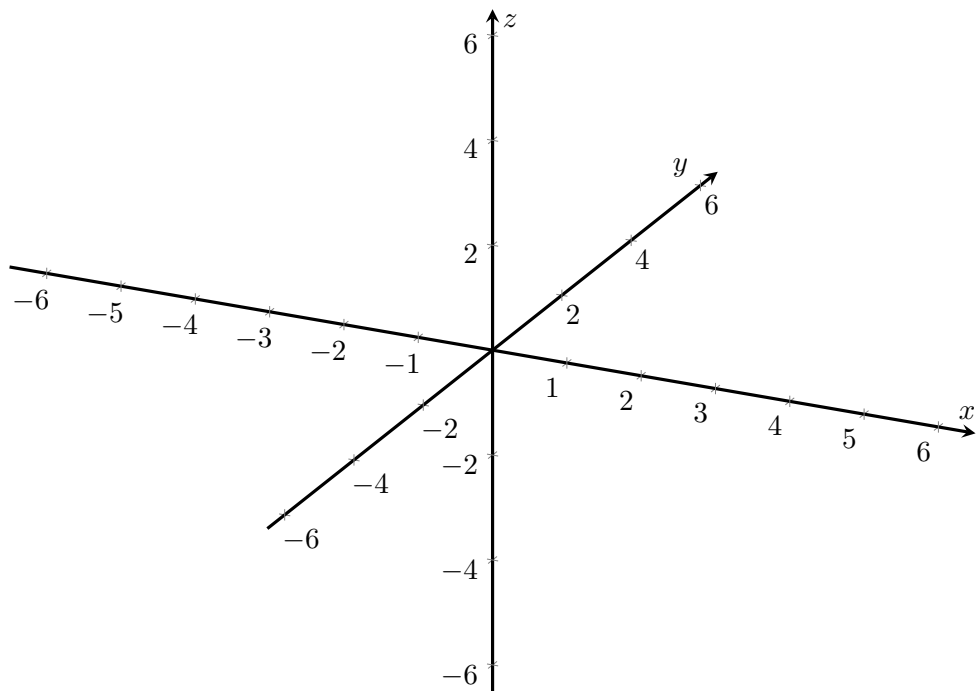
$$-4x^2 - y^2 + z^2 = 1.$$

- (a) Identify the horizontal traces of the surface in the planes $z = k$. If the answer depends on the value of k , be sure to specify which values of k give which answer.
- (b) Identify the vertical traces of the surface in the planes $x = k$. If the answer depends on the value of k , be sure to specify which values of k give which answer.
- (c) Identify the vertical traces of the surface in the planes $y = k$. If the answer depends on the value of k , be sure to specify which values of k give which answer.
- (d) Identify the surface.
- (e) Sketch the surface. Your sketch does not need to be quantitatively correct, but it should show the correct type of surface in the correct location with the correct orientation. If you feel like you need to, feel free to write a sentence to clarify the location and orientation.

7. Consider the space curve defined by the equations

$$x = \cos 8t, \quad y = \sin 8t, \quad z = e^t.$$

Describe and sketch this curve. Make sure to specify the location, orientation, size, direction of travel as t increases, and any other salient features.



8. If $\mathbf{r}(t) = \mathbf{a} \cos \omega t + \mathbf{b} \sin \omega t$, where \mathbf{a} and \mathbf{b} are constant vectors, show that $\mathbf{r}(t) \times \mathbf{r}'(t) = \omega \mathbf{a} \times \mathbf{b}$.

9. Consider the graph of the function $y = e^x$.

(a) Compute the curvature of this curve. The book contains the formula

$$\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

which you are welcome to use or not use.

(b) What happens to the curvature as $x \rightarrow -\infty$?

(c) What happens to the curvature as $x \rightarrow \infty$?

(d) At what point does the curve have maximum curvature?

10. The magnitude of the acceleration vector \mathbf{a} is 10 cm/s^2 . Use the figure to estimate the tangential and normal components of \mathbf{a} .

