Name: _____ ID: _____

1. Let $\mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

   (a) Compute $\mathbf{a} \times \mathbf{b}$.

   Solution: We compute

   $$\mathbf{a} \times = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ 1 & 2 & 3 \end{vmatrix} = \left(1 - \tfrac{1}{2}\right)\mathbf{i} + \left(\tfrac{1}{4} - \tfrac{3}{2}\right)\mathbf{j} + \left(1 - \tfrac{1}{3}\right)\mathbf{k} = \tfrac{1}{2}\mathbf{i} - \tfrac{5}{4}\mathbf{j} + \tfrac{2}{3}\mathbf{k}.$$

   (b) Verify that your anwer is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

   Solution: Two vectors are orthogonal if their dot product is zero. We compute

   $$\mathbf{a} \cdot \left(\tfrac{1}{2}\mathbf{i} - \tfrac{5}{4}\mathbf{j} + \tfrac{2}{3}\mathbf{k}\right) = \frac{1}{4} - \frac{5}{12} + \frac{1}{6} = \frac{1}{12}(3 - 5 + 2) = 0,$$
   $$\mathbf{b} \cdot \left(\tfrac{1}{2}\mathbf{i} - \tfrac{5}{4}\mathbf{j} + \tfrac{2}{3}\mathbf{k}\right) = \frac{1}{2} - \frac{5}{2} + 2 = 0.$$

2. Consider the line that passes through the point $(5, 1, 3)$ and is parallel to the vector $\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$.

   (a) Write down a vector equation for the line.

   Solution: A vector equation for the line is

   $$\mathbf{r} = 5\mathbf{i} + \mathbf{j} + 3\mathbf{k} + t(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}),$$

   which can optinally be simplified to

   $$\mathbf{r} = (5 + t)\mathbf{i} + (1 + 4t)\mathbf{j} + (3 - 2t)\mathbf{k}.$$

   (b) Write down parametric equations for the line.

   Solution: The parametric equations of the line are

   $$x = 5 + t, \qquad y = 1 + 4t, \qquad z = 3 - 2t.$$

   (c) Find two other points on the line.

   Solution: To find other points on the line, we plug in other values of t. For example, with $t = 1$, we find that $(6, 5, 1)$ is on the line. With $t = -1$, we find that $(4, -3, 5)$ is on the line.