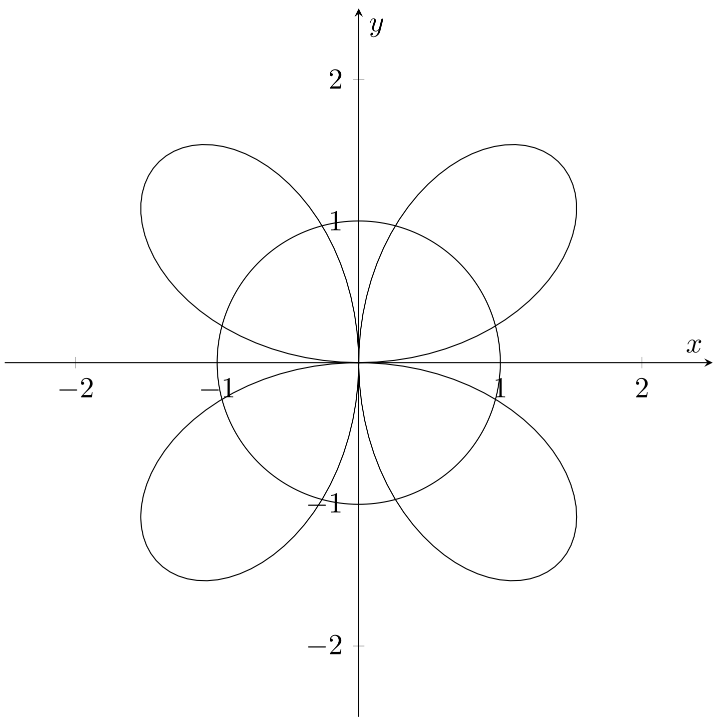
Name: ______________________________ ID: ______________

1. Find all points of intersection of the polar curves

$$r = 2\sin 2\theta, \qquad\qquad r = 1.$$



Solution: As in class, to find the points of intersection, we set $2\sin 2\theta = 1$, and simplify.

$$\begin{aligned} \sin 2\theta &= \frac{1}{2}, \\ 2\theta &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \ldots, \\ \theta &= \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \ldots. \end{aligned}$$

We obtain the intersection points

$$\left(1, \frac{\pi}{12}\right), \left(1, \frac{5\pi}{12}\right), \left(1, \frac{13\pi}{12}\right), \left(1, \frac{17\pi}{12}\right).$$

However, we clearly have eight intersection points, not four, so we note that if $2\sin 2\theta = -1$, we will also have a point on the circle of radius 1. Solving this equation, we find that

$$\begin{aligned} \sin 2\theta &= -\frac{1}{2}, \\ 2\theta &= \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19}{6}, \frac{23\pi}{6}, \ldots, \\ \theta &= \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19}{12}, \frac{23\pi}{12}, \ldots. \end{aligned}$$

Adding those points in, our final answer is

$$\left(1, \frac{\pi}{12}\right), \left(1, \frac{5\pi}{12}\right), \left(1, \frac{7\pi}{12}\right), \left(1, \frac{11\pi}{12}\right), \left(1, \frac{13\pi}{12}\right), \left(1, \frac{17\pi}{12}\right), \left(1, \frac{19\pi}{12}\right), \left(1, \frac{23\pi}{12}\right).$$

Alternatively, we could exploit the 90° rotational symmetry of the problem. After finding the points in the first quadrant, namely, $\left(1, \frac{\pi}{6}\right)$ and $\left(1, \frac{5\pi}{6}\right)$, we could add $\frac{\pi}{2}$ to the angles to obtain the two points in the second quadrant, add another $\frac{\pi}{2}$ to those to get the points in the third quadrant, and fianlly add another $\frac{\pi}{2}$ to get the two points in the fourth quadrant, giving us the same answer as above.

2. Consider the parabola with equation $y^2 + 10x = 0$.

   The textbook contains the equation $x^2 = 4py$, which you may use or not use.

   (a) Find the focus and directrix of the parabola.

Solution: The formula $x^2 = 4py$ doesn't match our equation very well, so we swap the roles of x and y, obtaining

$$y^2 = 4px.$$

To get the equation $y^2 + 10x = 0$ to this form, we subtract $10x$ from both sides, obtaining

$$y^2 = -10x.$$

Comparing the two equations, we see that $4p = -10$, so $p = -\frac{5}{2}$.
Thus, our focus is $\frac{5}{2}$ units to the left of the origin, at $(-\frac{5}{2}, 0)$, and our directrix is $\frac{5}{2}$ units to the right of the origin, at $x = \frac{5}{2}$.

   (b) Draw the parabola, focus, and directrix.

Solution: We note that $\frac{5}{2} = 2.5$, allowing us to correctly place our focus and directrix. Plugging in a point or two lets us realize that the parabola opens very slowly to the left. For example, if $y = 4$, then $x = -y^2/10 = -16/10 = -1.6$.