Name: ______________________________ ID: ______________

1. Write down inequalities that describe the solid upper hemisphere of the sphere of radius 2 centered at the origin.

   It may help to visualize the situation. Take an orange and cut it in half. Take half of the orange, and set it face down on a plate. Your task is to describe all of the points in this half of the orange.

Solution: As discussed in class, the solid ball of radius 2 can be described by the inequality $x^2 + y^2 + z^2 \leq 4$. Equivalently, these are all the points (x, y, z) whose distance from the origin is at most 2, so $\sqrt{x^2 + y^2 + z^2} \leq 2$.

However, the problem asks about the solid upper hemisphere, so we must add a further restriction that $z \geq 0$. Only the part of the orange above the plate counts. Our answer is thus

$$x^2 + y^2 + z^2 \leq 4, \quad z \geq 0.$$

An alternate approach is the following. If $x^2 + y^2 + z^2 \leq 4$, then, isolating z, we have $z \leq \sqrt{4 - x^2 - y^2}$. As before, $z \geq 0$, so we obtain an answer of

$$0 \leq z \leq \sqrt{4 - x^2 - y^2}.$$

2. Let $\mathbf{a} = \langle 4, 0, 3 \rangle$ and $\mathbf{b} = \langle -2, 1, 5 \rangle$.

(a) Compute $\mathbf{a} + \mathbf{b}$.

Solution: We compute

$$\mathbf{a} + \mathbf{b} = \langle 4 - 2, 0 + 1, 3 + 5 \rangle = \langle 2, 1, 8 \rangle.$$

(b) Compute $\mathbf{a} - \mathbf{b}$.

Solution: We compute

$$\mathbf{a} - \mathbf{b} = \langle 4 - (-2), 0 - 1, 3 - 5 \rangle = \langle 6, -1, -2 \rangle.$$

(c) Compute $3\mathbf{b}$.

Solution: We compute

$$3\mathbf{b} = \langle 3(-2), 3(1), 3(5) \rangle = \langle -6, 3, 15 \rangle.$$

(d) Compute $2\mathbf{a} + 5\mathbf{b}$.

Solution: Using the above techniques, we compute

$$\begin{aligned} 2\mathbf{a} &= \langle 8, 0, 6 \rangle, \\ 5\mathbf{b} &= \langle -10, 5, 25 \rangle \\ 2\mathbf{a} + 5\mathbf{b} &= \langle -2, 5, 31 \rangle. \end{aligned}$$